Class : 12

Register Number

FIRST REVISION EXAMINATION, JANUARY - 2025

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

20 x 1 = 20

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If A is a 3× 3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$

(1) A (2) B (3) I_3 (4) B^T

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

3. The area of the triangle formed by the complex numbers z, iz, and z + iz in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

4. The principal argument of $(\sin 40° + i \cos 40°)^5$ is

(1) −110° (2) −70° (3) 70° (4) 110°

5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

6. The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is

(1) $\pi - x$ (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) $x - \pi$

7. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

8. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

(1) $[-1, 1]$ (2) $[\sqrt{2}, 2]$ (3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}]$

9. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $15 < m < 65$ (2) $35 < m < 85$ (3) $-85 < m < -35$ (4) $-35 < m < 15$

10. Let C be the circle with centre at (1,1) and radius = 1. If T is the circle centered at (0, y) passing through the origin and touching the circle C externally, then the radius of T is equal to

(1) $\frac{\sqrt{3}}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

11. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

12. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

13. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq 0$ and $\vec{a}.\vec{b}\neq 0$, then $\vec{a}$ and $\vec{c}$ are

(1) perpendicular (2) parallel

(3) inclined at an angle $\frac{\pi}{3}$ (4) inclined at an angle $\frac{\pi}{6}$

14. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$ then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

15. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1) 0 (2) n (3) < n (4) r

16. If $z+\frac{1}{z}+1=0$, then $z^{2003}+\frac{1}{z^{2003}}$ is equal to-

(1) 1 (2) -1 (3) 0 (4) None of these

17. If the system of equations $x+2y-3z=1$, $(k+3)z=3$, $(2k+1)x+z=0$ is inconsistent, then the value of k is

(1) -3 (2) $\frac{1}{2}$ (3) 0 (4) 2

18. Latus rectum of ellipse $4x^2+9y^2-8x-36y+4=0$ is

(1) 8/3 (2) 4/3 (3) $\frac{\sqrt{5}}{3}$ (4) 16/3

19. If $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}x$, then x is equal to

(1) 0 (2) $\frac{\sqrt{5}-4\sqrt{2}}{9}$ (3) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (4) $\frac{\pi}{2}$

20. The value of $z=(1+i)^8+(1-i)^8$ is.

(1) 16 (2) - 16 (3) 32 (4) - 32

PART - II

1. Answer any 7 questions **7 x 2 = 14**
2. Each question carries 2 marks
3. Question number 30 is compulsory

21. Verify the property $(A^T)^{-1}=(A^{-1})^T$ with $A=\begin{bmatrix}2 & 9\\ 1 & 7\end{bmatrix}$.

22. Construct a cubic equation with roots 2, $\frac{1}{2}$ and 1.

23. Examine the position of the point (2,3) with respect to the circle $x^2+y^2-6x-8y+12=0$.

24. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If $c_1 = 1$ and $c_2 = 2$, find c_3 such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

25. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

26. Simplify : $i^{59} + \frac{1}{i^{59}}$

27. If k is real ,discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$ in terms of k.

28. Identify the type of conic section for $3x^2 + 3y^2 - 4x + 3y + 10 = 0$

29. Find the magnitude and the direction cosines of the torque about the point (2,0,−1) of a force whose line $2\hat{i} + \hat{j} - \hat{}$ action passes through the origin.

30. If $\vec{x}.\vec{a} = 0$, $\vec{x}.\vec{b} = 0$, $\vec{x}.\vec{c} = 0$, $\vec{x} \neq 0$. Prove that $\vec{a}$, $\vec{b}$, $\vec{c}$ are Coplanar .

PART – III

7 x 3 = 21

1. **Answer any 7 questions**
2. **Each question carries 3 marks**
3. **Question number 40 is compulsory**

31. Find the rank of the matrix $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

32. Find the square root of 6−8i .

33. Find the domain of $g(x) = 2\sin^{-1}(2x - 1) - \frac{\pi}{4}$.

34. Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 6)$.

35. Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z + 11 = 3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$.

36. Find the condition on a , b ,and c so that the following system of linear equations has one parameter family of soluti $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

37. If z= x+ iy , find the rectangular form of $Im(3z + 4\bar{z} - 4i)$

38. Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

39. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

40. Find the value of $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$

PART - IV

7 x 5 = 35

1. **Answer all the questions**
2. **Each question carries 5 marks**

41. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points (−6,8), (−2,−12) , and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian elimination method.)

(OR)

b) If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

42. a) If $\frac{1+z}{1-z}=\cos 2\theta+i\sin 2\theta$, show that $z=i\tan\theta$.

(OR)

b) Let z_1, z_2, and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$.

Prove that $\left|\frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_1+Z_2+Z_3}\right|=r$

43. a) Solve the following equations : $x^4-10x^3+26x^2-10x+1=0$.

(OR)

b) Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

44. a) Find the equation of the circle passing through the points (1,1), (2,−1) and (3,2).

(OR)

b) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

45. a) Find the value of $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$

(OR)

b) Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}, -1<x<1$

46. a) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$,

(OR)

b) Find the parametric form of vector equation of the straight line passing through (-1,2,1) and parallel to the straight line $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+t(\hat{i}-2\hat{j}+\hat{k})$ and hence find the shortest distance between the lines.

47. a) Prove that the line $5x+12y=9$ is a tangent to hyperbola $x^2-9y^2=9$. Also find the point of contact.

(OR)

b) If $a=\cos 2\alpha+i\sin 2\alpha, b=\cos 2\beta+i\sin 2\beta, c=\cos 2\gamma+i\sin 2\gamma$, prove $\sqrt{abc}+\frac{1}{\sqrt{abc}}=2\cos(\alpha+\beta+\gamma)$ and $\frac{a^2b^2+c^2}{abc}=2\cos 2(\alpha+\beta-\gamma)$.