IMPORTANT FIVE MARK QUESTIONS (TENTATIVE)

12^{TH} MATHS

Touch the sum and get the SOLUTION video

Note : * IMPORTANT AND EASY PROBLEM ONLY HERE. FIRST PRACTICE THE FOLLOWING AND THEN THE OTHER PROBLEMS.

CHAPTER-1 APPLICATIONS OF MATRICES AND DETERMINANTS

1 EXERCISE 1.1

3. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

2 EXERCISE1.1

5. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$, prove that $A^{-1}=A^{T}$.

3 **Example 1.24**

If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

4 EXERCISE1.3

2. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$.

5 **EXERCISE 1.4**

1. Solve the following systems of linear equations by Cramer's rule:

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

6 **Example 1.34**

Investigate for what values of λ and μ the system of linear equations

$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

7 EXERCISE1.6

2. Find the value of k for which the equations $kx-2y+z=1,\quad x-2ky+z=-2,\quad x-2y+kz=1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

8 EXERCISE1.6

3. Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8,\ 2x+3y+\lambda z=\mu$, have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

9 Example 1.39

By using Gaussian elimination method, balance the chemical reaction equation:

$C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.

(The above is the reaction that is taking place in the burning of organic compound called isoprene.)

10 EXERCISE1.7

2. Determine the values of λ for which the following system of equations $x+y+3z=0,\ 4x+3y+\lambda z=0,\ 2x+y+2z=0$ has

(i) a unique solution (ii) a non-trivial solution.

11 EXERCISE1.7

3. By using Gaussian elimination method, balance the chemical reaction equation:

$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

CHAPTER-2 COMPLEX NUMBERS

1 **Example 2.8**

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

2 **Example 2.14**

Show that the points 1, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

3 7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

(ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

EXERCISE 2.4

4 **Example 2.15**

Let z_1, z_2, and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3 \neq 0$.

Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

5 EXERCISE 2.5

2. For any two complex numbers z_1 and z_2, such that $|z_1|=|z_2|=1$ and $z_1z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number.

6 EXERCISE 2.6

2. If $z=x+iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$.

7 **Example 2.27**

If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, then show that $x^2+y^2=1$.

8 6. If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$.

EXERCISE 2.7

9 **Example 2.34**

Solve the equation $z^3+8i=0$, where $z \in \mathbb{C}$.

10 **Example 2.35**

Find all cube roots of $\sqrt{3}+i$.

11 **Example 2.36**

Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then find z_2 and z_3.

12 EXERCISE 2.8

4. If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that

(i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ (ii) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$

(iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$ (iv) $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$.

13 5. Solve the equation $z^3+27=0$.

14 6. If $\omega\neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

CHAPTER -3 THEORY OF EQUATION

1 **Example 3.15**

If $2+i$ and $3-\sqrt{2}$ are roots of the equation

$$x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0,$$

2 EXERCISE 3.3

5. Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

3 **Example 3.28**

Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$.

4 EXERCISE 3.3

5. Solve the equations

(i) $6x^4-35x^3+62x^2-35x+6=0$ (ii) $x^4+3x^3-3x-1=0$

5 7. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

CHAPTER-4 INVERSE TRIGONOMETRICAL FUNCTIONS

1 EXERCISE 4.2

8. Find the value of

(i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right)$ (ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

2,3 EXERCISE 4.3

4. Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$.

(iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$.

4 **Example 4.23**

If $a_1, a_2, a_3, \dots a_n$ is an arithmetic progression with common difference d,

prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\dots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$.

5 **Example 4.28**

Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

6 **Example 4.29**

Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$.

7 EXERCISE 4.5

3. Find the value of

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$ (iii) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

8 EXERCISE 4.5

4. Prove that

(i) $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$ (ii) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$

9 EXERCISE 4.5

5. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

10 EXERCISE 4.5

6. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$.

11

8. Simplify: $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$.

EXERCISE 4.5

12 EXERCISE 4.5

9. Solve:

(i) $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$ (ii) $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$, $a>0,\ b>0$

13 EXERCISE 4.5

9.

(iii) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$ (iv) $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12},\ x>0$

14 EXERCISE 4.5

10. Find the number of solution of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$

CHAPTER -5 2D ANALYTICAL GEOMETRY

1 **Example 5.10**

Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

2 EXERCISE 5.1

6. Find the equation of the circle through the points $(1,0)$, $(-1,0)$, and $(0,1)$.

3 EXERCISE 5.4

3. Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact.

4 EXERCISE 5.5

1. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides.

5 EXERCISE 5.5

2. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

6 3. At a water fountain, water attains a maximum height of $4m$ at horizontal distance of $0.5m$ from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of $0.75m$ from the point of origin.

7 5. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.

8 6. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$. The tower is $150m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

9 7. A rod of length $1.2m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3m$ from the end in contact with x-axis is an ellipse. Find the eccentricity.

10 8. Assume that water issuing from the end of a horizontal pipe, $7.5m$ above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5m$ below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

11 9. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

12 10. Points A and B are $10km$ apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

13 **Example 5.40**

Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

14 **Example 5.31**

A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway? (Fig. 5.6)

CHAPTER -6 VECTOR ALGEBRA

1 **Example 6.3**

By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

2 **Example 6.5**

Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

3 **Example 6.6 (Apollonius's theorem)**

If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $|\overline{AB}|^2+|\overline{AC}|^2=2(|\overline{AD}|^2+|\overline{BD}|^2)$. A

4 **Example 6.7**

Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. A

5 EXERCISE 6.1

8. If G is the centroid of a ΔABC, prove that

(area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) $=\frac{1}{3}$ (area of ΔABC).

6 EXERCISE 6.1

9. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

7 EXERCISE 6.1

10. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$,

8 **Example 6.23**

If $\vec{a}=\vec{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$, verify that

(i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

(ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$

9 EXERCISE 6.3

4. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$, verify that

(i) $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$ (ii) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

10 **Example 6.37**

Find the coordinates of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r}=(\hat{i}-4\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+\hat{k})$. Also, find the shortest distance from the point to the straight line.

11 EXERCISE 6.5

7. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.

12 **Example 6.43**

Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\hat{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$.

13 **Example 6.44**

Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

14 **EXERCISE 6.7**

1. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

15 EXERCISE 6.7

2. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1)$, $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

16 EXERCISE 6.7

3. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

17 EXERCISE 6.7

4. Find the non-parametric form of vector equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

18 EXERCISE 6.7

5. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=8$.

19 EXERCISE 6.7

6. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2),(-1,-2,6)$, and $(6,-4,-2)$.

20 EXERCISE 6.7

7. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=\left(6\hat{i}-\hat{j}+\hat{k}\right)+s\left(-\hat{i}+2\hat{j}+\hat{k}\right)+t\left(-5\hat{i}-4\hat{j}-5\hat{k}\right)$.

21 2. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar.Also, find the plane containing these lines.

22 **Example 6.46**

Show that the lines $\vec{r}=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)+s\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+t\left(\hat{i}+4\hat{j}+7\hat{k}\right)$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines.

23 EXERCISE 6.9

8. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x+2y+3z=2$.

CHAPTER -7 APPLICATIONS OF DIFFERENTIAL CALCULUS

1 **Example 7.9**

Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

2 **Example 7.10 (Two variable related rate problem)**

A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

3 EXERCISE 7.1

8. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

4 EXERCISE 7.1

9. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,

(i) how fast is the top of the ladder moving down the wall?

(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

5 EXERCISE 7.1

10. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

6 **Example 7.13**

Find the equation of the tangent and normal at any point to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t, t \in \mathbb{R}$.

7 **Example 7.14**

Find the angle between $y = x^2$ and $y = (x-3)^2$.

8 **Example 7.17**

If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then,

show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

9 **Example 7.18**

Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

10 EXERCISE 7.2

8. Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t, t \in \mathbb{R}$ at any point on the curve.

11 EXERCISE 7.2

9. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$.

12 EXERCISE 7.2

10. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

13 EXERCISE 7.5

12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1+\frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$

14 **Example 7.65**

Prove that among all the rectangles of the given area square has the least perimeter.

15 EXERCISE 7.8

5. A rectangular page is to contain 24 cm² of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

16 EXERCISE 7.8

7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

17 EXERCISE 7.8

8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.

18 EXERCISE 7.8

9. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

19 EXERCISE 7.8

12. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

CHAPTER -8 DIFFERENTIALS AND PARTIAL DERIVATIVES

1 EXERCISE 8.1

6. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

2 EXERCISE 8.2

4. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

3 **Example 8.15**

Let $u(x, y) = e^{-2y}\cos(2x)$ for all $(x, y) \in \mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

4 EXERCISE 8.5

2. For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$.

(ii) $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$

5 EXERCISE 8.5

6. Let $w(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, (x, y, z) \neq (0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$.

6 EXERCISE 8.5

7. If $V(x,y)=e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$.

7 EXERCISE 8.5

8. If $w(x,y)=xy+\sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\partial x}=\frac{\partial^2 w}{\partial x\partial y}$.

8 EXERCISE 8.5

9. If $v(x,y,z)=x^3+y^3+z^3+3xyz$, show that $\frac{\partial^2 v}{\partial y\partial z}=\frac{\partial^2 v}{\partial z\partial y}$.

9 **Example 8.22**

If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

10 EXERCISE 8.7

2. Prove that $f(x,y)=x^3-2x^2y+3xy^2+y^3$ is homogeneous; what is the degree? Verify Euler's Theorem for f.

11 EXERCISE 8.7

3. Prove that $g(x,y)=x\log\left(\frac{y}{x}\right)$ is homogeneous; what is the degree? Verify Euler's Theorem for g.

12 EXERCISE 8.7

5. If $v(x,y)=\log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$.

13 EXERCISE 8.7

6. If $w(x,y,z)=\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$.

CHAPTER -9 APPLICATIONS OF INTEGRAL CALCULUS

1 **Example 9.15**

Evaluate : $\int_{-4}^{4}|x+3|\,dx$.

2 2. Evaluate the following integrals using properties of integration :

EXERCISE 9.3

(vi) $\int_{0}^{1}|5x-3|\,dx$

3 (x) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{1}{1+\sqrt{\tan x}}\,dx$

EXERCISE 9.3

4 **Example 9.30**

Evaluate $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}\,dx$

5 **Example 9.29**

Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx.$

6 **Example 9.14**

Evaluate : $\int_0^{1.5}\left[x^2\right] dx$, where $[x]$ is the greatest integer function.

7

(i) $\int_0^{\frac{\pi}{2}} \frac{dx}{1+5\cos^2 x}$ (ii) $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$

EXERCISE 9.5

8 **Example 9.36**

Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x+5\cos^2 x}$.

9 **Example 9.53**

Find the area of the region bounded by $x-$ axis, the curve $y=|\cos x|$, the lines $x=0$ and $x=\pi$.

10 **Example 9.55**

Find the area of the region bounded between the parabola $x^2=y$ and the curve $y=|x|$.

11 **Example 9.56**

Find the area of the region bounded by $y=\cos x, y=\sin x$, the lines $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

12 EXERCISE 9.8

5. Find the area of the region bounded between the curves $y=\sin x$ and $y=\cos x$ and the lines $x=0$ and $x=\pi$.

13 EXERCISE 9.8

6. Find the area of the region bounded by $y=\tan x$, $y=\cot x$ and the lines $x=0$, $x=\frac{\pi}{2}, y=0$.

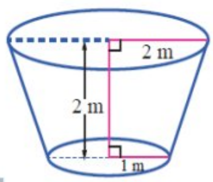
14 EXAMPLE 9.62 Find the volume of a sphere of radius a.

15 EXAMPLE 9.63 Find the volume of a right-circular cone of base radius r and height h.

16 EXAMPLE 9.64 Find the volume of the spherical cap of height *h* cut of from a sphere of radius *r*.

17 EXAMPLE 9.66 Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ about the major axis.

18 EXERCISE 9.9 6. Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the Figure.



19 EXERCISE 9.9 The region enclosed between the graphs of $y=x$ and $y=x^2$ is denoted by R, Find the volume generated when R is rotated through 360° about x-axis.

20 EXERCISE 9.8

8. Father of a family wishes to divide his square field bounded by $x=0$, $x=4$, $y=4$ and $y=0$ along the curve $y^2=4x$ and $x^2=4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

CHAPTER -10 DIFFERENTIAL EQUATIONS

1 EXERCISE 10.3

6. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

2 **Example 10.6**

Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

3 EXERCISE 10.5

1. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} - F - kV$, where k is a constant. Express V in terms of t given that $V = 0$ when $t = 0$.

4 EXERCISE 10.5

2. The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} - g\left(1 - \frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x.

5 EXERCISE 10.5

3. ஒரு வளைவரையின் சாய்வு $\frac{y-1}{x^2+x}$ ஆகும். வளைவரை $(1,0)$ எனும் புள்ளி வழிச் செல்லுமெனில், அதன் சமன்பாட்டைக் காண்க.

6 EXERCISE 10.6

3. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1,0)$.

7 EXERCISE 10.6

8. $(x^2+y^2)dy - xy\,dx$. It is given that $y(1)-1$ and $y(x_0)-e$. Find the value of x_0.

8 **Example 10.26**

Solve $ye^y dx = (y^3 + 2xe^y)dy$.

9 15. $\frac{dy}{dx} + \frac{3y}{x} - \frac{1}{x^2}$, given that $y = 2$ when $x = 1$

EXERCISE 10.7

10 **Example 10.27**

The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

11 **Example 10.28**

A radioactive isotope has an initial mass 200mg, which two years later is 50mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

12 EXERCISE 10.8

1. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

13 EXERCISE 10.8

2. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

14 EXERCISE 10.8

3. The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E - Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

15 EXERCISE 10.8

4. The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

16 EXERCISE 10.8

5. Suppose a person deposits ₹10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

17 EXERCISE 10.8

6. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

18 EXERCISE 10.8

9. A pot of boiling water at $100^\circ C$ is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to $80^\circ C$, and another 5 minutes later it has dropped to $65^\circ C$. Determine the temperature of the kitchen.

19 EXERCISE 10.8

10. A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.

20 **Example 10.30**

A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (*Brine is a high-concentration solution of salt (usually sodium chloride) in water*) runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

CHAPTER -11 PROBABILITY DISTRIBUTIONS

1 EXERCISE 11.2

2. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

(i) the probability mass function (ii) the cumulative distribution function

(iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$

2 **Example 11.8**

A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find $P(3 \le X < 6)$ (iv) Find $P(X \ge 4)$.

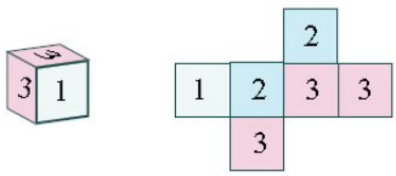


Fig. 11.7

3 **Example 11.10**

A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

4 EXERCISE 11.2

6. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

5 EXERCISE 11.2

7. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0 \\ \frac{1}{2} & \text{for } 0 \le x < 1 \\ \frac{3}{5} & \text{for } 1 \le x < 2 \\ \frac{4}{5} & \text{for } 2 \le x < 3 \\ \frac{9}{10} & \text{for } 3 \le x < 4 \\ 1 & \text{for } 4 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $P(X \ge 2)$.

6 **Example 11.9**

Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \le x < -1 \\ 0.60 & -1 \le x < 0 \\ 0.90 & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \ge -1)$.

7 EXERCISE 11.3

3. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k (ii) the distribution function

(iii) the probability that daily sales will fall between 300 litres and 500 litres?

8 **Example 11.14**

The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$

Find (i) Distribution function (ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \le X)$

9 **Example 11.15**

Let X be a random variable denoting the life time of an electrical equipment having probability density function

$$f(x)=\begin{cases} k e^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\le 0. \end{cases}$$

Find (i) the value of k (ii) Distribution function (iii) $P(X<2)$

(iv) calculate the probability that X is at least for four unit of time (v) $P(X=3)$.

10 EXERCISE 11.3

4. The probability density function of X is given by $f(x)=\begin{cases} k e^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

Find (i) the value of k (ii) the distribution function (iii) $P(X<3)$

(iv) $P(5\le X)$ (v) $P(X\le 4)$.

11 EXERCISE 11.3

5. If X is the random variable with probability density function $f(x)$ given by,

$$f(x)=\begin{cases} x+1, & -1\le x<0 \\ -x+1, & 0\le x<1 \\ 0 & \text{otherwise} \end{cases}$$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5\le X\le 0.5)$

12 **Example 11.12**

If X is the random variable with probability density function $f(x)$ given by,

$$f(x)=\begin{cases} x-1, & 1\le x<2 \\ -x+3, & 2\le x<3 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5\le X\le 2.5)$

13 EXERCISE 11.5

6. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) exactly 10 will have a useful life of at least 600 hours;

(ii) at least 11 will have a useful life of at least 600 hours;

(iii) at least 2 will *not* have a useful life of at least 600 hours.

14 **Example 11.22**

On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

CHAPTER -12 DISCRETE MATHEMATICS

1 EXERCISE 12.1

9. (i) Let $M=\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix}: x \in R-\{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.

(ii) Let $M=\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix}: x \in R-\{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the existence of identity, existence of inverse properties for the operation $*$ on M.

2 EXERCISE 12.1

10. (i) Let A be $\mathbb{Q}\setminus\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ binary on A? If so, examine the commutative and associative properties satisfied by $*$ on A.

(ii) Let A be $\mathbb{Q}\setminus\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A.

3 **Example 12.9**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

4 **Example 12.10**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A=\{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.

5 **Example 12.7**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$m*n=m+n-mn;\ m,n\in\mathbb{Z}$

6 **Example 12.19**

Using the equivalence property, show that $p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$.

7 EXERCISE 12.2

15. Prove that $p\rightarrow(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table.

8 EXERCISE 12.2

13. Using truth table check whether the statements $\neg(p\vee q)\vee(\neg p\wedge q)$ and $\neg p$ are logically equivalent.

IMPORTANT 2,3 MARK QUESTIONS (TENTATIVE)

12^{TH} MATHS

Touch the sum and get the SOLUTION video

Note : * IMPORTANT AND EASY PROBLEM ONLY HERE. FIRST PRACTICE THE FOLLOWING AND THEN THE OTHER PROBLEMS.

CHAPTER-1 APPLICATIONS OF MATRICES AND DETERMINANTS

1 EXERCISE 1.1

8. If $\text{adj}(A)=\begin{bmatrix}2 & -4 & 2\\ -3 & 12 & -7\\ -2 & 0 & 2\end{bmatrix}$, find A.

Example 1.5

Find a matrix A if $\text{adj}(A)=\begin{bmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{bmatrix}$.

2 EXERCISE 1.1

9. If $\text{adj}(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

Example 1.6

If $\text{adj}\,A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}.

3 EXERCISE 1.1

7. If $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

Example 1.9

Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{bmatrix}0 & -3\\ 1 & 4\end{bmatrix}$, $B=\begin{bmatrix}-2 & -3\\ 0 & -1\end{bmatrix}$.

4 EXERCISE 1.2

2. Find the rank of the following matrices by row reduction method:

(i) $\begin{bmatrix}1 & 1 & 1 & 3\\ 2 & -1 & 3 & 4\\ 5 & -1 & 7 & 11\end{bmatrix}$ (iii) $\begin{bmatrix}3 & -8 & 5 & 2\\ 2 & -5 & 1 & 4\\ -1 & 2 & 3 & -2\end{bmatrix}$

Example 1.18

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

5

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(i) $2x+5y=-2,\ x+2y=-3$

(ii) $2x-y=8,\ 3x+2y=-2$

Example 1.22

Solve the following system of linear equations, using matrix inversion method: $5x+2y=3,\ 3x+2y=5$.

6

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule:

(i) $5x-2y+16=0, x+3y-7=0$

(ii) $\frac{3}{x}+2y=12, \frac{2}{x}+3y=13$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

CHAPTER-2 COMPLEX NUMBERS

1 **Example 2.8**

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real

EXERCISE 2.4

7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

2 **Example 2.11**

Which one of the points $i, -2+i$, and 3 is farthest from the origin?

EXERCISE 2.5

3. Which one of the points $10-8i$, $11+6i$ is closest to $1+i$.

3 EXERCISE 2.5

4. If $|z|=3$, show that $7 \leq |z+6-8i| \leq 13$.

5. If $|z|=1$, show that $2 \leq |z^2-3| \leq 4$.

6. If $|z|=2$, show that $8 \leq |z+6+8i| \leq 12$.

Example 2.13

If $|z|=2$ show that $3 \leq |z+3+4i| \leq 7$

4 EXERCISE 2.5

10. Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$.

Example 2.17

Find the square root of $6-8i$.

5 EXERCISE 2.5

9. Show that the equation $z^3+2\bar{z}=0$ has five solutions.

Example 2.16

Show that the equation $z^2=\bar{z}$ has four solutions.

6

EXERCISE 2.6

1. If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$ show that the locus of z is real axis.

5. Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases:

(i) $|z-4|=16$ (ii) $|z-4|^2-|z-1|^2=16$.

Example 2.21

Obtain the Cartesian form of the locus of z in each of the following cases.

(i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$

7

EXERCISE 2.7

1. Write in polar form of the following complex numbers

(i) $2+i2\sqrt{3}$ (ii) $3-i\sqrt{3}$ (iii) $-2-i2$

Example 2.23

Represent the complex number (i) $-1-i$ (ii) $1+i\sqrt{3}$ in polar form.

8 **Example 2.32**

Find the cube roots of unity.

Example 2.33

Find the fourth roots of unity.

Example 2.29

Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

Example 2.31

Simplify (i) $(1+i)^{18}$ (ii) $(-\sqrt{3}+3i)^{31}$.

9 ## EXERCISE 2.8

1. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

2. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$.

8. If $\omega \neq 1$ is a cube root of unity, show that

(i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$.

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots(1+\omega^{2^{11}})=1$.

CHAPTER-3 THEORY OF EQUATIONS

1 EXERCISE 3.1

5. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$.

7. If α, β, and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the value of $\sum\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

Example 3.3

If α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

Example 3.4

Find the sum of the squares of the roots of $ax^4+bx^3+cx^2+dx+e = 0$.

2 EXERCISE 3.1

9. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$.

10. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

3 **Example 3.7**

If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.

EXERCISE 3.2

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$, in terms of k.

4 EXERCISE 3.2

4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

Example 3.10

Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

5 EXERCISE 3.2

5. Prove that a straight line and parabola cannot intersect at more than two points.

Example 3.14

Prove that a line cannot intersect a circle at more than two points.

6. EXERCISE 3.3

7. Solve the equation : $x^4-14x^2+45=0$

Example 3.16

Solve the equation $x^4-9x^2+20=0$.

7. EXERCISE 3.3

2. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

Example 3.19

Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

8

EXERCISE 3.5

1. Solve the following equations

(i) $\sin^2 x-5\sin x+4=0$ (ii) $12x^3+8x=29x^2-4$

Example 3.29

Find solution, if any, of the equation

$$2\cos^2 x-9\cos x+4 = 0$$

9. EXERCISE 3.6

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

Example 3.30

Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

CHAPTER-4 INVERSE TRIGONOMETRICAL FUNCTIONS

1 EXERCISE 4.1

4. Find the value of (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

Example 4.3

Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

2 EXERCISE 4.2

5. Find the value of

(i) $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

(iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

Example 4.6

Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

3 EXERCISE 4.2

7. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

8. Find the value of

(i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$ (ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

4 EXERCISE 4.3

4. Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$.

Example 4.10

Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

5 EXERCISE 4.4

2. Find the value of

(i) $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$ (ii) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$

(iii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$

Example 4.14

If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

6 EXERCISE 4.5

3. Find the value of

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$ (iii) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

8. Simplify: $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$.

Example 4.19

Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

Example 4.26

Show that $\cot(\sin^{-1}x)=\frac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\neq 0$

Example 4.27

Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$.

CHAPTER-5 2D ANALYTICAL GEOMETRY

1 EXERCISE 5.1

4. Find the equation of the circle with centre $(2,3)$ and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$.

12. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

2 EXERCISE 5.2

6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

Example 5.15

Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

3 EXERCISE 5.4

5. Find the equation of the tangent at $t=2$ to the parabola $y^2=8x$. (Hint: use parametric form)

8. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

4. **Example 5.32**

The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

Example 5.34

The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

Example 5.36

A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is $40\,cm$ wide from rim to rim and $30\,cm$ deep. The bulb is located at the focus .

(1) What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

CHAPTER-6 APPLICATIONS OF VECTOR ALGEBRA

1 **Example 6.1 (Cosine formulae)**

With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a^2=b^2+c^2-2bc\cos A$ (ii) $b^2=c^2+a^2-2ca\cos B$

(iii) $c^2=a^2+b^2-2ab\cos C$

2 **Example 6.2**

With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a=b\cos C+c\cos B$ (ii) $b=c\cos A+a\cos C$

(iii) $c=a\cos B+b\cos A$

3 **Example 6.4**

With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

4 EXERCISE 6.1

3. Prove by vector method that an angle in a semi-circle is a right angle.

5 6. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$.

6 EXERCISE 6.3

3. Prove that $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]=0$.

Example 6.18

If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}]=-[\vec{a},\vec{b},\vec{c}]$.

7 **Example 6.19**

Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

8. EXERCISE 6.4

5. Find the angle between the following lines.

(i) $\vec{r}=(4\hat{i}-\hat{j})+t(\hat{i}+2\hat{j}-2\hat{k})$, $\vec{r}=(\hat{i}-2\hat{j}+4\hat{k})+s(-\hat{i}-2\hat{j}+2\hat{k})$

(ii) $\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}$, $\vec{r}=4\hat{k}+t(2\hat{i}+\hat{j}+\hat{k})$.

(iii) $2x=3y=-z$ and $6x=-y=-4z$.

Example 6.29

Find the angle between the lines $\vec{r}=(\hat{i}+2\hat{j}+4\hat{k})+t(2\hat{i}+2\hat{j}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.

9. EXERCISE 6.5

2. Show that the lines $\vec{r}=(6\hat{i}+\hat{j}+2\hat{k})+s(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(3\hat{i}+2\hat{j}-2\hat{k})+t(2\hat{i}+4\hat{j}-5\hat{k})$ are skew lines and hence find the shortest distance between them.

5. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

Example 6.36

Find the shortest distance between the two given straight lines $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$.

10 EXERCISE 6.9

3. Find the angle between the line $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(6\hat{i}+3\hat{j}+2\hat{k}\right)=8$

4. Find the angle between the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}-2\hat{k}\right)=3$ and $2x-2y+z=2$.

Example 6.47

Find the acute angle between the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=11$ and $4x-2y+2z=15$.

Example 6.48

Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x-y+z=5$.

11 EXERCISE 6.9

8. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x+2y+3z=2$.

Example 6.55

Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}+4\hat{k}\right)=38$.

IMPORTANT 2,3 MARK QUESTIONS (TENTATIVE)

12TH MATHS

Touch the sum and get the SOLUTION video

Note : * IMPORTANT AND EASY PROBLEM ONLY HERE. FIRST PRACTICE THE FOLLOWING AND THEN THE OTHER PROBLEMS.

CHAPTER-7 APPLICATIONS OF DIFFERENTIAL CALCULUS

1 EXERCISE 7.1

2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground?

4. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.
5. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres.

2 EXERCISE 7.3

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :
   (i) $f(x) = x^2 - x, x \in [0,1]$
4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
   (i) $f(x) = x^3 - 3x + 2, x \in [-2,2]$

3 EXERCISE 7.3

6. A race car driver is kilometer stone 20. If his speed never exceeds 150 km/hr, what is the maximum kilometer he can reach in the next two hours.

7. Suppose that for a function $f(x), f'(x) \le 1$ for all $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$.

10. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

4 EXERCISE 7.4

1. **Write the Maclaurin series expansion of the following functions:**
   (i) e^x (ii) $\sin x$ (iii) $\cos x$
   (iv) $\log(1-x)$; $-1 \le x < 1$ (v) $\tan^{-1}(x)$; $-1 \le x \le 1$

2. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$.

5 EXERCISE 7.5

3. $\lim_{x\to\infty} \frac{x}{\log x}$

6. $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

Example 7.42

Evaluate : $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m\in N$.

6 EXERCISE 7.6

1. Find the absolute extrema of the following functions on the given closed interval.

(iii) $f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}$; $[-1,1]$

7 EXERCISE 7.6

2. Find the intervals of monotonicities and hence find the local extremum for the following functions:

(iii) $f(x)=\frac{e^x}{1-e^x}$

Example 7.51

Find the intervals of monotonicity and hence find the local extrema for the function $f(x)=x^{\frac{2}{3}}$.

Example 7.56

Find the intervals of monotonicity and local extrema of the function $f(x)=\frac{x}{1+x^2}$.

8 EXERCISE 7.7

1. Find intervals of concavity and points of inflexion for the following functions:

(iii) $f(x)=\frac{1}{2}(e^x-e^{-x})$

Example 7.58

Determine the intervals of concavity of the curve $y=3+\sin x$.

9 EXERCISE 7.8

1. Find two positive numbers whose sum is 12 and their product is maximum.
2. Find two positive numbers whose product is 20 and their sum is minimum.

10

EXERCISE 7.9

1. Find the asymptotes of the following curves :

(i) $f(x)=\frac{x^2}{x^2-1}$ (ii) $f(x)=\frac{x^2}{x+1}$

(iv) $f(x)=\frac{x^2-6x-1}{x+3}$ (v) $f(x)=\frac{x^2+6x-4}{3x-6}$

CHAPTER-8 DIFFERENTIAL AND PARTIAL DERIVATIVES

1 EXERCISE 8.1

1. Let $f(x)=\sqrt[3]{x}$. Find the linear approximation at $x=27$. Use the linear approximation to approximate $\sqrt[3]{27.2}$.

2. Use the linear approximation to find approximate values of

$$\text{(i) } (123)^{\frac{2}{3}} \qquad \text{(ii) } \sqrt[4]{15} \qquad \text{(iii) } \sqrt[3]{26}$$

Example 8.1

Find the linear approximation for $f(x)=\sqrt{1+x}, x\geq -1$, at $x_0=3$. Use the linear approximation to estimate $f(3.2)$.

Example 8.2

Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

2 EXERCISE 8.1

6. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T=2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

7. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

3 EXERCISE 8.2

6. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

11. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

Example 8.7

If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

4 EXERCISE 8.3

1. Evaluate $\lim\limits_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y)=\frac{3x^2-xy}{x^2+y^2+3}$.

2. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\frac{x^3+y^2}{x+y+2}\right)$. If the limit exists.

3. Let $f(x,y)=\frac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for $(x,y)\neq(0,0)$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

4. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\frac{e^x\sin y}{y}\right)$, if the limit exists.

Example 8.8

Let $f(x,y)=\dfrac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y)\in\mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$.

5 EXERCISE 8.4

3. If $U(x,y,z)=\dfrac{x^2+y^2}{xy}+3z^2y$, find $\dfrac{\partial U}{\partial x},\dfrac{\partial U}{\partial y}$, and $\dfrac{\partial U}{\partial z}$.

4. If $U(x,y,z)=\log(x^3+y^3+z^3)$, find $\dfrac{\partial U}{\partial x}+\dfrac{\partial U}{\partial y}+\dfrac{\partial U}{\partial z}$.

Example 8.15

Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

6. EXERCISE 8.5

1. If $w(x,y)=x^3-3xy+2y^2,\ x,y\in\mathbb{R}$, find the linear approximation for w at $(1,-1)$.

Example 8.17

Let $U(x,y,z)=x^2-xy+3\sin z,\ x,y,z\in\mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$.

7 EXERCISE 8.5

4. Let $V(x,y,z)=xy+yz+zx,\ x,y,z\in\mathbb{R}$. Find the differential dV.

Example 8.16

If $w(x,y,z)=x^2y+y^2z+z^2x,\ x,y,z\in\mathbb{R}$, find the differential dw.

8. EXERCISE 8.6

1. If $u(x,y)=x^2y+3xy^4,\ x=e^t$ and $y=\sin t$, find $\dfrac{du}{dt}$ and evaluate it at $t=0$.

Example 8.20

Let $g(x,y)=2y+x^2, x=2r-s, y=r^2+2s, r,s\in\mathbb{R}$. Find $\dfrac{\partial g}{\partial r},\dfrac{\partial g}{\partial s}$.

9. EXERCISE 8.7

4. If $u(x,y)=\dfrac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{3}{2}u$.

Example 8.21

Show that $F(x,y)=\dfrac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

CHAPTER-9 APPLICATIONS OF INTEGRAL CALCULUS

1

Example 9.7

Evaluate : $\int_0^1 [2x]\,dx$ where $[\cdot]$ is the greatest integer function.

Example 9.14

Evaluate : $\int_0^{1.5} \left[x^2\right] dx$, where $[x]$ is the greatest integer function.

2 EXERCISE 9.3

2. Evaluate the following integrals using properties of integration :

(ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\cos x + \tan^3 x + 1)\,dx$ (x) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$

Example 9.26

Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$.

Example 9.29

Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$.

3 EXERCISE 9.4

3. $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ 4. $\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx$

Example 9.34

Evaluate : $\int_{-1}^{1} e^{-\lambda x}(1-x^2)\,dx$.

4 EXERCISE 9.7

Evaluate the following (iii) $\int_0^{\frac{\pi}{4}} \sin^6 2x\,dx$ (iv) $\int_0^{\frac{\pi}{6}} \sin^5 3x\,dx$

(v) $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^4 x\,dx$ (vi) $\int_0^{2\pi} \sin^7 \frac{x}{4}\,dx$

Example 9.37

Evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right)dx$

Example 9.38

Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.

5 EXERCISE 9.7

Evaluate the following (ii) $\int_{0}^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\, dx$

2. If $\int_{0}^{\infty} e^{-\alpha x^2} x^3 dx = 32, \ \alpha > 0$, find α

6 **Example 9.50**

Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

Example 9.52

Find the area of the region bounded by $x-$ axis, the sine curve $y = \sin x$, the lines $x = 0$ and $x = 2\pi$.

7 ## EXERCISE 9.9

1. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$, $y = 0$ and $x = 1$.

2. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = e^{-2x}$ $y = 0$, $x = 0$ and $x = 1$

Example 9.62

Find the volume of a sphere of radius a.

CHAPTER-10 DIFFERENTIAL EQUATIONS

1 ## EXERCISE 10.3

7. Find the differential equation corresponding to the family of curves represented by the equation $y - \text{A}e^{8x} + \text{B}e^{-8x}$, where A and B are arbitrary constants.

Example 10.3

Form the differential equation by eliminating the arbitrary constants A and B from $y = \text{A}\cos x + \text{B}\sin x$.

2 ## EXERCISE 10.3

4. Find the differential equation of the family of all the parabolas with latus rectum $4a$ and whose axes are parallel to the x-axis.

Example 10.5

Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

3 **EXERCISE 10.3**

6. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

Example 10.6

Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

4 **EXERCISE 10.4**

4. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

8. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2 y = 0$.

Example 10.7

Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

Example 10.8

Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.

5 EXERCISE 10.5

4. Solve the following differential equations:

(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ (ii) $ydx + (1+x^2)\tan^{-1}x\, dy = 0$

Example 10.11

Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$.

Example 10.12

Find the particular solution of $(1+x^3)dy - x^2 y dx = 0$ satisfying the condition $y(1) = 2$.

CHAPTER-11 PROBABILITY DISTRIBUTIONS

1 **EXERCISE 11.2**

1. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

3. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

Example 11.6

A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

2

EXERCISE11.3

1. The probability density function of X is given by $f(x)=\begin{cases} k x e^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$.

Find the value of k.

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}(x^2+x) & 0\le x<1 \\ 1, & 1\le x<\infty \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3\le X\le 0.6)$

Example 11.13

If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & x<0 \\ x, & 0\le x<1 \\ 1, & 1\le x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2\le X\le 0.7)$.

4

EXERCISE 11.4

2. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

Example 11.17

Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance.

EXERCISE 11.4

5. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. It is known that the pdf of X is

$$f(x)=\begin{cases}\frac{1}{30} & 0<x<30\\ 0 & \text{elsewhere}\end{cases}$$

Obtain and interpret the expected value of the random variable X.

6. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}.$$

Find the expected life of this electronic equipment.

7. The probability density function of the random variable X is given by

$$f(x)=\begin{cases}16xe^{-4x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

find the mean and variance of X.

Example 11.18

Find the mean and variance of a random variable X, whose probability density function is

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\ge 0\\ 0 & \text{otherwise}\end{cases}$$

EXERCISE 11.5

2. The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

3. Using binomial distribution find the mean and variance of X for the following experiments
   (i) A fair coin is tossed 100 times, and X denote the number of heads.
   (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

9. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

Example 11.20

A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

CHAPTER-12 DISCRETE MATHEMATICS

1

EXERCISE 12.1

2. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

3. Let $*$ be defined on $\mathbb{R}$ by $(a*b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3*\left(\frac{-7}{15}\right)$.

Example12.1

Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary):

(i) $a*b = a + 3ab - 5b^2; \forall a, b \in \mathbb{Z}$ (ii) $a*b = \left(\frac{a-1}{b-1}\right), \forall a, b \in \mathbb{Q}$

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8. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \to q) \equiv p \wedge \neg q$.

9. Prove that $q \to p \equiv \neg p \to \neg q$

10. Show that $p \to q$ and $q \to p$ are not equivalent

11. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

Example 12.16

Construct the truth table for $(p \,\overline{\vee}\, q) \wedge (p \,\overline{\vee}\, \neg q)$.

Example 12.17

Establish the equivalence property: $p \to q \equiv \neg p \vee q$