XII-FP3-24 5

Full Portion Test - 3

Standard XII

MATHEMATICS

Time: 3.00 hrs. Marks: 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2) Use Blue or Black ink to write.

PART - I 20x1=20

Note : i) Answer all the questions.
ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$
a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is
a) 15 b) 12 c) 14 d) 11

3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
a) $\frac{1}{2}$ b) 1 c) 2 d) 3

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is
a) $\text{cis}\frac{2\pi}{3}$ b) $\text{cis}\frac{4\pi}{3}$ c) $-\text{cis}\frac{2\pi}{3}$ d) $-\text{cis}\frac{4\pi}{3}$

5. The polynomial $x^3 + 2x + 3$ has
a) one negative and two real zeros b) one positive and two imaginary zeros
c) three real zeros d) no zeros

6. If $|x| \leq 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to
a) $\tan^{-1}x$ b) $\sin^{-1}x$ c) 0 d) π

7. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has
a) no solution b) unique solution c) two solutions d) infinite number of solutions

8. An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{\sqrt{3}}$

9. If P(x, y) be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is
a) 8 b) 6 c) 10 d) 12

10. If the distance of the point $(1,1,1)$ from the origin is half of its distance from the plane $x+y+z+k=0$, then the value of k are

a) ±3 b) ±6 c) $-3,9$ d) $3,-9$

11. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

12. The maximum value of the product of two positive numbers, when their sum of the squares is 200 is

a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is

a) $t=0$ b) $t=\frac{1}{3}$ c) $t=1$ d) $t=3$

14. If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u

15. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

a) 0.2% b) 0.4% c) 0.04% d) 0.08%

16. The value of $\int_{-4}^{4}\left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right)+\tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$ is

a) π b) 2π c) 3π d) 4π

17. If $f(x)=\int_0^x t\cos t\,dt$, then $\frac{df}{dx}=$

a) $\cos x - x\sin x$ b) $\sin x + x\cos x$ c) $x\cos x$ d) $x\sin x$

18. If p and q are the order and degree of the differential equation $y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=\cos x$, when

a) $p<q$ b) $p=q$ c) $p>q$ d) none of these

19. If in 6 trails, X is a binomial variable which follows the relation $9P(X=4)=P(X=2)$, then the probability of success is

a) 0.125 b) 0.25 c) 0.375 d) 0.75

20. The operation * defined by $a*b=\frac{ab}{7}$ is not a binary operation on

a) $\mathbb{Q}^+$ b) $\mathbb{Z}$ c) $\mathbb{R}$ d) $\mathbb{C}$

PART - II

Note: i) Answer any seven questions.
ii) Question no.30 is compulsory.

7x2=14

21. If $adj(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

22. Find the product $\frac{3}{2}\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right].\,6\left[\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right]$ in rectangular form.

23. Find the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$.

24. Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$.

25. Compute the limit $\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]$.

26. The relation between the number of words of y a person learns in x hours is given by $y=52\sqrt{x},\ 0\le x\le 9$. What is the approximate number of words learned when x changes from (i) 1 to 1.1 hour (ii) 4 to 4.1 hour?

27. Evaluate : $\int_0^{\infty} x^5 e^{-3x}dx$

28. Find the differential equation of the curve represented by $xy=ae^x+be^{-x}+x^2$.

29. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function.

$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}$. Find the expected life of this electronic equipment.

30. Find the intercepts cut off by the plane $\vec{r}.(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

PART - III

Note : i) Answer any seven questions.
ii) Question no. 40 is compulsory. 7x3=21

31. In a competitive examination, one mark is awarded for every correct answer while 1/4 mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)

32. Find the rectangular form of the complex number $\frac{\cos\pi/6-i\sin\pi/6}{(2\cos\pi/3+i\sin\pi/3)}$.

33. Solve the equation $\sin^2 x-5\sin x+4=0$.

34. Prove that $\frac{\pi}{2}\le\sin^{-1}x+2\cos^{-1}x\le\frac{3\pi}{2}$.

35. A circle of area 9π square units has two of its diameters along the lines $x+y=5$ and $x-y=1$. Find the equation of the circle.

36. Prove that $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}]=0$.

37. Using Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval for the function $f(x)=x^3-3x+2,\ x\in[-2,2]$.

38. Evaluate : $\int_0^1\frac{2x+7}{5x^2+9}dx$

39. Prove that $q\to p\equiv\neg p\to\neg q$

40. Obtain the equation of the curve whose slope at any point is equal to $y+2x$ and which passes through the origin.

PART - IV 7x5=35

Note: Answer all the questions.

41. a) If $ax^2 + bx + c$ is divided by $x+3, x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method)

(OR)

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

42. a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

(OR)

b) Find the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

43. a) Identify the type of conic and find centre, foci, vertices and directrices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$.

(OR)

b) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6m from the centre on either side.

44. a) Find the non parametric form of vector equation and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$.

(OR)

b) With usual notation, in any triangle ABC, prove that $a^2 = b^2 + c^2 - 2bc\cos A$ by vector method.

45. a) A conical water tank with vertex down of 12 m height has a radius of 5m at the top. If water flows into the tank at a rate of 10 cubic/min, how fast is the depth of the water increases when the water is 8 m deep?

(OR)

b) Prove that $g(x, y) = x\log(y/x)$ is homogeneous; what is the degree? Verify Euler's Theorem for g.

46. a) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

(OR)

b) Solve the differential equation $\left(1 + 3e^{y/x}\right)dy + 3e^{y/x}\left(1 - \frac{y}{x}\right)dx = 0$. Given that $y = 0$ when $x = 1$.

47. a) If X is the random variable with probability density function $f(x) = \begin{cases} x-1, & 1 \le x < 2 \\ -x+3, & 2 \le x < 3 \\ 0, & \text{otherwise} \end{cases}$.

Find (i) the distribution function F(x). (ii) $P(1.5 \le x \le 2.5)$

(OR)

b) Let A be $Q/\{1\}$. Define * on A by $x * y = x + y - xy$. Is * binary on A? If so, examine the commutative, associative, the existence of identity and existence of inverse properties.

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