PRE MODEL EXAM – MARCH 2025

Standard-12

PART-III-MATHEMATICS

PATTUKKOTTAI-PALANIAPPAN-MATHS

Reg No | | | | | | |

Time Allowed:2.30 Hours **Maximum Marks: 90**

Instructions: 1. Check the question paper for fairness of printing. If there is any Lack of fairness,inform the Hall Supervisor Immediately
2.Use Blue or Black Ink to write and underline and pencil to draw Diagrams.

PART-I

Note: **I) Answer all the questions.** 20x1=20
II) Choose the most appropriate answer from the given four alternatives And write the option code and the corresponding answer.

1. If A, B and C are invertible matrices of some order, then which one of the following is not true?
(1) $\text{adj}\, A = |A|\, A^{-1}$ (2) $\text{adj}(AB) = (\text{adj}\, A)(\text{adj}\, B)$ (3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2. If a, b, c are positive real numbers then the following system of equations in x, y and z,
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ \frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has
(1) infinitely many solutions (2) finitely many solutions (3) no solution (4) unique solution

3. The principal argument of $\frac{3}{-1+i}$ is
(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

4. If $x+iy=\left(-1+i\sqrt{3}\right)^{2019}$, then x is
(1) 2^{2019} (2) -2^{2019} (3) -1 (4) 1

5. If a, b, c are the roots of $x^3-px^2+qx-r=0$, find the value of $(a+b-c)(b+c-a)(c+a-b)$:
(1) p^3-8r (2) $4pq-p^3$ (3) $4pq-p^3-8r$ (4) $4pq-8r$

6. If $\cot^{-1}\left(\sqrt{\sin\alpha}\right)+\tan^{-1}\left(\sqrt{\sin\alpha}\right)=u$, then $\cos 2u$ is equal to
(1) $\tan^2\alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

7. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x=\frac{-9}{2}$ is
(1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

8. The radius of the auxiliary circle of the conic $9x^2+16y^2=144$ is
(1) $\sqrt{7}$ (2) 4 (3) 3 (4) 5

9. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}]=\frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is
(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then
(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

11. The function $\sin^4 x+\cos^4 x$ is increasing in the interval
(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

12. The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

13. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdx\,m^3$ (2) $0.03x\,m^3$ (3) $0.03x^2m^3$ (4) $0.03x^3m^3$

14. The value of $\int_0^{\pi} \sin^4 x\,dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

15. The value of $\int_{-1}^{2} |x|\,dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

16. If the solution of the differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(1) 2 (2) -2 (3) 1 (4) -1

17. The solution of $\frac{dy}{dx}=2^{y-x}$ is

(1) $2^x+2^y=C$ (2) $2^x-2^y=C$ (3) $\frac{1}{2^x}-\frac{1}{2^y}=C$ (4) $x+y=C$

18. A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviation of X is (1) 6 (2) 4 (3) 3 (4) 2

19 If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

20. Which one is the inverse of the statement $(p\vee q)\rightarrow(p\wedge q)$?

(1) $(p\wedge q)\rightarrow(p\vee q)$ (2) $\neg(p\vee q)\rightarrow(p\wedge q)$

(3) $(\neg p\vee\neg q)\rightarrow(\neg p\wedge\neg q)$ (4) $(\neg p\wedge\neg q)\rightarrow(\neg p\vee\neg q)$

PART-II

Answer any seven questions. Question no. 30 is compulsory 7x2=14

21. Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

22. Find $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

23. If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

24. For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$.

25. Evaluate the limit $\lim_{x\to0}\left(\frac{\sin x}{x^2}\right)$.

26. If $v(x,y)=x^2-xy+\frac{1}{4}y^2+7,\ x,y\in R$, find the differential dv.

27. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.

28. Solve $\sin\frac{dy}{dx}=a,\ y(0)=1$

29. Write down the (i) inverse statement, and (ii) contrapositive statement for the two statements p and q given below. p : The number of primes is infinite. q: Ooty is in Kerala.

30. If A and B are any two non-singular square matrices of order n, then $\text{adj}(AB)=(\text{adj } B)(\text{adj } A)$.

PART-III

Answer any seven questions. Question no. 40 is compulsory 7x3=21

31. If $A=\begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}$, verify that $A(\text{adj } A)=(\text{adj } A)A=|A|I_2$.

32. If $|z|=3$, show that $7\le|z+6-8i|\le13$.

33. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$.

34. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

35. Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane.

36. Suppose $f(x)$ is a differentiable function for all x with $f'(x)\le29$ and $f(2)=17$. What is the maximum value of $f(7)$?

37. Find the differential equation of the curve represented by $xy=ae^x+be^{-x}+x^2$.

38. If $X\sim B(n,p)$ such that $4P(X=4)=P(X=2)$ and $n=6$. Find the distribution, mean and standard deviation of X.

39. On $\mathbb{Z}$, define $*$ by $(m*n)=m^n+n^m:\forall m,n\in\mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?

40. If $\int_0^\infty e^{-\alpha x^2}x^3dx=32,\ \alpha>0$, find α

PART-1V

Answer all the questions: 7x5=35

41.a) Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

42. a) If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$.

(OR)

b) Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

43. a) Assume that water issuing from the end of a horizontal pipe, $7.5m$ above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\,m$ below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

b) A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?

(i) At what time the particle is at rest?

(ii) At what time the particle changes its direction?

(iii) Find the total distance travelled by the particle in the first 2 seconds.

44. a) Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$,

(OR)

b) If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x-1, & 1 \leq x < 2 \\ -x+3, & 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5 \leq X \leq 2.5)$

45. a) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

(OR)

b) Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

46. a) Determine the intervals of concavity of the curve $f(x) = (x-1)^3 \cdot (x-5), x \in \mathbb{R}$ and, points of inflection if any.

(OR)

b) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

47. a) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

(OR)

b) Show that $\int_0^1 \left(\tan^{-1}x + \tan^{-1}(1-x)\right) dx = \frac{\pi}{2} - \log_e 2$.