COMMON HALF YEARLY EXAMINATION - 2025

A

Standard XII

Reg.No. | | | | | | |

MATHEMATICS

Time : 3.00 hrs **Part - I** *Marks : 90*

I. Choose the correct answer: 20 x 1 = 20

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then A =

a) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ d) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If $A=\begin{bmatrix}1 & \tan\theta/2\\-\tan\theta/2 & 1\end{bmatrix}$ and $AB = I_2$, then B =

a) $\left(\cos^2\frac{\theta}{2}\right)A$ b) $\left(\cos^2\frac{\theta}{2}\right)A^T$ c) $\left(\cos^2\theta\right)I$ d) $\left(\sin^2\frac{\theta}{2}\right)A$

3. If $\sqrt{-1}=i$, $n\in N$ then

a) $i^{4n+3}=-1$ b) $i^{8n+2}=1$ c) $i^{100n+4}=-1$ d) $i^{4n+5}=1$

4. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is

a) –2 b) –1 c) 1 d) 2

5. The minimum degree of a polynomial equation with rational co-efficients having the roots $p+\sqrt{q}$ and $-i\sqrt{q}$ is

a) 2 b) 1 c) 3 d) 4

6. If $\cot^{-1}x=\frac{2\pi}{5}$ for some $x\in R$, the value of $\tan^{-1}x$ is

a) $-\pi/10$ b) $\pi/5$ c) $\pi/10$ d) $-\pi/5$

7. If $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

9. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

a) $2x + 1 = 0$ b) $x = -1$ c) $2x - 1 = 0$ d) $x = 1$

10. If $\vec{a},\vec{b},\vec{c}$ are three non coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) π

11. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then (α,β) is

a) (–5,5) b) (–6,7) c) (5,–5) d) (6,–7)

12. With usual notation which of the following is not equal to $\vec{a}.(\vec{b}\times\vec{c})$

a) $-\vec{a}.(\vec{c}\times\vec{b})$ b) $\vec{c}.(\vec{b}\times\vec{a})$ c) $-\vec{b}.(\vec{c}\times\vec{a})$ d) $(\vec{c}\times\vec{a}).\vec{b}$

13. The value of the limit $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$ is

a) 0 b) 1 c) 2 d) ∞

14. The number given by the mean value theorem for the function $\frac{1}{x}$, $x\in[1,9]$ is

a) 2 b) 2.5 c) 3 d) 3.5

15. The curve $y = ax^4 + bx^2$ with $ab > 0$

a) has no horizontal tangent b) is concave up
c) is concave down d) has no points of inflection

16. If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to

a) $xy\,e^{xy}$ b) $(1 + xy)\,e^{xy}$ c) $(1 + y)\,e^{xy}$ d) $(1 + x)\,e^{xy}$

17. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

18. If $\int_0^a \frac{1}{4+x^2}\,dx=\frac{\pi}{8}$ then a is

a) 4 b) 1 c) 3 d) 2

19. Which of the following is not correct?

a) $P\vee\mathbb{T}\equiv\mathbb{T}$ b) $P\vee\mathbb{F}\equiv P$ c) $P\wedge\mathbb{T}\equiv\mathbb{T}$ d) $P\wedge\neg P\equiv F$

20. Subtraction is not a binary operation in

a) R b) Z c) N d) Q

Part - II

II. **Answer any 7 questions. (Q.No.30 is compulsory)** **7 x 2 = 14**

21. If A is a non-singular matrix of odd power. Prove that |adj A| is positive.

22. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$. Construct a quadratic equation whose roots are α^2 and β^2.

23. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$
24. Determine whether the point (–2,1) lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$
25. Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x - y + z = 5$
26. Evaluate : $\lim_{x\to\pi/2}\frac{\sec x}{\tan x}$
27. If $\omega(x,y) = xy+\frac{e^y}{y^2+1}$ for all $(x,y)\in R^2$, calculate $\frac{\partial^2\omega}{\partial y\,\partial x}$
28. Evaluate : $\int_0^{\pi/2}\sin^{10}x\,dx$
29. If $A=\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, find $A\wedge B$
30. For any complex number Z with conjugate part, prove that $z^n-(\bar{z})^n$ is purely imaginary.

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) 7 x 3 = 21

31. Solve by matrix inversion method : $2x + 5y = -2$, $x + 2y = -3$
32. Simplify : $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$
33. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.
34. Find $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$
35. Find the equation of tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{64}=1$ which are parallel to $10x - 3y + 9 = 0$
36. With usual notations, in any triangle ABC, prove by vector method that : $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
37. Find the absolute extrema of $f(x) = x^2 - 12x + 10$ in $[1,2]$
38. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T=2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.
39. Prove that $\neg(\neg p)\equiv p$
40. Evaluate : $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

Part - IV

IV. Answer all the questions. 7 x 5 = 35

41. a) Determine the values of λ for which the following system of equations.
$x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$
(i) a unique solution (ii) a non-trivial solution

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

42. a) Find all zeros of polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

(OR)

b) If $a_1, a_2, a_3, \ldots\ldots\ldots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_n a_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1 a_n}$

43. a) Identify the type of conic and find centre, foci, vertices
$18x^2 + 12y^2 - 144x + 48y + 120 = 0$

(OR)

b) Prove by vector method that the perpendiculars (altitude) from the vertices to the opposite sides of a triangle are concurrent.

44. a) Find the non-parametric form of vector equation and cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

(OR)

b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally, then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

45. a) At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at horizontal distance of 0.75 m from the point of origin.

(OR)

b) Evaluate : $\int_0^{\pi/2} \frac{e^{-\tan x}}{\cos^6 x} dx$

46. a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

(OR)

b) Find the area of region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$

47. a) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

(OR)

b) Using truth table, prove that $q \to p \equiv \neg p \to \neg q$

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