REG NO :

KALAIMAGAL MATRIC HIGHER SECONDARY SCHOOL, MOHANUR .

SECOND FULL TEST – JANUARY 2025

STD: XII **TIME: 3Hrs**

DATE: **MATHEMATICS** **MARKS: 90**

PART - I **20 X 1 = 20**

NOTE: (i)Answer all the questions.(ii)Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer:

1. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the joint of 3×3 matrix A and $|A| = 4$, then x is

   (1) 15 (2) 12 (3) 14 (4) 11

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   (1) 17 (2) 14 (3) 19 (4) 21

3. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   (1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

4. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$ then the locus of z is

   (1) real axis (2) imaginary axis (3) ellipse (4) circle

5. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

6. e^{ix} is a periodic function with period

   (1) 0 (2) π (3) 2π (4) 4

7. If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$ then

   (1) $c = \frac{a}{m}$ (2) $c = \frac{m}{a}$ (3) $c^2 = a^2m^2 + m^2$ (4) $m = c$

8. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

   (1) $2x+1=0$ (2) $x=-1$ (3) $2x-1=0$ (4) $x=1$

9. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath}$, $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

   (1) 0 (2) 1 (3) 6 (4) 3

10. If the length of the perpendicular from the origin to the plane 2x+3y+ λz=1, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

11. What is the value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

12. The slant asymptote of $f(x) = \frac{x^2-6x+7}{x+5}$ is

(1) $x + y + 11 = 0$ (2) $x + y - 11 = 0$ (3) $x = -5$ (4) $y = x - 11$

13. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

14. $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx =$

(1) 0 (2) a (3) $\frac{a}{2}$ (4) $2a$

15. The value of $\int_0^\infty e^{-3x}x^2\,dx$ is

(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

16. The number of arbitrary constants in the particular solution of a differential equation of third order is

(1) 3 (2) 2 (3) 1 (4) 0

17. The order and degree of the differential equation $(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^2 = x\sin(\frac{d^2y}{dx^2})$ are

(1) 2, not defined (2) 2, 2 (3) 2,1 (4) 1,2

18. A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

19. If in 6 trials, X is a binomial variate which follows the relation $9P(X=4) = P(X=2)$, then the probability of success is

(1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75

20. The operation $*$ defined by $a*b = \frac{ab}{7}$ is not a binary operation on

(1) $\mathbb{Q}^+$ (2) $\mathbb{Z}$ (3) $\mathbb{R}$ (4) $\mathbb{C}$

PART - II 7 X 2 = 14

Answer any seven of the following questions.Q.No.30 is compulsory.

21. Find the rank of the following matrix $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ which are in row-echelon form .
22. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$ in the rectangular form.
23. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
24. Find the angle between the straight line $\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane $2x - y + z = 5$.
25. Explain why Rolle's theorem is not applicable to the function $f(x) = \tan x,\ x \in [0, \pi]$.
26. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from $2\,mm$ to $2.1\,mm$, how much is cross-sectional area increased approximately?
27. Evaluate: $\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$.
28. Show that the expressions $y = ae^x + be^{-x}$ is a solution of the differential equation $y'' - y = 0$.
29. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x} & for\ x > 0 \\ 0 & elsewhere \end{cases}$

    Find the expected life of this electronic equipment.
30. Find the centre and radius of the circle $3x^2 + 3y^2 - 12x + 6y - 9 = 0$.

PART - III 7 X 3 = 21

Answer any seven of the following questions. Q.No. 40 is compulsory.

31. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).
32. Show that the equation $z^2 = \bar{z}$ has four solutions.
33. Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$.
34. Find the vertex, focus, equation of directrix and length of the latus rectum of $x^2 = 24y$.
35. Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$.
36. Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on the given closed interval $[-1, 2]$.
37. If $v(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.
38. Find the area of the region bounded by the line $7x - 5y = 35$, x - axis and the lines $x = -2$ and $x = 3$.
39. Prove that $q \to p \equiv \neg p \to \neg q$.
40. Solve : $\frac{dy}{dx} + y\, cotx = sin2x$.

PART - IV 7 X 5 = 35

Answer all the questions.

41. (*a*) If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $\left[F(\alpha)\right]^{-1}=F(-\alpha)$.

(OR)

(b) Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

42. (*a*) If $2\cos\alpha=x+\dfrac{1}{x}$ and $2\cos\beta=y+\dfrac{1}{y}$ show that

(i) $\dfrac{x}{y}+\dfrac{y}{x}=2\cos(\alpha-\beta)$ (ii) $xy-\dfrac{1}{xy}=2i\sin(\alpha+\beta)$ **(OR)**

(b) Assume that water issuing from the end of a horizontal pipe, 7.5 *m* above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

43. (*a*) Solve: $\tan^{-1}\left(\dfrac{x-1}{x-2}\right)+\tan^{-1}\left(\dfrac{x+1}{x+2}\right)=\dfrac{\pi}{4}$ **(OR)**

(b) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

44. (*a*) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$. **(OR)**

(b) Evaluate : $\displaystyle\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$.

45. (*a*) For the function $f(x,y)=\tan^{-1}\left(\dfrac{x}{y}\right)$ find the f_x, f_y and show that $f_{xy}=f_{yx}$.

(OR)

(b) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

46. (*a*) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights (i) exactly 10 will have a useful life of at least 600 hours;
(ii) at least 11 will have a useful life of at least 600 hours;
(iii) at least 2 will *not* have a useful life of at least 600 hours.

(OR)

(b) Find the non parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

47. (*a*) Verify (i) closure property, (ii) commutative property, (iii) associative property,
(iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subs $A=\{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$. **(OR)**

(b) Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

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$\overline{XII}$ FULL TEST - 2 ANSWER KEY

ONE MARKS

	TYPE-A		TYPE-B
1	(4) 11	1	(3) 2π
2	(3) 19	2	(1) $c=\frac{a}{m}$
3	(1) z	3	(2) $x=-1$
4	(2) imaginary axis	4	(1) 0
5	(1) $-\frac{9}{8}$	5	(1) $2\sqrt{3}$
c 6	(3) 2π	6	(1) 0
c 7	(1) $c=\frac{a}{m}$	7	(4) $y=x-1$
8	(2) $x=-1$	8	(2) $-x+\frac{\pi}{2}$
9	(1) 0	9	(3) $\frac{a}{2}$
10	(1) $2\sqrt{3}$	10	(4) $\frac{2}{27}$
11	(1) 0	11	(4) 0
c 12	(4) $y=x-11$	12	(1) 2, not defined
13	(3) $-x+\frac{\pi}{2}$	13	(4) 2
c 14	(3) $\frac{a}{2}$	14	(2) 0.25
15	(4) $\frac{2}{27}$	15	(2) z
16	(4) 0	16	(4) 11
c 17	(1) 2, not defined	17	(3) 19
18	(4) 2	18	(1) z
19	(2) 0.25	19	(2) imaginary axis
20	(2) z	20	(1) $-\frac{9}{8}$

TWO MARKS

(21) $|A|=\begin{vmatrix}2&0&-7\\0&3&1\\0&0&1\end{vmatrix}=6\neq0$

$\rho(A)=3$

(22) $\frac{z_1}{z_2}=\frac{3-2i}{6+4i}\times\frac{6-4i}{6-4i}$

$=\frac{5}{26}-\frac{6}{13}i$

(23) Let $\cos^{-1}(-x)=t$

$x=-\cos t=\cos(\pi-t)$

$\cos^{-1}(-x)=\pi-\cos^{-1}(x)$

Yes, it is true

(24) $\theta=\sin^{-1}\left(\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right)$

$\theta=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(25) $f(x)=\tan x$ is not continuous at $x=\frac{\pi}{2}$ in $[0,\pi]$

(26) $A=\pi r^2\Rightarrow dA=2\pi r\,dr$

$=2\pi(2)(0.1)=0.4\pi\,mm^2$

(27) $I_n=\frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{2}{3}$

$I_7=\frac{6}{7}\times\frac{4}{5}\times\frac{2}{3}=\frac{16}{35}$

(28) $y'=ae^{x}-be^{-x}$

$y''=ae^{x}+be^{-x}$

$y''-y=0$

(29) $E(x)=\int_{-\infty}^{\infty}x\,f(x)\,dx$

$=3\int_0^{\infty}xe^{-3x}dx$

$=3\,\frac{1!}{3^2}=\frac{1}{3}$

(30) $x^2+y^2=4x+2y-3=0$

centre $=(-g,-f)=(2,-1)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{8}$

3 MARKS

(31) $x+y=100$

$4x-y=320$

$\Delta=-5,\ \Delta_x=-420,\ \Delta_y=80$

$x=84,\ y=16$

The correct answer is 84

(32) $z^2=\bar{z}\Rightarrow|z|^2=|z|$

$|z|(|z|-1)=0$

$\Rightarrow|z|=0,\ |z|=1$

$z^2=\frac{1}{z}\Rightarrow z^3=1$

There are 4 solutions

(33) $2\cos^2x-9\cos x+4=0$

$\cos x=4$, $\cos x=\frac{1}{2}$

is not possible

$x=2n\pi\pm\frac{\pi}{3}$

(34) Vertex (0,0)

Focus (0,6)

Directrix $y=-6$

L.L.R = 24

(35) $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}]$

$=(\vec{a}-\vec{b})\cdot[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})]$

$=(\vec{a}-\vec{b})\cdot[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]-[\vec{a}\,\vec{b}\,\vec{c}]=0$

(36) $f(x)=3x^4-4x^3$

$f'(x)=12x^3-12x^2$

$f'(x)=0\Rightarrow x=0,1$

$f(0)=0,\ f(1)=-1,\ f(-1)=7$

$f(2)=16$

Absolute max is 16

Absolute min is -1

(37) V is not homogeneous

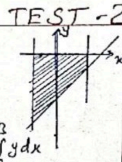
$e^v=\frac{x^2+y^2}{x+y}=f(x,y)$

f is homogeneous of deg 1

$x\frac{\partial v}{\partial x}e^v+y\frac{\partial v}{\partial y}e^v=e^v$

$x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$

(38)



$A=\int_{-2}^{3}y\,dx$

$=\int_{-2}^{3}\left(\frac{7x-35}{5}\right)dx$

$=\frac{1}{5}\left[\frac{7x^2}{2}-35x\right]_{-2}^{3}$

$=\frac{63}{2}$

(39)

p	q	¬p	¬q	q→p	¬p→¬q
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

$q\rightarrow p\equiv\neg p\rightarrow\neg q$

(40) $\frac{dy}{dx}+y\cot x=\sin 2x$

$I.F=e^{\int P dx}=e^{\int\cot x\,dx}=\sin x$

$y(I.F)=\int Q(I.F)dx+c$

$y\sin x=\int\sin 2x\sin x\,dx+c$

$=\int 2\sin^2x\cos x\,dx+c$

$=2\int t^2dt+c$

$=\frac{2\sin^3x}{3}+c$

5 MARKS

(41) (a) $F(\alpha)=\begin{bmatrix}\cos\alpha&0&-\sin\alpha\\0&1&0\\\sin\alpha&0&\cos\alpha\end{bmatrix}$

$|F(\alpha)|=1$

$[F(\alpha)]^{-1}=\begin{bmatrix}\cos\alpha&0&-\sin\alpha\\0&1&0\\\sin\alpha&0&\cos\alpha\end{bmatrix}$

$[F(\alpha)]^{-1}=F(-\alpha)$

(b) Given $3x^3-16x^2+23x-6=0$

$$\begin{array}{c|cccc}2&3&-16&23&-6\\&0&6&-20&6\\\hline&3&-10&3&0\end{array}$$

$3x^2-10x+3=0$

$x=\frac{1}{3},\ x=3$

∴ The roots are $\frac{1}{3},3,2$

(42) (a) $x=\cos\alpha+i\sin\alpha$

$y=\cos\beta+i\sin\beta$

(i) $\frac{x}{y}=\cos(\alpha-\beta)+i\sin(\alpha-\beta)$

$\frac{y}{x}=\cos(\alpha-\beta)-i\sin(\alpha-\beta)$

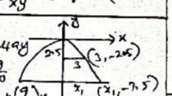
$\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$

(ii) $xy=\cos(\alpha+\beta)+i\sin(\alpha+\beta)$

$\frac{1}{xy}=\cos(\alpha+\beta)-i\sin(\alpha+\beta)$

$xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$

(b)



$x^2=-4ay$

$a=\frac{9}{10}$

$x^2=-4\left(\frac{9}{10}\right)y$

It passes $(x_1,-7.5)$

$x_1^2=27\Rightarrow x_1=3\sqrt{3}$

(43) (a) $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

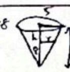
$\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4}$

$\frac{(x-1)(x+2)+(x-2)(x+1)}{(x-2)(x+2)-(x-1)(x+1)}=\tan\frac{\pi}{4}$

$2x^2-4=-3$

$x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}$

(b) $\frac{dv}{dt}=10,\ h=8$



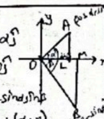
$V=\frac{1}{3}\pi r^2h$

$V=\frac{25\pi}{3\times144}h^3$

$\frac{dV}{dt}=\frac{25\pi}{3\times144}\times3h^2\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{9}{10\pi}$ m/min

(44) (a)



$\hat{a}=\cos\alpha\hat{i}+\sin\alpha\hat{j}$

$\hat{b}=\cos\beta\hat{i}-\sin\beta\hat{j}$

$\hat{b}\cdot\hat{a}=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\hat{b}\cdot\hat{a}=|\hat{b}||\hat{a}|\cos(\alpha+\beta)$

$=\cos(\alpha+\beta)$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(b) $I=\int_0^{\pi/4}\log(1+\tan\theta)d\theta$

$I=\int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$=\int_0^{\pi/4}\log\left(\frac{2}{1+\tan\theta}\right)d\theta$

$2I=\log 2\int_0^{\pi/4}d\theta$

$I=\frac{\pi}{8}\log 2$

(45) (a) $f_x=\frac{y}{x^2+y^2},\ f_y=\frac{-x}{x^2+y^2}$

$f_{xy}=\frac{x^2-y^2}{(x^2+y^2)^2},\ f_{yx}=\frac{x^2-y^2}{(x^2+y^2)^2}$

$f_{xy}=f_{yx}$

(b) $\frac{dA}{dt}\propto A$

$A=ce^{kt}$

$A=ce^{0.05t}$

when $t=0$, $A=10000$

$c=10000$

$P=10000e^{0.05t}$

when $t=\frac{3}{2}$

$P=10000e^{0.075}$

(46) (a) $n=12,\ p=0.9,\ q=0.1$

(i) $P(X=10)=\binom{12}{10}(0.9)^{10}(0.1)^2$

(ii) $P(X\geq11)=P(X=11)+P(X=12)$

$=\binom{12}{11}(0.9)^{11}(0.1)^1+\binom{12}{12}(0.9)^{12}(0.1)^0$

$=2.1(0.9)^{11}$

(b) $\vec{b}-\vec{a}=7\hat{i}+\hat{j}+5\hat{k}$

Non parametric

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times\vec{c}]=0$

$(\vec{r}-(2\hat{i}+2\hat{j}+\hat{k}))\cdot(-24\hat{i}-32\hat{j}+40\hat{k})=0$

$\vec{r}\cdot(3\hat{i}+4\hat{j}-5\hat{k})=9$

Cartesian

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$3x+4y-5z-9=0$

(47) (a)

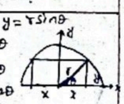
X_{11}	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

closure, commutative, associative is true

Identity element is 1

Inverse of 1 is 1, 3 is 4, 4 is 3, 5 is 9, 9 is 5.

(b) $x=2r\cos\theta,\ y=r\sin\theta$



$A(\theta)=r^2\sin2\theta$

$A'(\theta)=2r^2\cos2\theta$

$A''(\theta)=-4r^2\sin2\theta$

$A'(\theta)=0\Rightarrow\theta=\frac{\pi}{4}$

$A''(\theta)<0\Rightarrow$ Area is maximum

∴ length $=\sqrt{2}r$ cm, breadth $=\frac{r}{\sqrt{2}}$ cm

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