Full Portion Test - 1

Standard XII

MATHEMATICS

Time: 3.00 hrs. Marks: 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2) Use Blue or Black ink to write.

PART - I 20x1=20

Note: i) Answer all the questions.

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $|\operatorname{adj}(\operatorname{adj} A)| = |A|^9$, then the order of the square matrix A is

a) 3 b) 4 c) 2 d) 5

2. If $A^T A^{-1}$ is symmetric, then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. The area of the triangle formed by the complex numbers z, iz and $z + iz$ in the Argand's diagram is

a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ d) $2|z|^2$

4. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

a) -2 b) -1 c) 1 d) 2

5. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma \frac{1}{\alpha}$ is

a) $-\frac{q}{r}$ b) $-\frac{p}{r}$ c) $\frac{q}{r}$ d) $-\frac{q}{p}$

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

a) $|k| \leq 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \geq 6$

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in \mathbb{R}$, the value of $\tan^{-1} x$ is

a) $-\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{5}$

8. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$

9. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

10. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

11. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

12. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda>0$ is $\frac{1}{5}$, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 0 d) 1

13. The tangent to the curve $y^2-xy+9=0$ is vertical when

a) $y=0$ b) $y=\pm\sqrt{3}$ c) $y=\frac{1}{2}$ d) $y=\pm 3$

14. The minimum value of the function $|3-x|+9$ is

a) 0 b) 3 c) 6 d) 9

15. If $v(x,y)=\log(e^x+e^y)$, then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}$ is equal to

a) e^x+e^y b) $\frac{1}{e^x+e^y}$ c) 2 d) 1

16. If $f(x)=\frac{x}{x+1}$, then its differential is given by

a) $\frac{-1}{(x+1)^2}dx$ b) $\frac{1}{(x+1)^2}dx$ c) $\frac{1}{x+1}dx$ d) $\frac{-1}{x+1}dx$

17. If $f(x)=\int_0^x t\cos t\,dt$, then $\frac{df}{dx}=$ ....

a) $\cos x - x\sin x$ b) $\sin x + x\cos x$ c) $x\cos x$ d) $x\sin x$

18. The order and degree of $\left(\frac{d^2y}{dx^2}\right)^3=\sqrt{1+\frac{dy}{dx}}$

a) 6, 2 b) 2, 6 c) 2, 3 d) 3, 2

19. A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviation of X is

a) 6 b) 4 c) 3 d) 2

20. The proposition $p\wedge(\neg p\vee q)$ is

a) a tautology b) a contradiction

c) logically equivalent to $p\wedge q$ d) logically equivalent to $p\vee q$

PART - II

Note: i) Answer any seven questions.
ii) Question no.30 is compulsory. 7x2=14

21. If $\text{adj}A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}.

$-1(1-4)-2(1-4)+2(2-2)$

$-1(-3)-2(-3)$

$+3+6$

22. If $z_1 = 2 - i$ and $z_2 = -4 + 3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$.

23. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

24. The line $3x + 4y - 12 = 0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

25. Find the distance of a point $(2, 5, -3)$ from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 5$.

26. Evaluate : $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$.

28. If $\int_0^\infty e^{-\alpha x^2} x^3 dx = 32$, $\alpha > 0$, find α.

29. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

30. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

PART - III

Note : i) Answer any seven questions.
ii) Question no. 40 is compulsory.

7x3=21

31. Show that the equation $z^2 = \bar{z}$ has four solutions.

32. If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of in terms of $\Sigma \frac{\alpha}{\beta\gamma}$ the coefficients.

33. Find the domain of $\sin^{-1}(2 - 3x^2)$.

34. Find the equations of the two tangents that can be drawn from $(5, 2)$ to the ellipse $2x^2 + 7y^2 = 14$.

35. Show that the four points $(6, -7, 0), (16, -19, -4), (0, 3, -6), (2, -5, 10)$ lie on a same plane.

36. Compute the value of 'c' satisfied by Rolle's theorem for the function $f(x) = \log\left(\frac{x^2 + 6}{5x}\right)$ in the interval $[2, 3]$.

37. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

38. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}} dx$

39. Solve : $\frac{dy}{dx} = \tan^2(x + y)$

40. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

PART - IV

7x5=35

Note: Answer all the questions.

41. a) Solve the following system of linear equations by matrix inversion method.
$x + y + z - 2 = 0,\ 6x - 4y + 5z - 31 = 0,\ 5x + 2y + 2z = 13$

(OR)

b) By using Gaussian elimination method, balance the chemical reaction equations:
$C_2H_6 + O_2 \to H_2O + CO_2$

42. a) If $z = x + iy$ is a complex number such that $\text{Im}\left(\dfrac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

(OR)

b) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

43. a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

(OR)

b) Identify the type of conic and find centre, foci, vertices and directrices of the following.
$18x^2 + 12y^2 - 144x + 48y + 120 = 0$.

44. a) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i} + 2\hat{j} + \hat{k}) = 8$.

(OR)

b) Expand $\tan x$ in ascending powers of x upto 5^{th} power for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

45. a) If $U = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\tan u$.

(OR)

b) Evaluate the integral using properties of integration: $\displaystyle\int_{\pi/8}^{3\pi/8} \frac{1}{1+\sqrt{\tan x}}\,dx$

46. a) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

(OR)

b) Solve : $(1 + x^3)\dfrac{dy}{dx} + 6x^2y = 1 + x^2$

47. a) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

(OR)

b) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

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