XII-HP1

Half Portion Test - 1

Standard XII

MATHEMATICS

Time: 3.00 hrs. Marks: 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2) Use Blue or Black ink to write.

PART - I

20x1=20

Note:i) Answer all the questions.

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A=\begin{bmatrix}2 & 0\\1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\2 & 0\end{bmatrix}$ then $|\text{Adj}(AB)|=$

a) -40 b) -80 c) -60 d) -20

2. If $A=\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

a) 17 b) 14 c) 19 d) 21

3. If $A=\begin{bmatrix}a & 0 & 0\\0 & a & 0\\0 & 0 & a\end{bmatrix}$ then the value of $|\text{adj } A|$ is

a) a^{27} b) a^9 c) a^6 d) a^2

4. The value of x where $A=\begin{bmatrix}6 & x-2\\3 & x\end{bmatrix}$ has no inverse is

a) -2 b) 2 c) 0 d) 3

5. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

6. If z is a complex number such that $z\in\mathbb{C}\setminus\mathbb{R}$ and $z+\frac{1}{z}\in\mathbb{R}$, then $|z|$

a) 0 b) 1 c) 2 d) 3

7. The value of $\left[\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^{10}$ is

a) $\text{cis}\frac{2\pi}{3}$ b) $\text{cis}\frac{4\pi}{3}$ c) $-\text{cis}\frac{2\pi}{3}$ d) $-\text{cis}\frac{4\pi}{3}$

8. If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$, then the locus of z is

a) real axis b) imaginary axis c) ellipse d) circle

9. If α, β and γ are the zeros of x^3+px^2+qx+r, then $\Sigma\frac{1}{\alpha}$ is

a) $-\frac{q}{r}$ b) $-\frac{p}{r}$ c) $\frac{q}{r}$ d) $-\frac{q}{p}$

10. If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

11. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞

12. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

13. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) $[1, 2]$ b) $[-1, 1]$ c) $[0, 1]$ d) $[-1, 0]$

14. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

a) 4 b) 5 c) 2 d) 3

15. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

a) $2x+1=0$ b) $x=-1$ c) $2x-1=0$ d) $x=1$

16. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) $2ab$ b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

17. The radius of the circle passing through the point (6, 2) two of whose diameter are $x+y=6$ and $x+2y=4$ is

a) 10 b) $2\sqrt{5}$ c) 6 d) 4

18. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

a) 2 b) -1 c) 1 d) 0

19. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) -1

20. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

PART - II

Note: i) Answer any seven questions.
ii) Question no.30 is compulsory. 7x2=14

21. Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

22. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss - Jordan method.

23. If $z = x + iy$, find $\text{Im}(3z + 4\bar{z} - 4i)$ in rectangular form.

24. Find the modulus of the complex number $2i(3-4i)(4-3i)$.

25. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

26. Solve the equation $7x^3 - 43x^2 = 43x - 7$.

27. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

28. Find the equation of the circles that touch both the axes and passes through $(-4, -2)$ in general form.

29. Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i} + 4\hat{j} - 5\hat{k}$ about the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ acting through a point whose position vector is $4\hat{i} + 2\hat{j} - 3\hat{k}$.

30. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

PART - III

Note: i) Answer any seven questions.
ii) Question no. 40 is compulsory. 7x3=21

31. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

32. If $|z| = 2$, show that $3 \le |z + 3 + 4i| \le 7$.

33. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

34. Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{n}\right)$.

35. Find the domain of the function $\tan^{-1}\sqrt{9-x^2}$.

36. Find the equation of the ellipse with foci $(\pm 2, 0)$, vertices $(\pm 3, 0)$.

37. A parabolic communication antenna has a focus at 2 m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.

38. Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x - y + z = 8$.

39. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$.

40. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

PART - IV

Note: Answer all the questions. 7x5=35

41\. a) If the system of equations $px+by+cz=0$, $ax+qy+cz=0$, $ax+by+rz=0$ has a non-trivial solution and $p\neq a$, $q\neq b$, $r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

(OR)

b) Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have (i) no solution (ii) unique solution (iii) infinitely many solutions.

42\. a) Solve the system of linear equation $2x+3y-z=9$, $x+y+z=9$, $3x-y-z=-1$ using matrix inversion method.

(OR)

b) Find the cube roots of unity.

43\. a) Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.

(OR)

b) Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left[\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right]}$ in rectangular form.

44\. a) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

(OR)

b) Solve : $6x^4-35x^3+62x^2-35x+6=0$

45\. a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$ show that $x^2+y^2+z^2+2xyz=1$.

(OR)

b) Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

46\. a) Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, $(3,2)$.

(OR)

b) Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2=4ax$ is $(at_1t_2, a(t_1+t_2))$.

47\. a) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

(OR)

b) Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}$, $z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}$, $y-2=0$ intersect. Also find the point of intersection.

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