XII-FP4-23

Full Portion Test - 4

Standard XII

MATHEMATICS

Time: 3.00 hrs. Marks: 90

Instructions: 1) *Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.*
2) *Use Blue or Black ink to write and underline and pencil to draw diagrams.*

PART - I

20×1=20

Note: i) *Answer all the questions.*
ii) *Choose the most suitable answer from the given four alternatives and write the option code with the corresponding answer.*

1. If $|\text{adj}(\text{adj} A)| = |A|^9$, then the order of the square matrix A is
   a) 3 b) 4 c) 2 d) 5
2. If $A^T A^{-1}$ is symmetric, then $A^2 =$
   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$
3. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
   a) mn b) m+n c) m^n d) n^m
4. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, then the value of $\tan^{-1} x$ is
   a) $-\frac{\pi}{2}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$
5. The radius of the circle passing through the point (6, 2) two of whose diameter are $x+y=6$ and $x+2y=4$ is
   a) 10 b) $2\sqrt{5}$ c) 6 d) 4
6. If $x+y=k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
   a) 3 b) −1 c) 1 d) 9
7. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$, then |z| is
   a) $\frac{1}{2}$ b) 1 c) 2 d) 3
8. If $\frac{z-1}{z+1}$ is purely imaginary, then |z| is
   a) $\frac{1}{2}$ b) 1 c) 2 d) 3
9. The angle between the line $\vec{r} = (\hat{i}+2\hat{j}-3\hat{k}) + t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j})+4=0$ is
   a) 0° b) 30° c) 45° d) 90°
10. If the planes $\vec{r}\cdot(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}\cdot(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are
    a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$

11. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is
a) $(4, 11)$ b) $(4, -11)$ c) $(-4, 11)$ d($(-4, -11)$

12. The maximum value of the product of two positive numbers, when their sum of the squares is 200 is
a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

14. The value of $\int_{-\pi/4}^{\pi/4} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} \right) dx$ is
a) 4 b) 3 c) 2 d) 0

15. The value of $\int_0^{\pi} \sin^4 x \, dx$ is
a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

16. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
a) $y + \sin^{-1} x = c$ b) $x + \sin^{-1} y = 0$ c) $y^2 + 2\sin^{-1} x = C$ d) $x^2 + 2\sin^{-1} y = 0$

17. The order of the differential equation of all circles with centre at (h, k) and radius 'a' is
a) 2 b) 3 c) 4 d) 1

18. On a multiple - choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is
a) $\frac{11}{243}$ b) $\frac{3}{8}$ c) $\frac{1}{243}$ d) $\frac{5}{243}$

19. Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins ₹36, otherwise he loses ₹ k^2, where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in ₹ is
a) $\frac{19}{6}$ b) $-\frac{19}{6}$ c) $\frac{3}{2}$ d) $-\frac{3}{2}$

20. Which one of the following is not true?
a) Negation of a negation of a statement is the statement itself.
b) If the last column of the truth table contains only T then it is a tautology.
c) If the last column of its truth table contains only F then it is a contradiction.
d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

PART - II

7x2=14

...e: i) *Answer any seven questions.*
ii) *Question No. 30 is compulsory.*

If A is a non-singular matrix of odd order, prove that | adj A | is positive.

...implify: $\sum_{n=1}^{102} i^n$.

23. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma \frac{1}{\beta\gamma}$ in terms of coefficients.

24. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (in radians and degrees).

25. Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at $P(-3, 4)$.

26. Find the angle made by the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with the coordinate axes.

27. If the volume of a cube of side length x is $v = x^3$, find the rate of change of the volume with respect to x when x = 5 units.

28. Find the partial derivatives of the function $h(x, y, z) = x\sin(xy) + z^2 x$ at $\left(2, \frac{\pi}{4}, 1\right)$.

29. Evaluate : $\int_0^{2\pi} \sin^7 \frac{x}{4} \, dx$

30. Four fair coins are tossed once. Find the probability mass function, mean and variance for the number of heads occurred.

PART - III

7x3=21

Note: i) *Answer any seven questions.*
ii) *Question No. 40 is compulsory.*

31. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

32. Obtain the cartesian equation for the locus of $z = x + iy$ in $|z-4|^2 - |z-1|^2 = 16$.

33. Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$.

34. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

35. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \ell \vec{a} + m\vec{b} + n\vec{c}$, find the values of ℓ, m and n.

36. Write the Maclaurin series expansion of the function $f(x) = \cos^2 x$.

37. Let $f, g : (a, b) \rightarrow R$ be differentiable functions. Show that $d(fg) = fdg + gdf$.

38. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|} dx$

39. Establish the equivalence property connecting the bi-conditional with conditional $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

40. Solve : $\frac{dy}{dx} = (3x + y + 4)^2$

PART - IV

7x5=35

Note : Answer all the questions.

41. a) Determine the values of λ for which the system of equations $x+y+3z=0$, $4x+3y+\lambda z=0$, $2x+y+2z=0$ has (i) a unique solution (ii) a non-trivial solution. **(OR)**

b) Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

42. a) If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that (i) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$

(ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha+n\beta)$. **(OR)**

b) Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

43. a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x, y, z<1$, show that $x^2+y^2+z^2+2xyz=1$. **(OR)**

b) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

44. a) Find the coordinate of the foot of the perpendicular drawn from the point (−1, 2, 3) to the straight line $\vec{r}=(\hat{i}-4\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+\hat{k})$. Also, find the shortest distance from the given point to the straight line. **(OR)**

b) Find the equations of the tangents to the curve $y=1+x^3$ for which the tangent is orthogonal with the line $x+12y=12$.

45. a) Find the intervals of monotonicities and hence find the local extremum for the function $f(x)=\frac{x^3}{3}-\log x$.

(OR)

b) Let $w(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z)\neq(0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$.

46. a) Evaluate $\int_1^2 (4x^2-1)\,dx$ as the limits of sums. **(OR)**

b) The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx}=g\left(1-\frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x.

47. a) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2\le X<5)$ (iii) $P(3<X)$. **(OR)**

b) Solve : $\frac{dy}{dx}=\frac{\sin^2 x}{1+x^3}-\frac{3x^2}{1+x^3}y$

★★★★★