



Standard 12
MATHEMATICS
Part - I

Time: 3.00 Hours

Marks: 90

Answer all the questions

20×1=20

- 1) If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$
- a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$
- 2) In the system of equations with 3 unknowns, if $\Delta = 0$, and one of $\Delta x, \Delta y, \Delta z$ is non zero then the system is
- a) consistent
b) inconsistent
c) consistent with one parameter family of solutions
d) consistent with two parameter family of solutions
- 3) The conjugate of a complex number is $\frac{1}{i-2}$ then the complex number is
- a) $\frac{1}{i+2}$ b) $-\frac{1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$
- 4) If $x^2 + y^2 = 1$, then the value of $\frac{1+x+iy}{1+x-iy}$ is
- a) $x-iy$ b) $2x$ c) $-2iy$ d) $x+iy$
- 5) If α, β, γ are the roots of the equation $x^3 - 3x + 11 = 0$, then $\alpha + \beta + \gamma$ is
- a) 0 b) 3 c) -11 d) -3
- 6) If $\sin^{-1} x + \cot^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{2}$, then x is equal to
- a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$
- 7) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0, 3) is
- a) $x^2 + y^2 - 6y - 7 = 0$ b) $x^2 + y^2 - 6y + 7 = 0$
c) $x^2 + y^2 - 6y - 5 = 0$ d) $x^2 + y^2 - 6y + 5 = 0$
- 8) $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$, then
- a) $c = \frac{a}{m}$ b) $c = \frac{m}{a}$ c) $c^2 = a^2 m^2 + b^2$ d) $m = c$
- 9) If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then
- a) $c = \pm 3$ b) $c = \pm \sqrt{3}$ c) $c > 0$ d) $0 < c < 1$
- 10) The straight lines $\frac{x-3}{2} = \frac{y+5}{4} = \frac{z-1}{-13}$ and $\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+2}{2}$ are
- a) parallel b) perpendicular c) inclined at 45° d) none
- 11) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
- a) $y = 0$ b) $y = \pm \sqrt{3}$ c) $y = \frac{1}{2}$ d) $y = \pm 3$
- 12) The point of inflection of the curve $y = (x-1)^3$ is
- a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)

- 13) Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is
 a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$
- 14) The value of $\int_{-1}^2 |x| dx$ is
 a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$
- 15) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is
 a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$
- 16) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
 a) $y + \sin^{-1}x = C$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = C$ d) $x^2 + 2\sin^{-1}y = 0$
- 17) The order of the differential equation of all circles with centre at (h, k) and radius a is
 a) 2 b) 3 c) 4 d) 1
- 18) If $f(x) = \begin{cases} 2x, & 0 \leq x \leq a \\ 0, & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is
 a) 1 b) 2 c) 3 d) 4
- 19) $\text{var}(2x+5)$ is equal to
 a) 5 b) $\text{var}(2x)+5$ c) $4\text{var}(x)$ d) 0
- 20) The operation $*$ defined by $a*b = \frac{ab}{7}$ is not a binary operation on
 a) Q^+ b) Z c) R d) C

Part - II

Answer any 7 questions. Q.No. 30 is compulsory

7×2=14

21) If $\text{adj } A = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}

22) Simplify $\sum_{n=1}^{102} i^n$

23) Find the value of $\tan^{-1}\left(\tan \frac{5\pi}{4}\right)$

24) Find the general equation of the circle whose diameter is the line segment joining the points $(-4, -2)$ and $(1, 1)$

25) Find the intercepts cutoff by the plane $\vec{r} \cdot (6\hat{i} + 4\hat{j} + 3\hat{k}) = 12$ on the coordinate axes

26) A particle is fired straight up from the ground to reach a height of x feet in t seconds, where $s = 128t - 16t^2$, compute the maximum height of the particle reached?

27) Evaluate: $\int_0^5 x^5 e^{-3x} dx$

28) Solve: $\frac{dy}{dx} + 2y = e^{-x}$

29) If x is the random variable with distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

then find the probability density function $f(x)$.

30) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ then find $A \wedge B$

Part - III

Answer any 7 questions. Q.No. 40 is compulsory

7×3=21

- 31) Solve the system of linear equations by matrix inversion method
 $2x - y = 8$, $3x + 2y = -2$
- 32) Solve: $\sin^2 x - 5 \sin x + 4 = 0$
- 33) A concrete bridge is designed as a parabolic arch. The road over bridge is 40m long and the maximum height of the arch is 15m. Write an equation of the parabolic arch
- 34) A particle acted on by constant forces $8\hat{i} + 2\hat{j} - 6\hat{k}$ and $6\hat{i} + 2\hat{j} - 2\hat{k}$ is displaced from the point (1, 2, 3) to the point (5, 4, 1). Find the total work done by the forces
- 35) Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval (2, 7) and strictly decreasing in the interval (-2, 0).
- 36) Show that $f(x, y) = \frac{x^2 - y^2}{y^2 + 1}$ is continuous at every $(x, y) \in \mathbb{R}^2$
- 37) Evaluate: $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$
- 38) Solve: $\frac{dy}{dx} = (3x + y + 4)^2$
- 39) Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$
- 40) If $\sin\left(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1} x\right) = 1$, then find the value of x

Part - IV

Answer all the questions.

7×5=35

- 41) a) Solve the following systems of linear equations by Cramer's rule:
 $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$
 (OR)
 b) Solve: $(x^2 - 3y^2)dx + 2xy dy = 0$
- 42) a) Suppose Z_1 , Z_2 and Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$... If $Z_1 = 1 + i\sqrt{3}$ then find Z_2 and Z_3

(OR)

- b) Find the area of the region bounded by the parabola $y^2=x$ and the line $y=x-2$
- 43) a) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, If it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros

(OR)

- b) The probability density function of x is given by $f(x) = \begin{cases} k, & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$

find i) distribution function (ii) $P(x < 3)$ (iii) $P(2 < x < 4)$ (iv) $P(3 \leq X)$

44) a) Solve: $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$

(OR)

- b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.
- 45) a) For the ellipse, $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices and the foci. Also prove that the length of latus rectum is 2

(OR)

- b) If $w(x, y, z) = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

- 46) a) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$, $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

(OR)

- b) Find the angle of intersection of the curves $2y^2 = x^3$ and $y^2 = 32x$
- 47) a) Find the volume of a right circular cone of base radius r and height h .

(OR)

- b) Using the truth table, prove that $p \leftrightarrow q = (P \rightarrow q) \wedge (q \rightarrow p)$
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