CLASS: XII **Full Test** **TIME: 3.00 hrs**

SUBJECT: MATHEMATICS **MARKS: 90**

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I. Choose the correct answer: **20x1=20**

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

(1) $\frac{-3}{5}$ (2) $\frac{-4}{5}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

2. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

(1) real axis (2) circle (3) ellipse (4) imaginary axis

3. A is of order n, $\lambda \neq 0$ then adj (λA)=

(1) $\lambda^{n-1} adj(A)$ (2) $\lambda^{n-2} adj(A)$ (3) $\frac{1}{\lambda}\, adj(A)$ (4) $\lambda^n adj(A)$

4. The number of positive roots of the polynomial $\sum_{r=0}^{n} n_{c_r} (-1)^r x^r$ is

(1) 0 (2) $> n$ (3) $< n$ (4) n

5. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(1) no solution (2) two solutions

(3) unique solution (4) infinite number of solutions

6. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $35 < m < 85$ (2) $15 < m < 65$

(3) $-35 < m < 15$ (4) $-85 < m < -35$

7. The statement "A polynomial equation of degree n has exactly n roots which are either real or complex" is

(1) Fundamental theoem of Algebrs (2) Rational root theorem

(3) Descartes rule (4) Complex conjugate root theorem

8. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 5\hat{k}, \vec{c} = 3\hat{\imath} + 5\hat{\jmath} - \hat{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(1) $-17\hat{\imath} - 21\hat{\jmath} - 97\hat{k}$ (2) $17\hat{\imath} + 21\hat{\jmath} - 123\hat{k}$

(3) $-17\hat{\imath} - 21\hat{\jmath} + 97\hat{k}$ (4) $-17\hat{\imath} + 21\hat{\jmath} - 97\hat{k}$

9. The amplitude and period of $y = a \tan bx$ are respectively

(1) $|a|, \frac{\pi}{|b|}$ (2) $a, \frac{\pi}{b}$ (3) not defined, $\frac{\pi}{|b|}$ (4) not defined, $\frac{\pi}{b}$

10. Determine the truth value of each of the following statements:

(a) $4 + 2 = 5$ and $6 + 3 = 9$ (b) $3 + 2 = 5$ and $6 + 1 = 7$

(c) $4 + 5 = 9$ and $1 + 2 = 4$ (d) $3 + 2 = 5$ and $4 + 7 = 11$

	(a)	(b)	(c)	(d)
(1)	T	F	T	F
(2)	F	T	F	T
(3)	T	T	F	F
(4)	F	F	T	T

11. The non-parametric form of a vector equation passing through a point whose position vector is $\bar{a}$ and parallel to two vectors $\bar{u}$ and $\bar{v}$ is
(1) $[\bar{r}-\bar{u} \quad \bar{u} \quad \bar{v}\ ]=0$ (2) $[\bar{r}-\bar{a} \quad \bar{u} \quad \bar{v}\ ]=0$
(3) $[\bar{r}-\bar{v} \quad \bar{u} \quad \bar{v}\ ]=0$ (4) $[\bar{r}-\bar{u} \quad \bar{a} \quad \bar{v}]=0$

12. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
(1) 0.96 (2) 0.48 (3) 0.6 (4) 0.24

13. The order and degree of the differential eqation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2log\left(\frac{d^2y}{dx^2}\right)$ are
(1) 2, not defined (2) 3, 2 (3) 2,3 (4) 2,2

14. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then
(1) $P=Ce^{-kt}$ (2) $P=Ce^{kt}$ (3) $P=Ckt$ (4) $Pt=C$

15. If $\cos x$ is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, then P is
(1) $\log\sin x$ (2) $\cos x$ (3) $-\tan x$ (4) $\cot x$

16. The value of $\int_0^1 x(1-x)^{99}dx$ is
(1) $\frac{1}{11000}$ (2) $\frac{1}{10001}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10100}$

17. If Var (5x + 3) =
(1) 5 Var (X) (2) 5 (3) 25 (4) 25 Var X)

18. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
(1) 4.8 cu.cm (2) 0.45 cu.cm
(3) 2 cu.cm (4) 0.4 cu.cm

19. Let A be Q /{1}. Define * on A by x + y = x + y + xy. Existence of inverse is
(1) 0 (2) $\frac{a}{1-a}$ (3) $\frac{-a}{1+a}$ (4) does not exists

20. The curve $y=ax^4+bx^2$ with $ab>0$
(1) has no horizontal tangent (2) has no points of inflection
(3) is concave down (4) is concave up

II. Answer any seven questions: Q.No. 30 is compulsory :- 7x2=14

21. Find the principal argument $Argz$, when $z=\frac{-2}{1+i\sqrt{3}}$.
22. The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity.
23. The volume of the parallelopiped whose coterminus edges are $7\hat{\imath}+\lambda\hat{\jmath}-3\hat{k}, \hat{\imath}+2\hat{\jmath}-\hat{k}, -3\hat{\imath}+7\hat{\jmath}+5\hat{k}$ is 90 cubic units. Find the value of λ.
24. Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$.
25. Let $g(x)=x^2+\sin x$. Calculate the differential dg.
26. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}dx$.
27. Show that following expressions is a solution of the corresponding given differential equation. $y=2x^2$; $xy'=2y$

28. The time to failure in thousand hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x}, x > 0 \\ 0, \text{elsewhere} \end{cases}$. Find the expected life of this electronic equipment.

29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$

30. Find the value of $Sin^{-1}\left(-\frac{1}{2}\right) + Sec^{-1}(2)$.

III) Answer any Seven questions: Q.No. 40 is compulsory:- 7x3=21

31. Test for consistency and if possible, solve the following systems of equations by rank method. $x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4$

32. Obtain the Cartesian equation form of the locus of $z = x + iy$ in the following case: $Im[\,(1-i)z + 1] = 0$

33. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

34. Solve :$tan^{-1} 2\,x + tan^{-1} 3\,x = \frac{\pi}{4}$, if $6x^2 < 1$.

35. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.

36. Find two positive numbers whose product is 20 and their sum is minimum.

37. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$times the percentage error in the number.

38. Solve : $cos\,x \frac{dy}{dx} + y\,sin\,x = 1$

39. The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. He tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

40. Find the general equation of circle whose diameter is the line segment joining the points (-4, -2) and (-1, -1).

IV. Answer the following questions :- 7x5=35

41. (a) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$ has a non-trivial solution.

(OR)

(b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42. (a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and$z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$.

(OR)

(b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally if, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c}$.

43. (a) Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$at $(1, -3)$.

(OR)

(b) A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \leq X < 5)$ (iii) $P(3 < X)$

44. (a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

(OR)

(b) If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2)$.

45. (a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

(OR)

(b) Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

46. (a) Evaluate : $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$.

(OR)

(b) Verify whether the following compound propositions are tautologies or contradictions or contingency $(p \to q) \wedge (q \to r)) \to (p \to r)$

47. (a) Find the shortest distance between the straight line $\frac{x-6}{1} = \frac{2-y}{2} = \frac{z-2}{2}$ *and* $\frac{x+4}{3} = \frac{y}{-2} = \frac{1-z}{2}$.

(OR)

(b) Solve the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$.

BY

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