Part – III
(3 Mark Questions)

EXERCISE 1.1

1. Find the adjoint of the following: (each 3 marks)

(ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following: (each 3 marks)

(ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

4. If $A=\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2-3A-7I_2=O$. Hence find A^{-1}.

5. If $A=\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1}=A^T$.

6. If $A=\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA)=(adjA)A=|A|I$.

7. If $A=\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

8. If $\text{adj}(A)=\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

10. Find $adj(adj(A))$ if $adjA=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

11. $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1}=\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

12. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}=\begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

13. Given $A=\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB=C$.

EXERCISE 1.2

1. Find the rank of the following matrices by minor method: (each 3 marks)

(iv) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ (v) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

2. Find the rank of the following matrices by row reduction method: (each 3 marks)

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(i) $2x+5y=-2,\ \ x+2y=-3$ (ii) $2x-y=8,\ \ 3x+2y=-2$ (each 3 marks)

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule: (each 3 marks)

(i) $5x-2y+16=0, x+3y-7=0$

(ii) $\frac{3}{x}+2y=12, \frac{2}{x}+3y=13$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ?

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

(iii) $2x+2y+z=5,\quad x-y+z=1,\quad 3x+y+2z=4$

Example 1.3

Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

Example 1.5

Find a matrix A if $\text{adj}(A)=\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

Example 1.6

If $\operatorname{adj} A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}

Example 1.8

Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

Example 1.9

Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

Example 1.10

If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O$ Hence, find A^{-1}.

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Example 1.13

Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

Example 1.14

Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

Example 1.15 (each 3 marks)

Find the rank of each of the following matrices:

(i) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$.

Example 1.17

Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

Example 1.18

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

Example 1.19

Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

Example 1.22

Solve the following system of linear equations, using matrix inversion method:

$5x+2y=3,\ 3x+2y=5$.

EXERCISE 2.2

2. Given the complex number $z=2+3i$, represent the complex numbers (i) z, iz, and $z+iz$ (ii) $z, -iz$, and $z-iz$ in Argand diagram. (each 3 marks)

3. Find the values of the real numbers x and y, if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal

EXERCISE 2.3

1. If $z_1=1-3i,\ z_2=-4i$, and $z_3=5$, show that (each 3 marks)

   (i) $(z_1+z_2)+z_3=z_1+(z_2+z_3)$ (ii) $(z_1z_2)z_3=z_1(z_2z_3)$

2. If $z_1=3,\ z_2=-7i$, and $z_3=5+4i$, show that (each 3 marks)

   (i) $z_1(z_2+z_3)=z_1z_2+z_1z_3$ (ii) $(z_1+z_2)z_3=z_1z_3+z_2z_3$

3. If $z_1=2+5i,\ z_2=-3-4i,\ z_3=1+i$, find the additive and multiplicative inverse of $z_1,\ z_2$, and z_3 (each 3 marks)

EXERCISE 2.4

4. The complex numbers u, v, and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$.

   If $v=3-4i$ and $w=4+3i$, find u in rectangular form.

6. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$

   (i) real (ii) purely imaginary

7. Show that

   (ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real

EXERCISE 2.5

2. For any two complex numbers z_1 and z_2, such that $|z_1|=|z_2|=1$ and $z_1z_2 \neq -1$, then show that $\dfrac{z_1+z_2}{1+z_1z_2}$ is real number.

3. Which one of the points $10-8i, 11+6i$ is closed to $1+i$.

4. If $|z|=3$, show that $7 \leq |z+6-8i| \leq 13$.

5. If $|z|=1$, show that $2 \leq |z^2-3| \leq 4$.

6. If $|z|=2$, show that $8 \leq |z+6+8i| \leq 12$.

8. If the area of the triangle formed by the vertices z, iz, and $z+iz$ is 50 square units, find the value of $|z|$.

9. Show that the equation $z^3+2\bar{z}=0$ has five solutions.

EXERCISE 2.6

1. If $z=x+iy$ is a complex number such that $\left|\dfrac{z-4i}{z+4i}\right|=1$ show that the locus of z is real axis.

3. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases:

(i) $\left[\operatorname{Re}(iz)\right]^2=3$ (ii) $\operatorname{Im}[(1-i)z+1]=0$ (iii) $|z+i|=|z-1|$ (iv) $\bar{z}=z^{-1}$

(each 3 mar

5. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases:

(i) $|z-4|=16$ (ii) $|z-4|^2+|z-1|^2=16$

EXERCISE 2.7

1. Write in polar form of the following complex numbers (each 3 marks)

(i) $2+i2\sqrt{3}$ (ii) $3-i\sqrt{3}$ (iii) $-2-i2$ (iv) $\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$

2. Find the Cartesian form of the complex numbers (each 3 marks)

(i) $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$ (ii) $\dfrac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$

4. If $\dfrac{1+z}{1-z}=\cos 2\theta+i\sin 2\theta$, show that $z=i\tan\theta$.

EXERCISE 2.8

1. If $\omega \neq 1$ is a cube root of unity, show that $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+cw^2}{c+a\omega+b\omega^2}=-1$.

2. Show that $\left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^5+\left(\dfrac{\sqrt{3}}{2}-\dfrac{i}{2}\right)^5=-\sqrt{3}$.

7. Find the value of $\sum_{k=1}^{8}\left(\cos\dfrac{2k\pi}{9}+i\sin\dfrac{2k\pi}{9}\right)$.

8. If $\omega \neq 1$ is a cube root of unity, show that (each 3 marks)

(i) $\left(1-\omega+\omega^2\right)^6+\left(1+\omega-\omega^2\right)^6=128$

(ii) $(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\cdots\left(1+\omega^{2^{11}}\right)=1$

Example 2.2

Find the value of the real numbers x and y, if the complex numbers $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal

Example 2.4

Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

Example 2.8

Show that (ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

Example 2.11

Which one of the points $i, -2+i$ and 3 is farthest from the origin?

Example 2.13

If $|z|=2$ show that $3 \leq |z+3+4i| \leq 7$.

Example 2.14

Show that the points 1, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Example 2.16

Show that the equation $z^2=\bar{z}$ has four solutions.

Example 2.19

Show that $|3z-5+i|=4$, represents a circle, and find its centre and radius.

Example 2.20

Show that $|z+2-i|<2$ represents interior points of a circle. Find the centre and radius.

Example 2.21 (each 3 marks)

Obtain the Cartesian equation for the locus of z in each of the following cases.

(i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$.

Example 2.23

Represent the complex number (i) $-1-i$ (ii) $1+i\sqrt{3}$ in polar form.

Example 2.26

Find the quotient $\dfrac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

Example 2.28 (each 3 marks)

If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$.

Example 2.29

Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

Example 2.31

Simplify (i) $(1+i)^{18}$

EXERCISE 3.1

1. If the sides of a cubic box are increased by $1,2,3$ units respectively to form a rectangu cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
2. Construct a cubic equation with roots (each 3 marks)

   (i) 1,2, and 3 (ii) 1,1, and -2 (iii) $2,\frac{1}{2}$ and 1.

3. If α,β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equati whose roots are

   (i) $2\alpha,2\beta,2\gamma$ (ii) $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$ (iii) $-\alpha,-\beta,-\gamma$ (each 3 marks)

7. If $\alpha,\beta,$ and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the valu of $\sum\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.
8. If α,β,γ and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.
11. A 12 metre tall tree was broken into two parts. It was found that the height of the part whic was left standing was the cube root of the length of the part that was cut away. Formulate th into a mathematical problem to find the height of the part which was cut away.

EXERCISE 3.2

2. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ a a root.
3. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as root.
4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{}$ as a root.

EXERCISE 3.5

1. Solve the following equations:

   (i) $\sin^2 x-5\sin x+4=0$

EXERCISE 3.6

3. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.
5. Find the exact number of real zeros and imaginary zeros of the polynomia $x^9+9x^7+7x^5+5x^3+3x$.

Example 3.4

Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0,\ a \neq 0$.

Example 3.8

Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.

Example 3.9

Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

Example 3.10

Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

Example 3.13

Show that, if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

Example 3.25

Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$.

Example 3.29

Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$.

Example 3.30

Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary zeros.

EXERCISE 4.1

1. Find all the values of x such that

   (i) $-10\pi \leq x \leq 10\pi$ and $\sin x = 0$ (ii) $-3\pi \leq x \leq 3\pi$ and $\sin x = -1$. (each 3 marks)

3. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \leq x < 6\pi$.

6. Find the domain of the following

   (i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$. (each 3 marks)

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

EXERCISE 4.2

1. Find all values of x such that

   (i) $-6\pi \leq x \leq 6\pi$ and $\cos x = 0$ (ii) $-5\pi \leq x \leq 5\pi$ and $\cos x = -1$. (each 3 marks)

5. Find the value of

(i) $2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(-1)$

(iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$. (each 3 marks)

6. Find the domain of

(ii) $g(x)=\sin^{-1}x+\cos^{-1}x$.

7. For what values of x, the inequality $\frac{\pi}{2}<\cos^{-1}(3x-1)<\pi$ holds?

8. Find the value of

(ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

EXERCISE 4.3

4. Find the value of

(i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$

(iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$. (each 3 marks)

EXERCISE 4.4

2. Find the value of

(iii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$.

EXERCISE 4.5

2. Find the value of the expression in terms of x, with the help of a reference triangle.

(i) $\sin\left(\cos^{-1}(1-x)\right)$ (ii) $\cos\left(\tan^{-1}(3x-1)\right)$ (iii) $\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$. (each 3 mar

3. Find the value of

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$

(iii) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$. (each 3 marks)

4. Prove that

(i) $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$. (ii) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$ (each 3 marks)

8. Simplify : $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$.

Example 4.4

Find the domain of $\sin^{-1}(2-3x^2)$.

Example 4.7

Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

Example 4.10

Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$.

Example 4.14

If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

Example 4.15

Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x$, $|x|>1$.

Example 4.16

Prove that $\frac{\pi}{2}\le\sin^{-1}x+2\cos^{-1}x\le\frac{3\pi}{2}$.

Example 4.17 (each 3 marks)

Simplify (i) $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$ (ii) $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

(iii) $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$ (iv) $\sin^{-1}[\sin 10]$

Example 4.19

Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

Example 4.21

Prove that (i) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$ (ii) $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

Example 4.25

Solve $\sin^{-1}x>\cos^{-1}x$.

Example 4.26

Show that $\cot(\sin^{-1}x)=\frac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\ne 0$.

Example 4.27

Solve : $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$.

EXERCISE 5.1

1. Obtain the equation of the circles with radius 5 cm and touching x-axis at the origin in general form.

2. Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ standard form.

3. Find the equations of circles that touch both the axes and pass through $(-4,-2)$ in gene form.

4. Find the equation of the circles with centre $(2,3)$ and passing through the intersection of lines $3x-2y-1=0$ and $4x+y-27=0$.

7. A circle of area 9π square units has two of its diameters along the lines $x+y=5$ a $x-y=1$. Find the equation of the circle.

8. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.

12. If the equation $3x^2+(3-p)xy+qy^2-2px-8pq=0$ represents a circle, find p and q. A determine the centre and radius of the circle.

EXERCISE 5.2

1. Find the equation of the parabola in each of the cases given below : (each 3 marks)
   (i) focus $(4,0)$ and directrix $x=-4$.
   (ii) passes through $(2,-3)$, symmetric about y axis, open downwards and the vertex $(0,0$
   (iii) Vertex $(1-2)$ and focus $(4,-2)$.
   (iv) end points of latus rectum are $(4,-8),(4,8)$, open rightwords and the vertex $(0,0)$.

2. Find the equation of the ellipse in each of the cases given below : (each 3 marks)
   (i) Foci $(\pm 3,0)$, $e=\frac{1}{2}$
   (ii) foci $(0,\pm 4)$ and end points of major axis are $(0,\pm 5)$.
   (iii) length of latus rectum 8, eccentricity $=\frac{3}{5}$, major axis on x-axis and the centre $(0,0)$.
   (iv) length of latus rectum 4, distance between foci $4\sqrt{2}$, major axis as y-axis and tl centre $(0,0)$.

3. Find the equation of the hyperbola in each of the cases given below: (each 3 marks)
   (i) Foci $(\pm 2,0)$, $e=\frac{3}{2}$
   (ii) Centre $(2,1)$, one of the foci $(8,1)$ and corresponding directrix $x=4$.
   (iii) Passing through $(5,-2)$, length of the transverse axis along x-axis and of length units and the centre is $(0,0)$.

4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
   (i) $y^2=16x$ (ii) $x^2=24y$ (iii) $y^2=-8x$ (each 3 marks)

6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

7. Show that the absolute value of difference of the focal distances of any point P on th hyperbola is the length of its transverse axis.

Example 5.2

Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

Example 5.6

The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

Example 5.7

A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form.

Example 5.8

A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.

Example 5.9

Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

Example 5.11

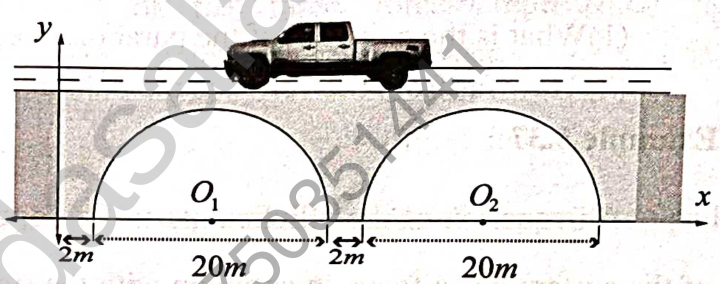
Find the equations of the tangent and normal to the circle $x^2+y^2=25$ at $P(-3,4)$.

Example 5.12

If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$ find c,

Example 5.13

A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$ use the figure to write the equations that model the arches.



Example 5.14

Find the Latus rectum of the parabola $y^2=4ax$.

Example 5.15

Find the Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Example 5.16

Find the equation of the parabola with focus $\left(-\sqrt{2},0\right)$ and directrix $x=\sqrt{2}$.

Example 5.17

Find the equation of the parabola whose vertex is $(5,-2)$ and focus $(2,-2)$.

Example 5.18

Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through $(3,6)$.

Example 5.20

Find the equation of the ellipse with foci $(\pm 2,0)$, vertices $(\pm 3,0)$.

directrix is $x=7$. Find the length of the major and minor axes of the ellipse.

Example 5.24

Find the equation of the hyperbola with vertices $(0,\pm 4)$ and foci $(0,\pm 6)$.

Example 5.32

The maximum and minimum distances of the Earth from the Sun respectively are $152\times$ km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the to the other focus.

Example 5.33

A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and maximum height of the arch is $15m$. Write an equation of the parabolic arch.

Example 5.34

The parabolic communications antenna has a focus at $2m$ distance from the vertex of antenna. Find the width of the antenna $3m$ from the vertex.

Example 5.35

The cross section of a parabolic mirror is the equation $y=\frac{1}{32}x^2$ that are used for sc energy. There is a heating tube located at the focus of each parabola; how high is this tube loca above the vertex of the parabola?

Example 5.36

A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabo bowl is $40cm$ wide from rim to rim and $30cm$ deep. The bulb is located at the focus

(1)What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

Example 5.37

An equation of the elliptical part of an optical lens system is $\frac{x^2}{16}+\frac{y^2}{9}=1$. The parabolic p of the system has a focus in common with the right focus of the ellipse .The vertex of the parabo is at the origin and the parabola opens to the right. Determine the equation of the parabola.

Example 5.38

A room $34m$ long is constructed to be a whispering gallery. The room has an ellipti ceiling, as shown. If the maximum height of the ceiling is $8m$, determine where the foci a located.

Example 5.39

If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeter where to the nearest centimeter, should the patient's kidney stone be placed so that the reflecte sound hits the kidney stone?

EXERCISE 6.1

1. Prove by vector method that if a line is drawn from the centre of a circle to the mid point of chord, then the line is perpendicular to the chord.
2. Prove by vector method that the median to the base of an isosceles triangle is perpendicula to the base.
3. Prove by vector method that an angle in a semi-circle is a right angle.

4. Prove by vector method that the diagonals of a rhombus intersect each other at right angles.
5. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle.
6. Prove by vector method that the area of quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$.
7. Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.
11. A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point $(1,2,3)$ to the point $(5,4,1)$. Find the total work done by the forces.
12. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.
13. Find the magnitude and direction cosines of the torque (moment) of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ acting through a point whose position vector is $4\hat{i}+2\hat{j}-3\hat{k}$.
14. Find the torque (moment) of the resultant of the three forces represented by $-3i+6\hat{j}-3\hat{k}, 4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting at the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$, about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$.

EXERCISE 6.2

4. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors represnted by concurrent edges of a parallelopiped of volume 4 cubic units, find the value of $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}+\vec{c}\right)\cdot\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}+\vec{a}\right)\cdot\left(\vec{a}\times\vec{b}\right)$.
5. Find the altitude of a parallelopiped determined by the vectors $\vec{a}=-2\hat{i}+5\hat{j}+3\hat{k}, \vec{b}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{c}=-3\hat{i}+\hat{j}+4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
10. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $\left[\vec{a},\vec{b},\vec{c}\right]^2=\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

EXERCISE 6.3

3. Prove that $\left[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}\right]=0$.
5. $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$ then find the value of $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{a}\times\vec{c}\right)$.
6. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors, show that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\vec{0}$.
7. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}, \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n.

8. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times\left(\hat{b}\times\hat{c}\right)=\frac{1}{2}\hat{b}$, the angle between $\hat{a}$ and $\hat{c}$.

EXERCISE 6.4

1. Find the non-parametric form of vector equation and Cartesian equations of the straight passing through the point with position vector $4\hat{i}+3\hat{j}-7\hat{k}$ and parallel to the v $2\hat{i}-6\hat{j}+7\hat{k}$.
2. Find the parametric form of vector equation and Cartesian equations of the straight passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4}=\frac{y+3}{5}=\frac{8-z}{6}$.
3. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz yz planes.
4. Find the direction cosines of the straight line passing through the points $(5,6,7)$ $(7,9,13)$. Also find the parametric form of vector equation and Cartesian equations of straight line passing through two given points.
6. The vertices of ΔABC are $A(7,2,1), B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$.
7. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.
8. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to e other, find the value of m.
9. Show that the points $(2,3,4),(-1,4,5)$ and $(8,1,2)$ are collinear.

EXERCISE 6.5

1. Find the parametric form of vector equation and Cartesian equations of a straight line passi through $(5,2,8)$ and is perpendicular to the straight lines $\vec{r}=\left(\hat{i}+\hat{j}-\hat{k}\right)+s\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ a $\vec{r}=\left(2\hat{i}-\hat{j}-3\hat{k}\right)+t\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.
3. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the val of m.

EXERCISE 6.6

1. Find the vector equation of a plane which is at a distance of 7 units from the orgin havir $3,-4,5$ as direction ratios of a normal to it.
2. Find direction cosines of the normal to the plane $12x+3y-4z=65$. Also, find the nor parametric form of vector equation of a plane and the length of the perpendicular to the plan from the origin.
3. Find the vector and Cartesian equations of the plane passing through the point with positio vector $2\hat{i}+6\hat{j}+3\hat{k}$ and normal to the vector $\hat{i}+3\hat{j}+5\hat{k}$.
4. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.
6. If a plane meets the coordinate axes at A,B,C such that the centroid of the triangle ABC is the point (u,v,w), find the equation of the plane.

EXERCISE 6.9

1. Find the Cartesian equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

3. Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+2\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(6\hat{i}+3\hat{j}+2\hat{k})=8$.

4. Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

5. Find the Cartesian equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes.

Example 6.4

With usual notations, in any triangle *ABC*, prove by vector method that

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Example 6.9

A particle acted upon by constant forces $2\hat{i}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point $(4,-3,-2)$ to the point $(6,1,-3)$. Find the total work done by the forces.

Example 6.10

A particle is acted upon by the forces $3\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

Example 6.11

Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{i}+\hat{j}-\hat{k}$ whose line of action passes through the origin.

Example 6.17

If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar.

Example 6.18

For any three vector $\vec{a},\vec{b},\vec{c}$ prove that $[\vec{a}+\vec{c},\vec{a}+b,\vec{a}+\vec{b}+\vec{c}]=[\vec{a},\vec{b},\vec{c}]$.

Example 6.19

Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

Example 6.20

Prove that $(\vec{a}\cdot(\vec{b}\times\vec{c}))\vec{a}=(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$.

Example 6.26

Find the vector (parametric) and Cartesian equations of the line passing through $(-4,2,-3)$ and is parallel to the line $\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$.

Example 6.28

Find the angle between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=\frac{z}{-1}$ with coordinate axes.

Example 6.29

Find the acute angle between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 4\hat{k}) + t(2\hat{i} + 2\hat{j} + \hat{k})$ and the straight passing through the points $(5,1,4)$ and $(9,2,12)$.

Example 6.30

Find the angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and whether they are parallel or perpendicular.

Example 6.31

Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$.

Example 6.33

Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

Example 6.34

Find the vectpr equation (parametric) of a straight line passing through the point intersection of the straight lines $\vec{r} = \hat{i} + 3\hat{j} - \hat{k} + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, perpendicular to both straight lines.

Example 6.35

Determine whether the pair of straight lines $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel.Find the shortest distance between them.

Example 6.36

Find the shortest distance between the given straight lines $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

Example 6.38

Find the vector and Cartesian equations of a plane which is at a distance of 12 units from t origin and perpendicular to $6\hat{i} + 2\hat{j} - 3\hat{k}$.

Example 6.40

Find the direction cosines of the normal and the length of the normal from the origin to tl plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$.

Example 6.41

Find the vector and Cartesian equations of the plane passing through the point with positic vector $4\hat{i} + 2\hat{j} - 3\hat{k}$ and normal to vector $2\hat{i} - \hat{j} + \hat{k}$.

Example 6.42

A variable plane moves in such a way that the sum of the reciprocals of its intercepts on th coordinate axes is a constant. Show that the plane passes through a fixed point

Example 6.52

Find the distance between the planes $\vec{r} \cdot (2\hat{i} - \hat{j} - 2\hat{k}) = 6$ and $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 6\hat{k}) = 27$.

Example 6.56

Find the point where the straight line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+t\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ meets the plane $x-y+z-5=0$.

EXERCISE 7.1

1. A point moves along a line in such a way that after t seconds its distance from the origin is $s=2t^2+3t$ metres.
   (i) Find the average velocity of the points between $t=3$ and $t=6$ seconds.
   (ii) Find the instantaneous velocities at $t=3$ and $t=6$ seconds.
5. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x)=\sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x=3$ and $x=27$ metres.

EXERCISE 7.2

2. Find the point on the curve $y=x^2-5x+4$ at which the tangent is parallel to the line $3x+y=7$.
3. Find the points on the curve $y=x^3-6x^2+x+3$ where the normal is parallel to the line $x+y=1729$.
4. Find the points on the curve $y^2-4xy=x^2+5$ for which the tangent is horizontal.
5. Find the tangent and normal to the following curves at the given points on the curve.
   (i) $y=x^2-x^4$ at $(1,0)$ (ii) $y=x^4+2e^x$ at $(0,2)$
   (iii) $y=x\sin x$ at $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ (iv) $x=\cos t, y=2\sin^2 t$ at $t=\frac{\pi}{3}$ (each 3 marks)
6. Find the equations of the tangents to the curve $y=1+x^3$ for which the tangent is orthogonal with the line $x+12y=12$.
7. Find the equations of the tangents to the curve $y=\frac{x+1}{x-1}$ which are parallel to the line $x+2y=6$.
8. Find the equation of tangent and normal to the curve given by $x=7\cos t$ and $y=2\sin t, t\in\mathbb{R}$ at any point on the curve.

EXERCISE 7.3

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions : (each 3 marks)
   (i) $f(x)=x^2-x, x\in[0,1]$ (ii) $f(x)=\frac{x^2-2x}{x+2}, x\in[-1,6]$
   (iii) $f(x)=\sqrt{x}-\frac{x}{3}, x\in[0,9]$
4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval : (each 3 marks)
   (i) $f(x)=x^3-3x+2, x\in[-2,2]$ (ii) $f(x)=(x-2)(x-7), x\in[3,11]$

5. Show that the value in the conclusion of the mean value theorem for

(i) $f(x)=\frac{1}{x}$ on a closed interval of positive numbers $[a,b]$ is $\sqrt{ab}$.

(ii) $f(x)=Ax^2+Bx+C$ on any interval $[a,b]$ is $\frac{a+b}{2}$. (each 3 marks)

6. A race car is racing at 20th km. If its speed never exceeds 150 km/hr, what is the ma distance he can cover in the next two hours?

7. Suppose that the function $f(x), f'(x)\le 1$ for all $1\le x\le 4$. Show that $f(4)-f(1)\le 3$.

8. Does there exist a differentiable function $f(x)$ such that $f(0)=-1, f(2)=4$ and f' for all x. Justify your answer.

9. Show that there lies a point on the curve $f(x)=x(x+3)e^{-\frac{\pi}{2}}, -3\le x\le 0$ where tangent is parallel to the x-axis.

10. Using mean value theorem prove that for, $a>0,\ b>0, \left|e^{-a}-e^{-b}\right|<|a-b|$.

EXERCISE 7.4

1. Write the Maclaurin series expansion of the following functions: (each 3 marks)

(i) e^x (ii) $\sin x$ (iii) $\cos x$

(iv) $\log(1-x); -1\le x<1$

2. Write the Taylor series expansion of the function $\log x$ about $x=1$ upto three no terms for $x>0$.

3. Expand $\sin x$ in ascending powers $x-\frac{\pi}{4}$ upto three non-zero terms.

4. Expand the polynomial $f(x)=x^2-3x+2$ in powers of $x-1$.

EXERCISE 7.5

Evaluate the following limits, if necessary use I'Hôpital Rule : (each 3 marks)

1. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$ 2. $\lim\limits_{x\to\infty}\frac{2x^2-3}{x^2-5x+3}$ 3. $\lim\limits_{x\to\infty}\frac{x}{\log x}$

4. $\lim\limits_{x\to\frac{\pi}{2}}\frac{\sec x}{\tan x}$ 5. $\lim\limits_{x\to\infty}e^{-x}\sqrt{x}$ 6. $\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

7. $\lim\limits_{x\to 1^+}\left(\frac{2}{x^2-1}-\frac{x}{x-1}\right)$

EXERCISE 7.6

1. Find the absolute extrema of the following functions on the given closed interval.

(i) $f(x)=x^2-12x+10$; $[1,2]$ (iii) $f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}$; $[-1,1]$

(iv) $f(x)=2\cos x+\sin 2x$; $\left[0,\frac{\pi}{2}\right]$ (each 3 marks)

EXERCISE 7.7

2. Find the local extrema for the following functions using second derivative test:

(iii) $f(x)=x^2e^{-2x}$

EXERCISE 7.8

2. Find two positive numbers whose product is 20 and their sum is minimum.

Example 7.3

A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1-0.1t)^2$, $0 \le t \le 10$. What is the rate at which the person forgets the words '2' days after learning?

Example 7.4

A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively?

Example 7.11

Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1,2)$.

Example 7.12

For what values of x the tangent of the curve $y = x^3 - 3x^2 + x - 2$ is parallel to the line $y = x$.

Example 7.13

Find the equation of the tangent and normal to the curve $x = 2\cos 3t$ and $y = 3\sin 2t, t \in R$.

Example 7.16

Find the angle of intersection of the curve $y = \sin x$ with the positive x-axis.

Example 7.19

Compute the value of 'c' satisfied by the Rolle's theorem for the function $f(x) = x^2(1-x)^2, x \in [0,1]$.

Example 7.20

Find the values in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$.

Example 7.21

Compute the value of 'c' satisfied by Rolle's theorem for the function $f(x) = \log\left(\frac{x^2+6}{5x}\right)$ in the interval $[2,3]$.

Example 7.22

Without actually solving show that the equation $x^4 + 2x^3 - 2 = 0$ has only one real root in the interval $(0,1)$.

Example 7.23

Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

there is a zero of the polynomial

$$na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1.$$

Example 7.24

Prove that there is a zero of the polynomial, $2x^3-9x^2-11x+12$ in the interval $(2,7)$ that 2 and 7 are the zeros of the polynomial $x^4-6x^3-11x^2+24x+28$.

Example 7.25

Find the values in the interval $(1,2)$ of the mean value theorem satisfied by the fu $f(x)=x-x^2$ for $1\le x\le 2$.

Example 7.26

A truck travels on a toll road with a speed limit of 80 km/hr. The truck completes a 164 km journey in 2 hours. At the end of the toll road the trucker is issued with a speed vi ticket. Justify this using the Mean Value Theorem.

Example 7.27

Suppose $f(x)$ is a differentiable function for all x with $f'(x)\le 29$ and $f(2)=17$ the is the maximum value of $f(7)$?

Example 7.28

Prove using mean value theorem that, $|\sin\alpha-\sin\beta|\le|\alpha-\beta|,\ \alpha,\ \beta\in\mathbb{R}$.

Example 7.29

A thermometer was taken from a freezer and placed in a boiling water. It took 22 secon the thermometer to raise from $-10°C$ to $100°C$. Show that the rate of change of temperat some time t is $5°C$ per second.

Example 7.37

If $\lim\limits_{\theta\to 0}\left(\dfrac{1-\cos m\theta}{1-\cos n\theta}\right)=1$, then prove that $m=\pm n$.

Example 7.38

Evaluate : $\lim\limits_{x\to 1^-}\left(\dfrac{\log(1-x)}{\cot(\pi x)}\right)$.

Example 7.39

Evaluate : $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$.

Example 7.40

Evaluate : $\lim\limits_{x\to 0^+} x\log x$.

Example 7.41

Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{x^2+17x+29}{x^4}\right)$.

Example 7.42

Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{e^x}{x^m}\right), m\in N$.

Example 7.46

Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval $(2,7)$ and strictly decreasing in the interval $(-2,0)$.

Example 7.48

Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3,2]$.

Example 7.49

Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0, 2\pi]$.

Example 7.52

Prove that the function $f(x) = x - \sin x$ is increasing on the real line. Also discuss for the existence of local extrema.

EXERCISE 8.1

2. Use the linear approximation to find approximate values of (each 3 marks)

   (i) $(123)^{\frac{2}{3}}$ (ii) $\sqrt[4]{15}$ (iii) $\sqrt[3]{26}$.

3. Find a linear approximation for the following functions at the indicated points.

   (i) $f(x) = x^3 - 5x + 12,\ x_0 = 2$ (ii) $g(x) = \sqrt{x^2 + 9},\ x_0 = -4$

   (iii) $h(x) = \dfrac{x}{x+1},\ x_0 = 1$. (each 3 marks)

4. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.

   (i) Absolute error (ii) Relative error

   (iii) Percentage error, in calculating the area (each 3 marks)

5. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following: (each 3 marks)

   (i) decrease in the volume (ii) change in the surface area

6. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\dfrac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of t.

7. Show that the percentage error in the n^{th} root of a number is approximately $\dfrac{1}{n}$ times the percentage error in the number.

EXERCISE 8.2

3. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare

   (i) $f(x) = x^3 - 2x^2; x = 2,\ \Delta x = dx = 0.5$

   (ii) $f(x) = x^2 + 2x + 3;\ x = -0.5,\ \Delta x = dx = 0.1$ (each 3 marks)

4. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

8. In a newly developed city, it is estimated that the voting population (in thousand) increase according to $V(t)=30+12t^2-t^3, 0\le t\le 8$ where t is the time in years. F approximate change in voters for the time change from 4 to $4\frac{1}{6}$ year.

10. A circular plate expands uniformly under the influence of heat. If it's radius increase 10.5cm to 10.75cm, then find an approximate change in the area and the appro percentage changes in the area.

11. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 c the differentials to find approximately how many cubic centimeters of paint is used t this cube. Also calculate the exact amount of paint used to paint this cube.

EXERCISE 8.3

5. Let $g(x,y)=\frac{x^2y}{x^4+y^2}$ for $(x,y)\neq(0,0)$ and $g(0,0)=0$.

   (i) Show that $\lim_{(x,y)\to(0,0)} g(x,y)=0$ along every line $y=mx, m\in\mathbb{R}$.

   (ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y)=\frac{k}{1+k^2}$ along every parabola $y=kx^2, k\in\mathbb{R}\setminus\{0\}$.

6. Show that $f(x,y)=\frac{x^2-y^2}{y^2+1}$ is continuous at every $(x,y)\in\mathbb{R}^2$.

7. Let $g(x,y)=\frac{e^y\sin x}{x}$, for $x\neq 0$ and $g(0,0)=1$. Show that g is continuous at $(0,0)$.

EXERCISE 8.4

1. Find the partial derivatives of the following functions at the indicated points.

   (i) $f(x,y)=3x^2-2xy+y^2+5x+2,(2,-5)$

   (ii) $g(x,y)=3x^2+y^2+5x+2,(1,-2)$

   (iii) $h(x,y,z)=x\sin(xy)+z^2x, \left(2,\frac{\pi}{4},1\right)$

   (iv) $G(x,y)=e^{x+3y}\ln(x^2+y^2),(-1,1)$ (each 3 marks)

4. If $u=\log(x^3+y^3+z^3)$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

8. If $w(x,y)=xy+\sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\partial x}=\frac{\partial^2 w}{\partial x\partial y}$.

EXERCISE 8.5

1. If $w(x,y)=x^3-3xy+2y^2, x,y\in\mathbb{R}$, find the linear approximation for w at $(1,-1)$.
2. Let $z(x,y)=x^2y+3xy^4, x,y\in\mathbb{R}$. Find the linear approximation for z at $(2,-1)$.
3. If $v(x,y)=x^2-xy+\frac{1}{4}y^2+7, \quad x, y\in\mathbb{R}$, find the differential dv.
4. Let $V(x,y,z)=xy+yz+zx, \quad x,y,z\in\mathbb{R}$: Find the differential dV.

EXERCISE 8.7

4. If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

Example 8.1

Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$.

Example 8.2

Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

Example 8.3

Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

Example 8.5

Let $f, g : (a,b) \to \mathbb{R}$ be differentiable functions. Show that $d(fg) = fdg + gdf$.

Example 8.7

If the radius of a sphere, with radius 10 cm, were to decrease by 0.1 cm, approximately how much would its volume decrease?

Example 8.8

Let $f(x, y) = \frac{3x - 5y + 8}{x^2 + y^2 + 1}$ for all $(x, y) \in \mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$.

Example 8.9

Consider $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$.

Example 8.10

Consider $g(x, y) = \frac{2x^2 y}{x^2 + y^2}$ if $(x, y) \neq (0,0)$ and $g(0,0) = 0$. Show that g is continuous on $\mathbb{R}^2$.

Example 8.16

If $w(x, y, z) = x^2 y + y^2 z + z^2 x, x, y, z \in \mathbb{R}$, find the differential dw.

Example 8.17

Let $U(x, y, z) = x^2 - xy + 3\sin z, x, y, z \in \mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$.

EXERCISE 9.3

1. Evaluate the following definite integrals :

(ii) $\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5}$ (iii) $\int_{0}^{1} \sqrt{\frac{1-x}{1+x}}\, dx$

(iv) $\int_{0}^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ (v) $\int_{0}^{\frac{\pi}{2}} \sqrt{\cos\theta}\, \sin^3\theta\, d\theta$ (vi) $\int_{0}^{1} \frac{1-x^2}{(1+x^2)^2} dx$

2. Evaluate the following integrals using properties of integration:

(i) $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ (iii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$

(iv) $\int_{0}^{2\pi} x\log\left(\frac{3+\cos x}{3-\cos x}\right)dx$ (x) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx$

EXERCISE 9.4

Evaluate the following:

1. $\int_{0}^{1} x^3e^{-2x}\,dx$ 2. $\int_{0}^{1} \frac{\sin(3\tan^{-1}x)\tan^{-1}x}{1+x^2}dx$

3. $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}dx$ 4. $\int_{0}^{\frac{\pi}{2}} x^2\cos 2x\,dx$

EXERCISE 9.5

Evaluate the following:

(1) $\int_{0}^{\frac{\pi}{2}} \frac{dx}{1+5\cos^2 x}$ (2) $\int_{0}^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$

EXERCISE 9.6

1. Evaluate the following:

(iii) $\int_{0}^{\frac{\pi}{4}} \sin^6 2x\,dx$ (iv) $\int_{0}^{\frac{\pi}{6}} \sin^5 3x\,dx$

(v) $\int_{0}^{\frac{\pi}{2}} \sin^2 x\cos^4 x\,dx$ (vi) $\int_{0}^{2\pi} \sin^7\frac{x}{4}\,dx$ (vii) $\int_{0}^{\frac{\pi}{2}} \sin^3\theta\cos^5\theta\,d\theta$ (viii) $\int_{0}^{1} x^2(1-x)$

EXERCISE 9.8

2. Find the area of the region bounded by $2x-y+1=0$, $y=-1$, $y=3$ and y-axis.

EXERCISE 9.9

1. Find the volume of the solid generated by revolving the region enclosed by $y=2x^2$, and $x=1$ about the x-axis, using integration.

Example 9.5

Evaluate : $\int_{0}^{3}(3x^2-4x+5)dx$.

Example 9.6

Evaluate : $\int_0^1 \frac{2x+7}{5x^2+9}dx$.

Example 9.8

Evaluate : $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x}dx$.

Example 9.9

Evaluate : $\int_0^9 \frac{1}{x+\sqrt{x}}\, dx$.

Example 9.23

If $f(x)=f(a+x)$, then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$

Example 9.26

Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$.

Example 9.29

Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$.

Example 9.31

Evaluate $\int_0^{\pi} x^2 \cos nx dx$, where n is a positive integer.

Example 9.32

Evaluate : $\int_0^1 e^{-2x}(2x^3-x-1)dx$.

Example 9.33

Evaluate : $\int_0^{2\pi} x^2 \sin nx dx$, where n is a positive integer.

Example 9.34

Evaluate : $\int_{-1}^1 e^{-\lambda x}(1-x^2)dx$.

Example 9.35

Evaluate $\int_b^{\infty} \frac{1}{a^2+x^2}dx, a>0, b\in\mathbb{R}$.

Example 9.38

Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix}\cos^4 x & 7\\ \sin^5 x & 3\end{vmatrix}dx$.

Example 9.39

Find the values of the following:

(i) $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x\,dx$.

Example 9.41

Evaluate $\int_0^1 x^5 \left(1-x^2\right)^5 dx$.

Example 9.42

Evaluate $\int_0^1 x^3 \left(1-x\right)^4 dx$.

Example 9.44

Evaluate $\int_0^{\infty} e^{-ax} x^n dx$, where $a>0$.

Example 9.48

Find the area of the region bounded by the line $7x-5y=35$, $x-$axis and the lines $x=-$ $x=3$.

Example 9.62

Find the volume of a sphere of radius *a*.

Example 9.67

Find, by integration, the volume of the solid generated by revolving about *y*-axis the re bounded between the parabola $x=y^2+1$, the *y*-axis, and the lines $y=1$ and $y=-1$.

EXERCISE 10.3

1. Find the differential equation of the family of (i) all non-vertical lines in a plane (i non-horizontal lines in a plane.
2. Form the differential equation of all straight lines touching the circle $x^2+y^2=r^2$.
3. Find the differential equation of the family of circles passing through the origin and ha their centres on the x-axis.
4. Find the differential equation of the family of all the parabolas with latus rectum $4a$ whose axes are parallel to the x-axis.
5. Find the differential equation of the family of parabolas with vertex at $(0,-1)$ and ha axis along the y-axis.
6. Find the differential equations of the family of all the ellipses having foci on the y-axis centre at the origin.
7. Find the differential equation corresponding to the family of curves represented by equation $y=Ae^{8x}+Be^{-8x}$, where A and B are arbitrary constants.

EXERCISE 10.4

3. The slope of the tangent to the curve at any point is the reciprocal of four times the ordi at that point. The curve passes through (2,5). Find the equation of the curve.
5. Show that $y=ax+\frac{b}{x}, x\neq 0$ is a solution of the differential equation $x^2y''+xy'-y=0$.

6. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.

7. Show that the differential equation representing the family of curves $y^2 = 2a\left(x + a^{\frac{2}{3}}\right)$, where a is a positive parameter, is $\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$.

EXERCISE 10.5

4. Solve the following differential equations:

(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(iii) $\sin\frac{dy}{dx} = a,\ y(0) = 1$ (iv) $\frac{dy}{dx} = e^{x+y} + x^3e^y$

EXERCISE 10.7

Solve the following Linear differential equations:

1. $\cos x\frac{dy}{dx} + y\sin x = 1$

Example 10.4

Find the differential equation of the family of circles passing through the points $(a,0)$ and $(-a,0)$.

Example 10.6

Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

Example 10.9

Show that $y = 2(x^2 - 1) + ce^{-x^2}$ is a solution of the differential equation $\frac{dy}{dx} + 2xy - 4x^3 = 0$.

Example 10.10

Show that $y = a\cos(\log x) + b\sin(\log x), x > 0$ is a solution of the differential equation $x^2y'' + xy' + y = 0$.

Example 10.11

Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$.

Example 10.16

Solve : $\frac{dy}{dx} = (3x + y + 4)^2$.

Example 10.20

Solve $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

EXERCISE 11.1

1. Suppose X is the number of tails occurred when three fair coins are tossed o simultaneously. Find the values of the random variable X and number of points in its inv images.

2. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the numbe taken fruits are apple, then find the values of the random variable and number of points i inverse images.

3. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Supp that we win ₹15for each red ball selected and we lose ₹10 for each black ball selected. denotes of winning amount, then find the values of X and number of points in its inv images.

4. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining t faces. The die is thrown twice. If X denotes the total score in two throws, find the value the random variable and number of points in its inverse images.

EXERCISE 11.2

1. Three fair coins are tossed simultaneously. Find the probability mass function for numbe heads occurred.

EXERCISE 11.3

1. The probability density function of X is given by $f(x)=\begin{cases} kxe^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\leq 0 \end{cases}$

   Find the value of k.

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & x<0 \\ \frac{1}{2}\left(x^2+x\right) & 0\leq x<1 \\ 1, & x\geq 1 \end{cases}$$

   then find (ii) $P(0.3\leq X\leq 0.6)$.

EXERCISE 11.4

3. If μ and σ^2 are the mean and variance of the discrete random variable X, a $E(X+3)=10$ and $E(X+3)^2=116$, find μ and σ^2.

5. A commuter train arrives punctually at a station every half an hour. Everyday in the morni a student leaves his home to the railway station. Let X denote the amount of time, minutes, that the student waits for the train from the time he reaches the railway station. T pdf of X is

$$f(x)=\begin{cases} \frac{1}{30} & 0<x<30 \\ 0 & \text{elsewhere} \end{cases}$$

   Obtain and interpret the expected value of the random variable X.

8. A lottery with 600 tickets gives one prize of ₹200, four prices of ₹100, and six prizes of ₹ If the ticket costs is ₹2, find the expected winning amount of a ticket.

EXERCISE 11.5

2. The probability that Q hits a target at any trial is $\frac{1}{4}$. He tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

5. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items.

8. If $X \sim B(n, p)$ such that $4P(X=4) = P(x=2)$ and $n=6$. Find the distribution, mean and standard deviation.

9. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution.

Example 11.1:

Suppose two coins are tossed once. If X denotes the number of tails, (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable, and number of elements in its inverse images.

Example 11.2:

Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10, and (iv) the number of elements in inverse image of X.

Example 11.3

An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of if inverse images.

Example 11. 4:

Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ₹ 30 for each black ball selected and the loss ₹ 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

Example 11.5

Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

Example 11.11

Find the constant C such that the function

$$f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

is a density function, and compute (i) $P(1.5 < X < 3.5)$ (ii) $P(X \le 2)$ (iii) $P(3 < X)$.

Example 11.13

If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases}0, & x<0\\ x, & 0\le x<1\\ 1, & 1\le x\end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2\le X\le 0.7)$.

Example 11.15

Let X be random variable denoting the life time of an electrical equipment having prob. density function

$$f(x)=\begin{cases}k e^{-2x} & \text{for } x>0\\ 0 & \text{for } x\le 0.\end{cases}$$

Find (i) the value of k (ii) Distribution function (iii) $P(X<2$

(iv) calculate the probability that X is at least for four units of life time (v) $P(X=3$

Example 11.16

Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance.

Example 11.19 (each 3 marks)

Find the binomial distribution function for each of the following.

(i) Five fair coins are tossed once and X denotes the number of heads.

(ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared.

EXERCISE 12.2

6. Construct the truth table for the following statements. (each 3 marks)

(i) $\neg p\wedge\neg q$ (ii) $\neg(p\wedge\neg q)$ (iii) $(p\vee q)\vee\neg q$ (iv) $(\neg p\rightarrow r)\wedge(p\leftrightarrow q)$

9. Prove that $q\rightarrow p\equiv\neg p\rightarrow\neg q$.