Part – II

(2 Mark Questions)

EXERCISE 1.1

1. Find the adjoint of the following:

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following:

(i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

9. If $\operatorname{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

EXERCISE 1.2

1. Find the rank of the following matrices by minor method: (each 2 marks)

(i) $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

EXERCISE 1.7

3. By using Gaussian elimination method, balance the chemical reaction equations:

$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

Example 1.4

If A is a nonsingular matrix of odd order, prove that $|\operatorname{adj} A|$ is positive.

Example 1.7

If A is symmetric, prove that then $\operatorname{adj} A$ is also symmetric.

Example 1.11

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Example 1.16

Find the rank of the following matrices which are in row-echelon form : (each 2 marks)

(i) $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Example 1.39

By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.

Chapter 2
Complex Numbers

EXERCISE 2.1

Simplify the following: (each 2 marks)

(1) $i^{1947} + i^{1950}$ (2) $i^{1948} - i^{-1869}$ (3) $\sum_{n=1}^{12} i^n$

(4) $i^{59} + \frac{1}{i^{59}}$ (5) $i.i^2.i^3 ... i^{2000}$ (6) $\sum_{n=1}^{10} i^{n+50}$

EXERCISE 2.2

1. Evaluate the following if $z = 5-2i$ and $w = -1+3i$ (each 2 marks)

(i) $z+w$ (ii) $z-iw$ (iii) $2z+3w$

(iv) zw (v) $z^2+2zw+w^2$ (vi) $(z+w)^2$

EXERCISE 2.4

1. Write the following in the rectangular form: (each 2 marks)

(i) $\overline{(5+9i)+(2-4i)}$ (ii) $\frac{10-5i}{6+2i}$ (iii) $\overline{3i} + \frac{1}{2-i}$

2. If $z = x+iy$, find the following in cartesian form. (each 2 marks)

(i) $\text{Re}\left(\frac{1}{z}\right)$ (ii) $\text{Re}(i\bar{z})$ (iii) $\text{Im}(3z+4\bar{z}-4i)$

3. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverses of z_1z_2 and $\frac{z_1}{z_2}$. (each 2 marks)

5. Prove the following properties: (each 2 marks)

(i) z is real if and only if $z = \bar{z}$ (ii) $\text{Re}(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$

7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

EXERCISE 2.5

1. Find the modulus of the following complex numbers (each 2 marks)

(i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$

10. Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$. (each 2 marks)

EXERCISE 2.6

4. Show that the following equations represent a circle, and find its centre and radius. (each 2 marks)

(i) $|z-2-i|=3$ (ii) $|2z+2-4i|=2$ (iii) $|3z-6+12i|=8$

EXERCISE 2.8

9. If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when (each 2 marks)

(i) $\theta=\frac{\pi}{3}$ (ii) $\theta=\frac{2\pi}{3}$ (iii) $\theta=\frac{3\pi}{2}$

Example 2.1

Simplify the following. (each 2 marks)

(i) i^7 (ii) i^{1729} (iii) $i^{-1924}+i^{2018}$ (iv) $\sum_{n=1}^{102} i^n$ (vi) $i\, i^2\, i^3 \cdots i^{40}$

Example 2.3

Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find the real and imaginary parts.

Example 2.5

If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in the rectangular form.

Example 2.6

If $z_1=3-2i$ and $z_2=6+4i$ find $\frac{z_1}{z_2}$ in the rectangular form.

Example 2.7

Find z^{-1}, if $z=(2+3i)(1-i)$.

Example 2.8

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real.

Example 2.9 (each 2 marks)

If $z_1=3+4i$, $z_2=5-12i$, $z_3=6+8i$, find $|z_1|, |z_2|, |z_3|, |z_1+z_2|, |z_2-z_3|$, and $|z_1+z_3|$.

Example 2.10 (each 2 marks)

Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$.

Example 2.17

Find the square root of $6-8i$.

Example 2.22 (each 2 marks)

Find the modulus and principal argument of the following complex numbers.

(i) $\sqrt{3}+i$ (ii) $-\sqrt{3}+i$ (iii) $-\sqrt{3}-i$ (iv) $\sqrt{3}-i$

Example 2.24

Find the principal argument $Arg\, z$, when $z=\frac{-2}{1+i\sqrt{3}}$.

Example 2.25

Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular from.

Example 3.1

If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

Example 3.2

If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

Example 3.3

If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

Example 3.11

Show that the equation $2x^2-6x+7=0$ cannot be satisfied by any real values of x.

Example 3.12

If $x^2+2(k+2)x+9k=0$ has equal roots, find k.

EXERCISE 4.1

2. Find the period and amplitude of

(i) $y=\sin 7x$ (ii) $y=-\sin\left(\frac{1}{3}x\right)$ (iii) $y=4\sin(-2x)$. (each 2 marks)

4. Find the values of (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$. (each 2 marks)

5. For what value of x does $\sin x=\sin^{-1}x$?

EXERCISE 4.2

2. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$.

3. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

4. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

8. Find the value of

(i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$.

EXERCISE 4.3

1. Find the domain of the following functions : (each 2 marks)

(i) $\tan^{-1}\left(\sqrt{9-x^2}\right)$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$.

2. Find the value of (i) $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ (ii) $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$. (each 2 marks)

3. Find the value of (i) $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ (ii) $\tan\left(\tan^{-1}(1947)\right)$

(iii) $\tan\left(\tan^{-1}(-0.2021)\right)$ (each 2 marks)

EXERCISE 4.4

1. Find the principal value of

(i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (ii) $\cot^{-1}\left(\sqrt{3}\right)$ (iii) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ (each 2 marks)

2. Find the value of

(i) $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$ (ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$ (each 2 marks)

EXERCISE 4.5

1. Find the value, if it exits. If not, give the reason for non-existence. (each 2 marks)

(i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}\left(\sin\left(-\frac{5\pi}{2}\right)\right)$ (iii) $\sin^{-1}[\sin 5]$.

Example 4.1

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (in radians and degrees).

Example 4.2

Find the principal value of $\sin^{-1}(2)$, if it exists.

Example 4.3

Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$. (each 2 marks)

Example 4.5

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Example 4.6

Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$. (each 2 marks)

Example 4.8

Find the principal value of $\tan^{-1}(\sqrt{3})$.

Example 4.9

Find (i) $\tan^{-1}(-\sqrt{3})$ (ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ (iii) $\tan\left(\tan^{-1}(2019)\right)$. (each 2 marks)

Example 4.12

Find the principal value of

(i) $\operatorname{cosec}^{-1}(-1)$ (ii) $\sec^{-1}(-2)$. (each 2 marks)

Example 4.13

Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

Example 4.18

Find the value of (i) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ (each 2 marks)

EXERCISE 5.1

5. Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter.

10. Determine whether the points $(-2,1),(0,0)$, and $(-4,-3)$ lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$. (each 2 marks)

11. Find the centre and radius of the following circles: (each 2 marks)

(i) $x^2+(y+2)^2=0$ (ii) $x^2+y^2+6x-4y+4=0$

(iii) $x^2+y^2-x+2y-3=0$ (iv) $2x^2+2y^2-6x+4y+2=0$

EXERCISE 5.3

Identify the type of conic sections. (each 2 marks)

(1) $2x^2-y^2=7$ (2) $3x^2+3y^2-4x+3y+10=0$ (3) $3x^2+2y^2=14$

(4) $x^2+y^2+x-y=0$ (5) $11x^2-25y^2-44x+50y-256=0$ (6) $y^2+4x+3y+4=0$

Example 5.1

Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units.

Example 5.3

Determine whether $x+y-1=0$ is the equation of a diameter of the circle $x^2+y^2-6x+4y+c=0$ for all possible values of c.

Example 5.4

Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$.

Example 5.5

Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

Example 5.25

Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

Example 5.27

The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity.

Example 5.28

Identify the type of the conic for the following equations: (each 2 marks)

(i) $16y^2=-4x^2+64$ (ii) $x^2+y^2=-4x-y+4$

(iii) $x^2-2y=x+3$ (iv) $4x^2-9y^2-16x+18y-29=0$

EXERCISE 6.2

1. If $a=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-2\hat{k}, \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.
2. Find the volume of the parallelopiped whose coterminous edges are represented by the vectors $-6\hat{i}+14\hat{j}+10\hat{k}, 14\hat{i}-10\hat{j}-6\hat{k}$ and $2\hat{i}+4\hat{j}-2\hat{k}$.
3. The volume of the parallelopiped whose coterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ.
6. Determine whether the three vectors $2\hat{i}+3\hat{j}+\hat{k}, \hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.
7. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\vec{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar.
8. If $\vec{a}=\hat{i}-\hat{k}, \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}, \vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, show that $\left[\vec{a}, \vec{b}, \vec{c}\right]$ independent of x and y.
9. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, prove that c is the geometric mean of a and b.

EXERCISE 6.3

1. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find (i) $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ (ii) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, (each 2 marks)

2. For any vector $\vec{a}$, prove that $\hat{i}\times\left(\hat{a}\times\hat{i}\right)+\hat{j}\times\left(\vec{a}\times\hat{j}\right)+\hat{k}\times\left(\vec{a}\times\hat{k}\right)=2\vec{a}$.

EXERCISE 6.4

5. Find the angle between the following lines: (each 2 marks)

(i) $\vec{r}=\left(4\hat{i}-\hat{j}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right), \vec{r}=\left(\hat{i}-2\hat{j}+4\hat{k}\right)+s\left(-\hat{i}-2\hat{j}+2\hat{k}\right)$

(ii) $\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}$ and $\vec{r}=4\hat{k}+t\left(2\hat{i}+\hat{j}+\hat{k}\right)$.

(iii) $2x=3y=-z$ and $6x=-y=-4z$.

EXERCISE 6.6

5. Find the intercepts cut off by the plane $\vec{r}\cdot\left(6\hat{i}+4\hat{j}-3\hat{k}\right)=12$ on the coordinate axes.

EXERCISE 6.9

6. Find the distance (length of the perpendicular) from the point $(1,-2,3)$ to the plane $x-y+z=5$.

Example 6.12

If $\vec{a}=-3\hat{i}-\hat{j}+5\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{c}=4\hat{j}-5\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.

Example 6.13

Find the volume of the parallelepiped whose coterminus edges (adjacent edges) are given by the vectors $2\hat{i}-3\hat{j}+4\hat{k}, \hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$.

Example 6.14

Show that the vectors $\hat{i}+2\hat{j}-3\hat{k}, 2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

Example 6.15

If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

Example 6.32

Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel.

Example 6.39

If the Cartesian equation of a plane is $3x-4y+3z=-8$, find the vector equation of the plane in the standard form.

Example 6.45

Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$.

Example 6.47

Find the acute angle between the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=11$ and $4x-2y+2z=15$.

Example 6.48

Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the pl $2x-y+z=5$.

Example 6.49

Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=5$.

Example 6.51

Find the distance between the parallel planes $x+2y-2z+1=0$ and $2x+4y-4z+5=0$.

EXERCISE 7.1

4. If the volume of a cube of side length x is $V=x^3$. Find the rate of change of the volur with respect to x when $x=5$ units.

6. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is cm find the rate of changing of the total area of the disturbed water?

EXERCISE 7.2

1. Find the slope of the tangent to the curves at the respective given points. (each 2 marks)

   (i) $y=x^4+2x^2-x$ at $x=1$ (ii) $x=a\cos^3 t, y=b\sin^3 t$ at $t=\frac{\pi}{2}$.

EXERCISE 7.3

1. Explain why Rolle's theorem is not applicable to the following functions in the respectiv intervals. (each 2 marks)

   (i) $f(x)=\left|\frac{1}{x}\right|,\ x\in[-1,1]$ (ii) $f(x)=\tan x, x\in[0,\pi]$

   (iii) $f(x)=x-2\log x,\ x\in[2,7]$

3. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals : (each 2 marks)

   (i) $f(x)=\frac{x+1}{x},\ x\in[-1,2]$ (ii) $f(x)=|3x+1|, x\in[-1,3]$

Example 7.2

The temperature in celsius in a long rod of length $10m$, insulated at both ends, is a function of length x given by $T=x(10-x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

Example 7.5 (each 2 marks)

A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s=128t-16t^2$.

(i) Compute the maximum height of the particle reached?

(ii) What is the velocity when the particle hits the ground?

Example 7.33

Evaluate : $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

Example 7.34

Compute the limit $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)$.

Example 7.35

Evaluate : $\lim_{x\to 0}\left(\frac{\sin mx}{x}\right)$.

Example 7.36

Evaluate : $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$.

Example 7.47

Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2,\infty)$.

Example 7.50

Find the intervals of monotonicity and hence find the local extrema for the function $f(x)=x^2-4x+4$.

EXERCISE 8.2

1. Find the differentials dy for each of the following functions: (each 2 marks)

   (i) $y=\frac{(1-2x)^3}{3-4x}$ (ii) $y=3(3+\sin(2x))^{2/3}$ (iii) $y=e^{x^2-5x+7}\cos(x^2-1)$

2. Find df for $f(x)=x^2+3x$ and evaluate it for

   (i) $x=2$ and $dx=0.1$ (ii) $x=3$ and $dx=0.02$ (each 2 marks)

6. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5mm and radius to the outside of the shell is 5.3mm, find the volume of the shell approximately.

7. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm, how much is cross-sectional area increased approximately?

9. The relation between number of words y a person learns in x hours is given by $y=52\sqrt{x}, 0\le x\le 9$. What is the approximate number of words learned when x changes from

   (i) 1 to 1.1 hour? (ii) 4 to 4.1 hour? (each 2 marks)

EXERCISE 8.3

1. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y)=\frac{3x^2-xy}{x^2+y^2+3}$.

2. Evaluate $\lim_{(x,y)\to(0,0)}\cos\left(\frac{x^3+y^2}{x+y+2}\right)$, if the limit exists.

3. Let $f(x,y)=\dfrac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for $(x,y)\neq(0,0)$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

4. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{e^x \sin y}{y}\right)$, if the limit exists.

EXERCISE 8.4

10. A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are $R(x,y)=80x+90y+0.04xy-0.05x^2-0.05y^2$ and $C(x,y)=8x+6y+2000$ respectively.

(i) Find the profit function $P(x,y)$,

(ii) Find $\dfrac{\partial P}{\partial x}(1200,1800)$ and $\dfrac{\partial p}{\partial y}(1200,1800)$.

Remark : Revenue function, cost function and profit functions are not defined.

EXERCISE 8.7

1. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree. (each 2 marks)

(i) $f(x,y)=x^2y+6x^3+7$

(ii) $h(x,y)=\dfrac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$

(iii) $g(x,y,z)=\dfrac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$

(iv) $U(x,y,z)=xy+\sin\left(\dfrac{y^2-2z^2}{xy}\right)$

Example 8.6

Let $g(x)=x^2+\sin x$. Calculate the differential dg.

Example 8.21

Show that $F(x,y)=\dfrac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

EXERCISE 9.3

1. Evaluate the following definite integrals :

(i) $\displaystyle\int_3^4 \frac{dx}{x^2-4}$

EXERCISE 9.6

1. Evaluate the following:

(i) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$ (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$

EXERCISE 9.7

1. Evaluate the following

(i) $\displaystyle\int_0^{\infty} x^5 e^{-3x}\, dx$

EXERCISE 9.8

1. Find the area of the region bounded by $3x-2y+6=0$, $x=-3$, $x=1$ and x-axis.

Example 9.7

Evaluate : $\int_0^{\frac{1}{2}} [2x]dx$ where $[\cdot]$ is the greatest integer function.

Example 9.20

Show that $\int_0^{\pi} g(\sin x)dx = 2\int_0^{\frac{\pi}{2}} g(\sin x)dx$, where $g(\sin x)$ is a function of $\sin x$.

Example 9.22

Show that $\int_0^{2\pi} g(\cos x)dx = 2\int_0^{\pi} g(\cos x)dx$, where $g(\cos x)$ is a function of $\cos x$.

Example 9.24

Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \, dx.$

Example 9.25

Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|} dx$.

Example 9.37

Evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^2 x+\cos^4 x\right)dx$

Example 9.47

Find the area of the region bounded by the line $6x+5y=30$, $x-$axis and the lines $x=-1$ and $x=3$.

EXERCISE 10.3

8. Find the differential equation of the curve represented by $xy=ae^{x}+be^{-x}+x^2$. Here a and b are arbitrary constants.

EXERCISE 10.4

1. Show that each of the following expressions is a solution of the corresponding given differential equation. (each 2 marks)

   (i) $y=2x^2$; $xy'=2y$

   (ii) $y=ae^{x}+be^{-x}$; $y''-y=0$

2. Find value of m so that the function $y=e^{mx}$ is a solution of the given differential equation. (each 2 marks)

   (i) $y'+2y=0$ (ii) $y''-5y'+6y=0$

4. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

8. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.

Example 10.2

Find the differential equation for the family of all straight lines passing through the origin.

Example 10.3

Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

Example 10.5

Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

Example 10.7

Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

Example 10.8

Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.

Example 10.22

Solve $\frac{dy}{dx} + 2y = e^{-x}$.

EXERCISE 11.3

2. The probability density function of X is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$ (each 2 marks)

Find (i) $P(0.2 \le X < 0.6)$ (ii) $P(1.2 \le X < 1.8)$ (iii) $P(0.5 \le X < 1.5)$

4. The probability density function of X is given by $f(x) = \begin{cases} ke^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ (each 2 marks)

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$
(iv) $P(5 \le X)$ (v) $P(X \le 4)$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$

then find (i) the probability density function $f(x)$

EXERCISE 11.4

1. For the random variable X with the given probability mass function as below, find the mean and variance. (each 2 marks)

(i) $f(x)=\begin{cases}\frac{1}{10} & x=2,5\\ \frac{1}{5} & x=0,1,3,4\end{cases}$ (ii) $f(x)=\begin{cases}\frac{4-x}{6} & x=1,2,3\end{cases}$

(iii) $f(x)=\begin{cases}2(x-1) & 1<x<2\\ 0 & \text{otherwise}\end{cases}$ (iv) $f(x)=\begin{cases}\frac{1}{2}e^{-\frac{x}{2}} & \text{for } x>0\\ 0 & \text{otherwise}\end{cases}$

6. The time to failure in thousand hours of an electronic equipment used in a manufactured computer has the density function

$$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

Find the expected life of this electronic equipment.

EXERCISE 11.5

1. Compute $P(X=k)$ for the binomial distribution, $B(n,p)$ where

(i) $n=6, p=\frac{1}{3}, k=3$ (ii) $n=10,\ p=\frac{1}{5},\ k=4$ (iii) $n=9,\ p=\frac{1}{2},\ k=7$ (each 2 marks)

3. Using binomial distribution find the mean and variance of X for the following experiments
   (i) A fair coin is tossed 100 times, and X denote the number of heads.
   (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.
   (each 2 marks)

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

Example 11.6

A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

Example 11.21 (each 2 marks or entire question 5 marks)

The mean and variance of a binomial variate X are respectively 2 and 1.5. Find
(i) $P(X=0)$ (ii) $P(X=1)$ (iii) $P(X\geq 1)$

EXERCISE 12.1

1. Determine whether $*$ is a binary operation on the sets given below:
   (i) $a*b=a\cdot|b|$ on $\mathbb{R}$ (ii) $a*b=\min(a,b)$ on $A=\{1,2,3,4,5\}$
   (iii) $(a*b)=a\sqrt{b}$ on $\mathbb{R}$. (each 2 marks)

2. On $\mathbb{Z}$, define $\otimes$ by $(m\otimes n)=m^n+n^m:\forall m,n\in\mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

3. Let $*$ be defined on $\mathbb{R}$ by $(a*b)=a+b+ab-7$. Is $*$ binary on $\mathbb{R}$? If so, find $3*\left(\frac{-7}{15}\right)$.

4. Let $A=\{a+\sqrt{5}b : a,b\in\mathbb{Z}\}$. Check whether the usual multiplication is a binary operation A.

6. Fill in the following table so that the binary operation $*$ on $A=\{a,b,c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

7. Consider the binary operation $*$ defined on the set $A=\{a,b,c,d\}$ by the following table :

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is it commutative and associative?

8. Let $A=\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix}$, $B=\begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix}$, $C=\begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$ be any three Boolea matrices of the same type. Find (i) $A\vee B$ (ii) $A\wedge B$ (iii) $(A\vee B)\wedge C$ (iv) $(A\wedge B)\vee C$. (each 2 marks)

EXERCISE 12.2

1. Let p : Jupiter is a planet and
   q : India is an island be any two simple statements. Give verbal sentence describing eacl of the following statements. (each 2 marks)
   (i) $\neg p$ (ii) $p\wedge\neg q$ (iii) $\neg p\vee q$ (iv) $p\rightarrow\neg q$ (v) $p\leftrightarrow q$
2. Write each of the following sentences in symbolic form using statement variable: p and q.
   (i) 19 is not a prime number and all the angles of a triangle are equal.
   (ii) 19 is a prime number or all the angles of a triangle are not equal
   (iii) 19 is a prime number and all the angles of a triangle are equal
   (iv) 19 is not a prime number (any two, 2 marks)
3. Determine the truth value of each of the following statements
   (i) If $6+2=5$, then the milk is white.
   (ii) China is in Europe or $\sqrt{3}$ is an integer
   (iii) It is not true that $5+5=9$ or Earth is a planet
   (iv) 11 is a prime number and all the sides of a rectangle are equal (any two, 2 marks)
4. Which one of the following sentences is a proposition? (any two, 2 marks)
   (i) $4+7=12$ (ii) What are you doing? (iii) $3^n\leq 81, n\in\mathbb{N}$
   (iv) Peacock is our national bird (v) How tall this mountain is!

5. Write the converse, inverse, and contrapositive of each of the following implication.

(i) If x and y are numbers such that $x = y$, then $x^2 = y^2$.

(ii) If a quadrilateral is a square then it is a rectangle (each 2 marks)

Example12.1

Examine the closure property for $a * b = a + 3ab - 5b^2; \forall a, b \in \mathbb{Z}$

Example 12.8

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

Example 12.11

Identify the valid statements from the following sentences (for two sentences – 2 marks).

Example 12.12 (each 2 marks)

Write the statements in words corresponding to $\neg p, p \wedge q, p \vee q$ and $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining.'

Example 12.13 (each 2 marks)

How many rows are needed to form truth tables for the following compound statements ?

(i) $p \vee \neg t \wedge (p \vee \neg s)$ (ii) $((p \wedge q) \vee (\neg r \vee \neg s)) \wedge (\neg t \wedge v)$

Example 12.15 (each 2 marks)

Write down the (i) conditional statement (ii) converse statement (iii) inverse statement, and (iv) contrapositive statement for the two statements p and q given below.

p : The number of primes is infinite. q : Ooty is in Kerala.

Example 12.17

Establish the equivalence property: $p \to q \equiv \neg p \vee q$.