Second Revision Test - 2025

Standard XII

MATHEMATICS

Time: 3.00 hrs. Marks: 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

20x1=20

Note:i) Answer all the questions.

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2 - A =$

a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

2. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 17 b) 14 c) 19 d) 21

3. If $|z-2+i| \le 2$, then the greatest value of $|z|$ is

a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

4. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

a) -110° b) -70° c) 70° d) 110°

5. A polynomial equation in x of degree n always has

a) n distinct roots b) n real roots c) n complex roots d) at most one root

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

7. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

8. $\sin(\tan^{-1} x), |x| < 1$ is equal to

a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

9. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

a) $4(a^2+b^2)$ b) $2(a^2+b^2)$ c) a^2+b^2 d) $\frac{1}{2}(a^2+b^2)$

10. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are $(11,2)$, the coordinates of the other end are

a) $(-5,2)$ b) $(-3,2)$ c) $(5,-2)$ d) $(-2,5)$

11. If 1, 2, 3 are the direction ratios of a straight line, then its direction cosines are

a) $\frac{1}{14},\frac{2}{14},\frac{3}{14}$ b) $-\frac{1}{14},-\frac{2}{14},-\frac{3}{14}$ c) $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ d) $\frac{1}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-3}{\sqrt{14}}$

12. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

13. The value of $\lim_{x\to 0}(x^2-2x+5)$

a) -5 b) 0 c) 5 d) 0.5

14. The maximum value of the function x^2e^{-2x}, $x>0$ is

a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

15. If $w(x,y)=x^y$, $x>0$, then $\frac{\partial w}{\partial x}$ is equal to

a) $x^y\log x$ b) $y\log x$ c) yx^{y-1} d) $x\log y$

16. If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\,\partial y}$ is equal to

a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

17. The value of $\int_{-\pi/2}^{\pi/2}\sin^2 x\cos x\,dx$ is

a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{2}{3}$

18. The value of $\int_0^a(\sqrt{a^2-x^2})^3dx$ is

a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

19. If $f(x)=\begin{cases}2x & 0\le x\le a\\ 0 & \text{otherwise}\end{cases}$ is a probability density function of a random variable, then the value of a is

a) 1 b) 2 c) 3 d) 4

20. On a multiple-choice exam with 3 possible destructive for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

a) $\frac{11}{243}$ b) $\frac{3}{8}$ c) $\frac{1}{243}$ d) $\frac{5}{243}$

PART - II

Note: i) Answer any seven questions.
ii) Question no. 30 is compulsory. 7x2=14

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

22. Find the modulus of the following complex number. $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$

23. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

24. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.

25. Find the distance between the parallel planes $x+2y-2z+1=0$ and $2x+4y-4z+5=0$.

26. Find the slope of the tangent to the following curve at the given point.
$y=x^4+2x^2-x$ at $x=1$.

27. Let $g(x)=x^2+\sin x$. Calculate the differential dg.

28. Find the differential equation of all straight lines touching the circle $x^2+y^2=r^2$.

29. Compute $P(X=k)$ for the binomial distribution, $B(n,p)$ where $n=6, p=\frac{1}{3}, k=3$.

30. Evaluate : $\int_{-\pi/4}^{\pi/4}\sin^2 x\,dx$.

PART - III

Note: i) Answer any seven questions.
ii) Question no. 40 is compulsory 7x3=21

31. Verify the property $(A^T)^{-1}=(A^{-1})^T$ with $A=\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

32. Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33. If p and q are the roots of the equation $\ell x^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}}=0$.

34. Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}, -1<x<1$.

35. Find the equation of the tangent to the parabola $y^2=16x$ perpendicular to $2x+2y+3=0$.

36. A particle is acted upon by the forces $3\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

37. Evaluate : $\int_0^2 \frac{dx}{4\sin^2 x+5\cos^2 x}$

38. Solve : $(1+x^2)\frac{dy}{dx}=1+y^2$

39. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

40. Prove that In an algebraic structure the identity element (if exists) must be unique.

PART - IV

Note: Answer all the questions. 7x5=35

41. a) Solve the following system of linear equations by Cramer's rule.
$3x+3y-z=11,\ 2x-y+2z=9,\ 4x+3y+2z=25$

(OR)

b) Evaluate the following limit, if necessary use ℓ' Hôpital Rule. $\lim_{x\to\pi/2}(\sin x)^{\tan x}$.

42. a) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$.

(OR)

b) Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

43. a) Solve: $6x^4-35x^3+62x^2-35x+6=0$

(OR)

b) Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$ about the major axis.

44. a) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}(-\sqrt{2})$.

(OR)

b) Solve the following differential equation: $(x^3+y^3)dy-x^2y\,dx=0$.

45. a) Find the centre, foci and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$.

(OR)

b) A radioactive isotope has an initial mass 200 mg, after two years it is decreased by 50 mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotopes to fall to half its original value)

46. a) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

(OR)

b) A random variables X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2<X<6)$ (ii) $P(2\le X<5)$ (iii) $P(X\le 4)$ (iv) $P(3<X)$.

47. a) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,-2), (-1,-2,6)$ and $(6,4,-2)$.

(OR)

b) Using the equivalence property, show that $p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$.

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