HTVM

HALF YEARLY EXAMINATION - 2024
MATHEMATICS

12 - Std

Time : 3.00 Hrs Marks : 90

I. Choose the correct answer: 20 X 1 = 20

1. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & otherwise \end{cases}$ is the p.d.f of the random variable, then the value of a is (a) 4 (b) 3 (c) 2 (d) 1

2. Subtraction is not a binary operation in
(a) N (b) Z (c) R (d) Q

3. If A is a 3 x 3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, $BB^T =$
(a) I_3 (b) B^T (c) A (d)

4. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

5. If $cot^{-1}(\sqrt{\sin\alpha}) + tan^{-1}(\sqrt{\sin\alpha}) = u$, then $cos2u =$
(a) 0 (b) π (c) $tan^{-1}x$ (d) $sin^{-1}x$

6. The equationof the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$having centre at(0,3) is
(a) $x^2 + y^2 - 6y - 5 = 0$ (b) $x^2 + y^2 - 6y + 5 = 0$
(c) $x^2 + y^2 - 6y - 7 = 0$ (d) $x^2 + y^2 - 6y + 7 = 0$

7. The angle between the line$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + s(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath}) + 4 = 0$ is
(a) 90° (b) 0° (c) 45° (d) 30°

8. The point of inflection of the curve $y = (x-1)^3$is
(a) (1,0) (b) (1,1) (c) (0,0) (d) (0,1)

9. The value of $\int_0^a (\sqrt{a^2 - x^2})^3 dx$ is
(a) $\frac{3\pi a^2}{8}$ (b) $\frac{3\pi a^4}{8}$ (c) $\frac{\pi a^3}{16}$ (d) $\frac{3\pi a^4}{16}$

10. The order and degree(if exists) of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ are respectively
(a) 1,1 (b) 1,2 (c) 2,1 (d) 2,2

11. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$ then A is

(a) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ (c) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

12. If $|z-2+i|\le 2$ then the greatest value of $|z|$ is

(a) $\sqrt{3}-2$ (b) $\sqrt{3}+2$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

13. If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

(a) $a\ge 0$ (b) $a>0$ (c) $a<0$ (d) $a\le 0$.

14. The value of $sin^{-1}(\cos x), 0\le x\le \pi$ is

(a) $x-\pi$ (b) $\frac{\pi}{2}-x$ (c) $\pi-x$ (d) $\pi-x$

15. The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ And $y^2-14y+45=0$ is

(a) (9,4) (b) (4,9) (c) (4,7) (d) (7,4)

16. The volume of the parallelepiped withits edges represented by the vectors $\hat{\imath}+\hat{\jmath}, \hat{\imath}+2\hat{\jmath}$ and $\hat{\imath}+\hat{\jmath}+\pi\hat{k}$.

(a) π (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

17. The tangent to the curve $y^2-xy+9=0$ is vertical when

(a) $y=\frac{1}{2}$ (b) $y=\pm 3$ (c) $y=\pm\sqrt{3}$ (d) $y=0$

18. The approximate change in the volume V of a cube of side x meters caused by increasing the side by 1% is

(a) $0.03x^2m^3$ (b) $0.03x^3m^3$ (c) $0.3xdxm^2$ (d) $0.03xm^3$

19. The value of $\int_{-1}^{2}|x|\,dx$ is (a) $\frac{5}{2}$ (b) $\frac{1}{2}$ (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

20. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$ Then P is (a) $\tan x$ (b) $\cot x$ (c) $\cos x$ (d) $log(\sin x)$

II Answer any 7 questions. Q.No. 30 is compulsory: 7 x 2 = 14

21. If A is a non-singular matrix of order, prove that $|adj\,A|$ is positive.

22. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$.

23. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

24. Find the value of $sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$.

25. Find centre and radius of $2x^2+2y^2-6x+4y+2=0$.

26. Find the angle between the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + t(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ and the plane $\vec{r} \cdot (6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) = 8$.

27. Show that the percentage error in the n^{th} root of a number is approximately 1/n times the percentage error in the number.

28. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$. Compute the maximum height of the particle reached.

29. Find the area of the region bounded by the line $6x + 5y = 30$, x-axis and the lines $x = -1$ and $x = 3$.

30. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

III Answer any 7 questions. Q.No.40 is compulsory: 7 x 3 = 21

31. Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$ has a non-trivial solution.

32. Find the values of the real numbers x and y, if the complex numbers $(3 - i)x - (2 - i)y + 2i + 5$ and $2x + (-1 + 2i)y + 3 + 2i$ are equal.

33. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

34. Find the value of $tan^{-1}(-1) + cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$.

35. Find the local extrema for $f(x) = x^2e^{-2x}$ using second derivative test.

36. Evaluate: $\int_0^{2\pi} x \log\left(\frac{3+\cos x}{3-\cos x}\right) dx$.

37. Solve: $\frac{dy}{dx} = e^{x+y} + x^3e^y$.

38. Two balls are choosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs.15 for each red ball selected and we lose Rs.10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images.

39. Construct the truth table for $(p\bar{V}q)\Lambda(p\bar{V}\neg q)$.

40. Find the torque of the resultant of the three forces represented by $-3\hat{\imath} + 6\hat{\jmath} - 3\hat{k}, 4\hat{\imath} - 10\hat{\jmath} - 12\hat{k}$ and $4\hat{\imath} + 7\hat{\jmath}$ acting at the point with position vector $8\hat{\imath} - 6\hat{\jmath} - 4\hat{k}$, about the point with position vector $18\hat{\imath} + 3\hat{\jmath} - 9\hat{k}$.

IV. Answer ALL the Questions: $7 \times 5 = 35$

41. (a) Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, \ x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution. (OR)

(b) Find the foot of the perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also, find the equation of the perpendicular.

42. (a) If $z = x + iy$ and $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$. (OR)

(b) Prove that among all the rectangles of the given area square has the least perimeter.

43. (a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection. (OR) (b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

44. (a) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. (OR)

(b) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

45. (a) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, -2, 3) and parallel to the straight line passing through the points (2, 1, -3) and (-1, 5, -8). (OR)

(b) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{4sin^2x + 5cos^2x}$.

46. (a) Solve: $\left(1 + e^{\frac{x}{y}}\right)dx + 2e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$. (OR)

(b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on z_5 using table corresponding to addition modulo 5.

47. (a) Solve: $\left(y - e^{sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1 - x^2} = 0$. (OR)

(b) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.