First Revision Exam - Jan. 2025
Maths

Part - I

1) a) -4
2) c) $a<0$
3) $|\alpha| \leq \frac{1}{\sqrt{2}}$
4) d) 1
5) b) Only (ii)
6) b) -1
7) a) $cis\left(-\frac{2\pi}{5}\right)$
8) c) 0
9) a) $\frac{\pi}{6}$
10) b) $c = \pm\sqrt{3}$
11) c) 3
12) a) $f(a) = f(b)$
13) b) $2xu$
14) d) 9
15) c) 1, 1
16) a) $\frac{1}{x+1}$
17) d) $\frac{1}{3}$
18) b) Multiplication
19) a) $S \to R$
20) d) $\sqrt{5}$ is an irrational number.

Part - II

21) $\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{2i}{2} = i$

$\left(\frac{1-i}{1+i}\right) = -i$, $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1-i}\right)^3 = i^3 - (-i)^3$

$= i - i$

$= \boxed{-2i}$

22) $|adj A| = 9$; $A^{-1} = \pm\frac{1}{\sqrt{|adj A|}} adj(A)$

$A = \pm\frac{1}{\sqrt{9}}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \Rightarrow A^{-1} = \pm\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

23) $\vec{a}\cdot(\vec{b}\times\vec{c}) \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 3 & 2 & 1 \end{vmatrix} = 24$

24) $f(x)$ is not Continuous at $x=0$ in $[-1,1]$ so, Rolle's theorem is not applicable to the functions

25) $n \Rightarrow$ (odd) 7 ; $I = \frac{n-1}{n}\times\frac{n-3}{n-2}\times\frac{n-5}{n-4}\times\cdots\times\frac{2}{3}$

$I_7 = \frac{6}{7}\times\frac{4}{5}\times\frac{2}{3} \Rightarrow \frac{16}{35}$

26) $\frac{\partial u}{\partial x} = \frac{3x^2}{x^3+y^3+z^3}$, $\frac{\partial u}{\partial y} = \frac{3y^2}{x^3+y^3+z^3}$

$\frac{\partial u}{\partial z} = \frac{3z^2}{x^3+y^3+z^3}$,

$\therefore \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2)}{x^3+y^3+z^3}$

27) $y^2 = 4ax$, 'a' is an arbitrary Constant

Diff. w.r.to x, $2y\frac{dy}{dx} = 4a$

$a = \frac{y}{2}\frac{dy}{dx}$

Substituting the Value of a in $y^2 = 4ax$

$y^2 = \cancel{4}^{2}\left(\frac{y}{\cancel{2}}\right)\left(\frac{dy}{dx}\right)x$

$\therefore$ we get $\frac{dy}{dx} = \frac{y}{2x}$ as the required differential equation.

28) Since $2-\sqrt{3}$ is a root, $2+\sqrt{3}$ is also a root.

$x^2 - (S.R)x + P.R = 0$. $S.R = 4$, $P.R = 1$

$\therefore x^2 - 4x + 1 = 0$

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29) Let $a+\sqrt{5}b,\ c+\sqrt{5}d \in A$,
Using usual multiplication we have,
$(a+\sqrt{5}b)(c+\sqrt{5}d) = ac+5bd+\sqrt{5}(bc+ad) \in A$
Hence usual multiplication is binary on A

30) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
$(-4,-2)$ & $(-1,-1)$
$x_1\ y_1 \quad x_2\ y_2$
$(x+4)(x+2)+(y+1)(y+1)=0$
$x^2+4x+2x+8+y^2+y+y+1=0$
$x^2+y^2+6x+2y+9=0$

Part - II

31) $\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15+2 = 17$
$\Delta_x = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48+14 \Rightarrow -34$
$\Delta_y = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35+16 = 51$
$x = \frac{\Delta x}{\Delta} = -\frac{34}{17} = -2$, $y = \frac{\Delta y}{\Delta} = \frac{51}{17} = 3$
$\boxed{x=-2}$, $\boxed{y=3}$

32) $z = (\cos\theta + i\sin\theta)$, $z^n = \cos n\theta + i\sin n\theta$
$\frac{1}{z^n} = \cos n\theta - i\sin n\theta$
$z^n + \frac{1}{z^n} = \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta}$
$\boxed{z^n + \frac{1}{z^n} = 2\cos n\theta}$ &
$z^n - \frac{1}{z^n} = \cancel{\cos n\theta} + i\sin n\theta - \cancel{\cos n\theta} + i\sin n\theta$
$\boxed{z^n - \frac{1}{z^n} = 2i\sin n\theta}$

34) $\hat{i}\times(\vec{a}\times\hat{i}) = (\hat{i}\cdot\hat{i})\vec{a} - (\hat{i}\cdot\vec{a})\hat{i} = \vec{a} - (\vec{a}\cdot\hat{i})\hat{i}$
$\hat{j}\times(\vec{a}\times\hat{j}) = (\hat{j}\cdot\hat{j})\vec{a} - (\hat{j}\cdot\vec{a})\hat{j} = \vec{a} - (\vec{a}\cdot\hat{j})\hat{j}$
$\hat{k}\times(\vec{a}\times\hat{k}) = (\hat{k}\cdot\hat{k})\vec{a} - (\hat{k}\cdot\vec{a})\hat{k} = \vec{a} - (\vec{a}\cdot\hat{k})\hat{k}$
$\therefore (\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}))$
$= \vec{a} - a_1\hat{i} + \vec{a} - a_2\hat{j} + \vec{a} - a_3\hat{j}$
$= 3\vec{a} - (a_1\hat{i} + a_2\hat{j} + a_3\hat{j})$
$= 3\vec{a} - \vec{a}$ $\quad [\because \text{Let } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
$= 2\vec{a}$

33) $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$
$= \sin^{-1}\left(\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{6\pi}{9}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{3\pi-\pi}{3}\right)\right) \Rightarrow \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$
$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$
$= \frac{\pi}{3} \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

35) $2x + 8y\frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = -\frac{x}{4y}$ at (a,b)
$\therefore m_1 = -\frac{a}{4b}$
$2x - 4y\frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = \frac{x}{2y}$ at (a,b)
$\therefore m_2 = \frac{a}{2b}$ $\quad \therefore$ To prove that $m_1 \times m_2 = -1$
$m_1 \times m_2 = \left(-\frac{a}{4b}\right)\times\left(\frac{a}{2b}\right) = \frac{-a^2}{8b^2}$
applying the ratio of proportions $\frac{a^2}{-32} = \frac{b^2}{-4}$
$\frac{a^2}{b^2} = \frac{32}{4} = 8$. $\therefore m_1 \times m_2 = -1$
substituting $m_1 \times m_2 \Rightarrow -\frac{1}{8}\times 8 = -1$
Hence the curves cut Orthogonally.

36) $L(x) = f(x_0) + f'(x_0)(x - x_0)$
$x_0 = 3$, $\Delta x = 0.2$
$f'(x) = \frac{1}{2\sqrt{1+x}}$ and hence $f'(3) = \frac{1}{4}$
$L(x) = 2 + \frac{1}{4}(x-3) = \frac{x}{4} + \frac{5}{4}$
$\sqrt{4.2} = \frac{3.2}{4} + \frac{5}{4} = 2.050$.

37) The given differential Equation is $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$
$\int\frac{dy}{\sqrt{1-y^2}} = \int\frac{dx}{\sqrt{1-x^2}}$
$\sin^{-1}y = \sin^{-1}x + c$

38) Mean $= \int_1^2 x f(x)dx = \int_1^2 2x(x-1)dx = \frac{5}{3}$

$E(x^2) = \int_1^2 x^2 f(x)dx = \int_1^2 2x^2(x-1)dx = \frac{17}{6}$

$Var(X) = E(x^2) - [E(x)]^2 = \frac{17}{6} - \left(\frac{5}{3}\right)^2$

$= \frac{17}{6} - \frac{25}{9} = \frac{1}{18}$

39) $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ (by Commutative)

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$ (by Distributive)

$\Rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$ (by Distributive)

$\Rightarrow F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$ (by complement Law)

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$; (by Identity Law)

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$; (by commutative Law)

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

40) Let $I = \int_0^{\pi/2} \frac{f(\cos x)}{f(\sin x) + f(\cos x)} dx$ ——— ①

$$\boxed{\int_0^a f(x)dx = \int_0^a f(a-x)dx}$$

① ⇒ $I = \int_0^{\pi/2} \frac{f(\cos(\pi/2 - x))}{f(\sin(\pi/2 - x)) + f(\cos(\pi/2 - x))} dx$

$I = \int_0^{\pi/2} \frac{f(\sin x)}{f(\cos x) + f(\sin x)} dx$ ——— ②

① + ② ⇒ $2I = \int_0^{\pi/2} \frac{f(\cos x) + f(\sin x)}{f(\cos x) + f(\sin x)} dx$

$2I = \int_0^{\pi/2} dx \Rightarrow 2I = [x]_0^{\pi/2}$

$2I = \left[\frac{\pi}{2} - 0\right]$

$$\boxed{I = \frac{\pi}{4}}$$

Hence Proved.

Part - IV

41) Let $x, y \in Q - \{1\}$

Since x and y are rational, $x + y - xy$ is also a rational. To prove binary, it is to prove $x + y - xy \neq 1$

If not, assume $x + y - xy = 1$

$\Rightarrow x - 1 + y - xy = 0$

$\Rightarrow (x-1)(1-y) = 0$

$\Rightarrow$ either $x = 1$ or $y = 1$

Which is a contradiction. ∴ $*$ binary on A

ii) $x * y = x + y - xy = y + x - yx = y * x$

$*$ is Commutative.

iii) $(x * y) * z = x * (y * z)$

$x + y + z - xy - yz - zx + xyz = x + y + z - xy - yz - zx + xyz$

$*$ is associative

iv) $x * e = x$

$x + e - xe = x$

$e - xe = 0$

$e(1-x) = 0$

$e = 0$

is identity

$x * x^{-1} = 0$

$x + x^{-1} - xx^{-1} = 0$

$x^{-1}(1-x) = -x$

$x^{-1} = \frac{x}{x-1} \in Q - \{1\}$

The inverse axiom is also satisfied.

b) $\frac{1}{3}$ is a solution for the reciprocal Equation of even degree. Hence 3 also is a solution

$(3x-1)(x-3) = 3x^2 - 10x + 3$ is a factor of $6x^4 - 5x^3 - 38x^2 - 5x + 6$.

∴ $6x^4 - 5x^3 - 38x^2 - 5x + 6 = (3x^2 - 10x + 3)(2x^2 + px + 2)$

Equate x term

$-5 = -20 + 3p$

$\boxed{p = 5}$

∴ The other factor is $2x^2 + 5x + 2 = 0$

$(2x+1)(x+2) = 0$

$x = -\frac{1}{2}, -2$

Hence the roots are $\frac{1}{3}, 3, -\frac{1}{2}, -2$

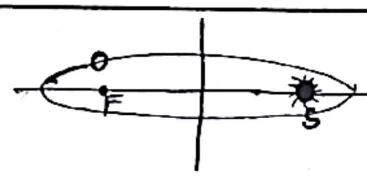
42) a) $\Sigma f(x) = 1 \Rightarrow 30k = 1 \Rightarrow k = \frac{1}{30}$

i) $P(2 < x < 6) = \frac{17}{30}$ ii) $P(2 \leq X < 5) = \frac{13}{30}$

iii) $P(X \leq 4) = \frac{14}{30}$, iv) $P(3 < X) = \frac{21}{30}$

b) $AS = 94.5 \times 10^6$ Km,

$SA' = 152 \times 10^6$ Km



$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

Substracting $2c = 57.5 \times 10^6$

$= 575 \times 10^5$ Km

Distance of the Sun from the other focus is $SS' = 575 \times 10^5$ Km.

43) (a) $\overline{AD} \perp \overline{BC}$

$\overrightarrow{OA} \perp \overrightarrow{BC}$

$\overrightarrow{OA} \cdot (\overrightarrow{OC} - \overrightarrow{OB}) = 0$

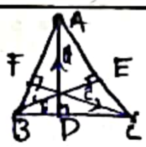
$\vec{a} \cdot (\vec{c} - \vec{b}) = 0$ — (1)

$\overline{BE} \perp \overline{CA}$

$\overrightarrow{OB} \perp \overrightarrow{CA}$

$\overrightarrow{OB} \cdot (\overrightarrow{OA} - \overrightarrow{OC}) = 0$

$\vec{b} \cdot (\vec{a} - \vec{c}) = 0$



$\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} = 0$ — (1) $\vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{c} = 0$ — (2)

(1)+(2) $\Rightarrow$ $\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{c} = 0$

$\vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} = 0$

$\vec{c} \cdot (\vec{a} - \vec{b}) = 0$

$\overrightarrow{OC} \cdot (\overrightarrow{OA} - \overrightarrow{OB}) = 0$

$\overrightarrow{CF} \perp BA$.

The perpendicular (altitudes) from the Vertices to the opposite sides of a triangle are concurrent.

b) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) \Rightarrow \tan^{-1}\left[\dfrac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2}\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\dfrac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2}\left(\frac{x+1}{x+2}\right)} = 1$, which on simplification gives $2x^2 - 4 = -3$

$x^2 = \frac{1}{2}$ gives $x = \pm\frac{1}{\sqrt{2}}$

44) a) $[A/B] = \left[\begin{array}{ccc|c} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{array}\right] R_1 \leftrightarrow R_2$

$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right]$ $R_3 \to R_3 - R_2$ $\therefore \rho(A) = 2$, $\rho([A,B]) = 3$

$\therefore$ The system is inconsistent, No solution.

b) $\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t\vec{c}$

$\vec{r} = (-\hat{i} + 2\hat{j}) + s(3\hat{i} - \hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$

Non parametric from $(\vec{r} - \vec{a}) \cdot ((\vec{b} - \vec{a}) \times \vec{c}) = 0$

$(\vec{b} - \vec{a}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 3$

cartesian equation $x + 2y + 3z = 3$

45) a) $z^3 = -27$

$z = 3(-1)^{1/3}$

$z = 3(\cos\pi + i\sin\pi)^{\frac{1}{3}}$

$z = 3[\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi)]^{1/3}$, $k \in z$

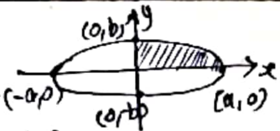
$z = 3\left[\cos\left(\frac{2k\pi + \pi}{3}\right) + i\sin\left(\frac{2k\pi + \pi}{3}\right)\right]$, $k = 0, 1, 2$

$k = 0$; $z = (\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})(3)$

$k = 1$; $z = (\cos\pi + i\sin\pi)(3)$

$k = 2$; $z = (\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})(3)$

b) $y = \frac{b}{a}\sqrt{a^2 - x^2}$



$A = 4\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$

$A = \frac{4b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$

$A = \frac{4b}{a} \times \frac{\pi a^2}{4}$

$A = \pi ab$

46) a) Let P be the principal at any time 't'

$\frac{dP}{dt} \propto P \Rightarrow P = ce^{kt}$; $k = 0.05$

$\Rightarrow P = ce^{0.05t}$

when $t = 0$, $P = 10000$

$\therefore c = 10000$

$P = 10000\,e^{0.05t}$

when $t = 18$ months i.e., $t = \frac{3}{2}$

$\boxed{P = 10000\,e^{0.075}}$

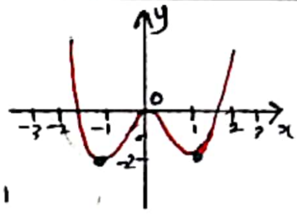
b) $f'(x) = 24x^5 - 24x^3$

$f'(x) = 24x^3(x^2 - 1)$

$f'(x) = 24x^3(x+1)(x-1)$

$f'(x) = 0 \Rightarrow x = -1, 0, 1$

Critical number $x = -1, 0, 1$



$f''(x) = 120x^4 - 72x^2 \Rightarrow 24x^2(5x^2 - 3)$

$f''(-1) = 48$, $f''(0) = 0$, $f''(1) = 48$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $f'(x)$	$-$	$+$	$-$	$+$
Monotonicity	strictly decreasing	strictly Increase	Strictly decreasing	Strictly Increasing

47) a) $4x^2 + 36y^2 + 40x - 288y + 532 = 0$

$4(x^2 + 10x + 25 - 25) + 36(y^2 - 8y + 16 - 16) = -532$

$4(x+5)^2 + 36(y-4)^2 = 144$

$\div 144$ $\frac{(x+5)^2}{36} + \frac{(y-4)^2}{4} = 1$; centre $(-5, 4)$

Vertices are $(1, 4)$ and $(-11, 4)$

$c^2 = a^2 - b^2 = 36 - 4 = 32$ & $c = \pm 4\sqrt{2}$

Foci $(-5 - 4\sqrt{2}, 4)$ and $(-5 + 4\sqrt{2}, 4)$

Length of the major axis $2a = 12$ units & Length of the minor axis $2b = 4$ units

b) $V(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$

$e^V = \frac{x^2 + y^2}{x + y} = f$ (say) $\therefore f(x, y) = \frac{x^2 + y^2}{x + y}$

f is homogeneous function of degree 1

By Euler's theorem $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

$x\frac{\partial(e^V)}{\partial x} + y\frac{\partial(e^V)}{\partial y} = e^V$

$e^V\left[x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}\right] = e^V \Rightarrow x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 1$