Class : 12

Register Number

SECOND REVISION EXAMINATION - 2025

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I **20 x 1 = 20**

1. **Answer all the questions by choosing the correct answer from the given 4 alternatives**
2. **Write question number, correct option and corresponding answer**
3. **Each question carries 1 mark**

1. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$.

(iv) $A(adjA) = (adjA)A = |A|I$

(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i), (ii) and (iv)

2. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

3. The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

4. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1) 0 (2) n (3) < n (4) r

5. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

6. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse is

(1) $\frac{\sqrt{2}}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

7. The circle passing through (1, −2) and touching the axis of x at (3, 0) passing through the point

(1) (−5, 2) (2) (2, −5) (3) (5, −2) (4) (−2, 5)

8. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(1) $\vec{a}$ (2) $\vec{b}$ (3) $\vec{c}$ (4) $\vec{0}$

9. A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(1) $\frac{3}{25}$ radians/sec (2) $\frac{4}{25}$ radians/sec (3) $\frac{1}{5}$ radians/sec (4) $\frac{1}{3}$ radians/sec

10. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdxm^3$ (2) $0.03xm^3$ (3) $0.03x^2m^3$ (4) $0.03x^3m^3$

11. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of f(1) is

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) $\frac{3}{4}$

12. The area between $y^2 = 4x$ and its latus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

13. The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1\cdot2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1\cdot2\cdot3}\left(\frac{dy}{dx}\right)^3 + \ldots$ is

(1) 2 (2) 3 (3) 1 (4) 4

14. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(1) $x\phi\left(\frac{y}{x}\right) = k$ (2) $\phi\left(\frac{y}{x}\right) = kx$ (3) $y\phi\left(\frac{y}{x}\right) = k$ (4) $\phi\left(\frac{y}{x}\right) = ky$

15. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

(1) $i+2n, i = 0,1,2\ldots n$ (2) $2i-n, i = 0,1,2\ldots n$ (3) $n-i, i = 0,1,2\ldots n$ (4) $2i+2n, i = 0,1,2\ldots n$

16. If $a * b = \sqrt{a^2 + b^2}$ on the real numbers then $*$ is

(1) commutative but not associative (2) associative but not commutative

(3) both commutative and associative (4) neither commutative nor associative

17. arg z lies in

(1) $-\pi \le \theta \le \pi$ (2) $0 \le \theta \le \pi$ (3) $0 \le \theta \le 2\pi$ (4) $-\pi < \theta \le \pi$

18. Sum of the squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$ is

(1) 10 (2) −10 (3) 5 (4) 12

19. Area of the region bounded by the curve $y = \cos x$, x – axis, $x = 0$ and $x = \pi$ is

(1) 0 (2) 2 (3) 1 (4) $\frac{1}{2}$

20. Determine the order and degree of the differential equation $3\left(\frac{d^2y}{dx^2}\right) = \left[4 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$

(1) 4 and 2 (2) 2 and 4 (3) 2 and 1 (4) 1 and 2

PART - II

1. **Answer any 7 questions** 2. **Each question carries 2 marks** 3. **Question number 30 is compuls** **7 x 2 = 14**

21. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$.

22. Write in polar form of the following complex numbers : $-2 - i2$

23. If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P., prove that $9pqr = 27r^2 + 2p$. Assume $p, q, r \neq 0$.

24. If $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

25. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals:

$$f(x) = \frac{x+1}{x}, x \in [-1, 2]$$

26. Show that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$

27. Determine the order and degree (if exists) of the following differential equations: $dy + (xy - \cos x)dx = 0$

28. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where $n = 9, p = \frac{1}{2}, k = 7$

29. If the probability mass function $f(x)$ of a random variable X is

X	1	2	3	4
f(x)	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$

30. Find the length of Latus rectum of the parabola $y^2 = 4ax$.

PART - III

1. Answer any 7 questions 2. Each question carries 3 marks 3. Question number 40 is compulsory **7 x 3 = 21**

31. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

32. Form the polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

33. Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

34. Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$.

35. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$, find the value of c.

36. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of $x - 1$.

37. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3+y^2}{x+y+2}\right)$. If the limit exists.

38. Solve: $\frac{dy}{dx}=\sqrt{4x+2y-1}$.

39. Construct the truth table for the statement $(\neg p\to r)\wedge(p\leftrightarrow q)$

40. Evaluate: $\int_{-2}^{2}|x+1|\,dx$

PART - IV

1. Answer all the questions 2. Each question carries 5 marks **7 x 5 = 35**

41. a) Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (OR)

b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

42. a) Solve $(x-4)(x-7)(x-2)(x+1)=16$ (OR)

b) A semi elliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

43. a) Solve the equation $z^3+8i=0$, where $z\in C$. (OR)

b) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

44. a) Find parametric form of vector equation , and Cartesian equations of the plane passing through the points (2,2,1) , (1,-2,3) and parallel to the straight line passing through the points (2,1,-3) and (-1,5,-8).(OR)

b) Evaluate the following limits, if necessary use l'Hôpital Rule: $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}$

45. a) For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. (OR)

b) Show that $\int_0^1(\tan^{-1}x+\tan^{-1}(1-x))dx=\frac{\pi}{2}-\log_e 2$.

46. a) Find the area of the region bounded by $y=\tan x$, $y=\cot x$ and the lines $x=0$, $x=\frac{\pi}{2}$, $y=0$. (OR)

b) Solve $(1+x^3)\frac{dy}{dx}+6x^2y=1+x^2$.

47. a) The probability density function of X is given by $f(x)=\begin{cases}ke^{-\frac{x}{3}} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$.

Find (i) the value of k (ii) the distribution function (iii) $P(X<3)$ (iv) $P(5\le X)$ (v) $P(X\le 4)$. (OR)

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A=\{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.