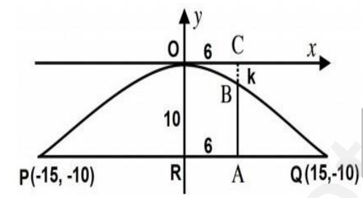
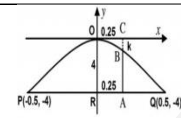
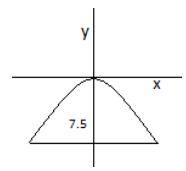
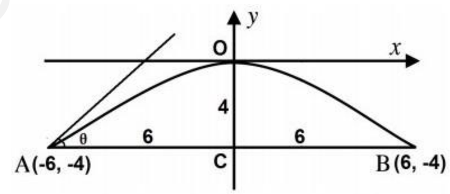
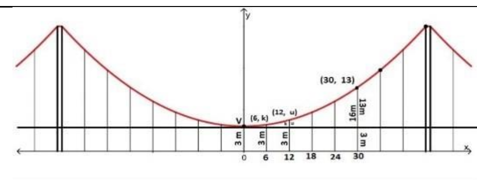
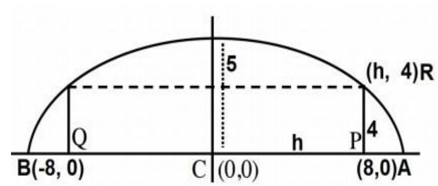
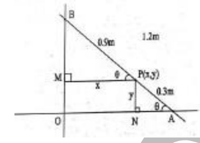
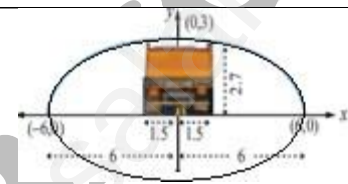
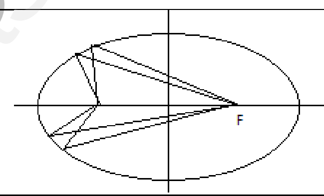
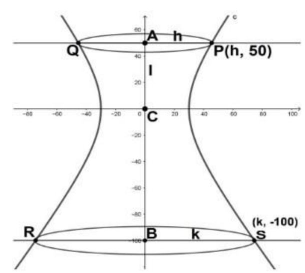
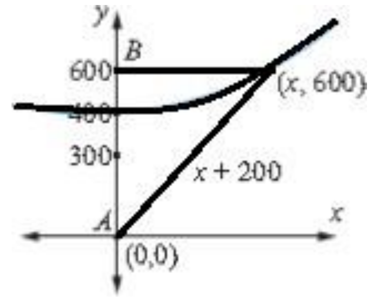
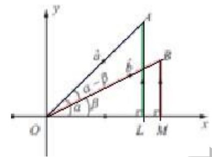
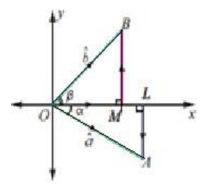
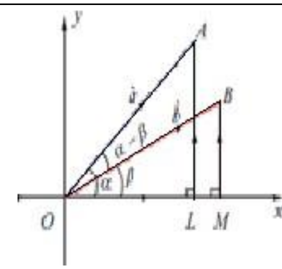
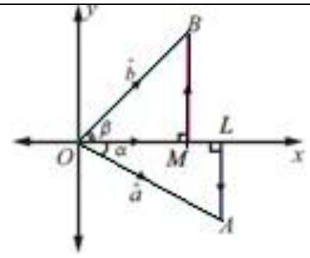
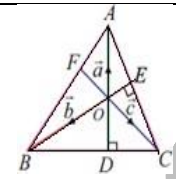
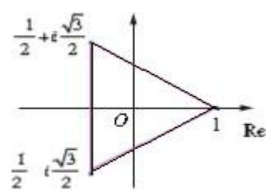
12STANDARDMATHEMATICS(Volume-I)

(5MarkquestionandanswerforslowlearnersOnly)

1)**Abridgehasaparabolicarchthatis10mhighatthecentre and 30m wide at the bottom. Find the height of the arch 6m from the centre on either sides.**		**Equationofpaparabola** $x^2 = -4ay$ a=$\frac{225}{40}$ **height** = 8.4m
2)**Atawaterfountain,waterattainsamaximumheightof4*m* at horizontal distance of 0 5 . *m* from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 075.*m* from the point of origin.**		**Equationofpaparabola** $x^2 = -4ay$ a=$\frac{0.25}{16}$ height = 3m
3)**Assume that water issuingfromthe end of ahorizontal pipe, 75.*m*abovetheground,describesaparabolicpath.Thevertex oftheparabolicpathisattheendofthepipe.Ataposition 25. *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?**		**Equationofpaparabola** $x^2 = -4ay$ a = $\frac{9}{10}$ height = $3\sqrt{3}$m
4) **On lighting a rocket cracker it gets projected in a parabolic path andreaches amaximumheight of 4*m*whenitis6*m* away fromthepointofprojection.Finallyitreachestheground 12*m* away from the starting point. Find the angle of projection.**		**Equationofpaparabola** $x^2 = -4ay$ a = $\frac{9}{4}$ Angle = $\tan^{-1}\left(\frac{4}{3}\right)$
5)**Parabolic cable of a 60*m* portion of the roadbed of a suspensionbridgearepositionedasshownbelow.Vertical Cables are to be spaced every 6*m* along this portion of the roadbed.Calculatethelengthsoffirsttwoofthesevertical cables from the vertex.**		**Equationofpaparabola** $x^2 = 4ay$ 4a=$\frac{900}{13}$ length = 5.08m

6)**A tunnel through a mountain for a four lane highway is to haveaellipticalopening.Thetotalwidthofthehighway(notthe opening) is to be 16m and the height at the edges of the road mustbesufficientforatruck4mhightoclearifthehighestpoint of the opening is to be 5m approximately. how wide must the opening be ?**		**EquationofEllipse** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $b^2=25$,a=13.33 **wide= 26.7m**
7)**A rod of length** 1 2 . *m* **moves with its ends always touching thecoordinateaxes.Thelocusofapoint** *P***ontherod,whichis** 0 3 . *m* **from the end in contact with** *x* **-axis is an ellipse. Find the eccentricity.**		**EquationofEllipse** $\frac{x^2}{0.9^2}+\frac{y^2}{0.3^2}=1$ **eccentricity=**$\frac{2\sqrt{2}}{3}$
8)**A semielliptical archway over a one-way road has a height of 3*m*andawidthof** 12*m*.**Thetruckhasawidthof3*m*andaheight of** 27.*m*. **Will the truck clear the opening of the archway?**		**EquationofEllipse** $\frac{x^2}{6^2}+\frac{y^2}{3^2}=1$ **height=2.9m**
9)**If theequationoftheellipseis** $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ **(*x*and*y*are measuredincentimeters)wheretothenearestcentimeter, shouldthepatient'skidneystonebeplaced sothat thereflected soundhitsthekidney stone?**		$a^2=484$ $b^2=64$ $ae=\sqrt{a^2-b^2}$ $ae=\sqrt{420}=20.5$cm
10)**CrosssectionofaNuclearcoolingtowerisin theshapeofa hyperbolawithequation** $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$**Thetoweris150*m*talland the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centreofthehyperbola.Findthediameterofthetopandbaseof the tower.**		**EquationofHyperbola** $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$ **Topdiameter=90.82m** **Basediameter=148.98m**

11)**Twocoastguardstationsarelocated 600kmapartatpoints *A*(0,0) and *B*(0,600) . A distress signal from a ship at *P* is received at slightly different times by two stations. It is determinedthattheshipis200kmfartherfromstation *A*thanit isfromstation*B*.Determinetheequationofhyperbolathat passes through the location of the ship.**		$a^2=10000$ $b^2=80000$ EquationofHyperbola $\frac{(y-300)^2}{10000}-\frac{x^2}{80000}=1$
12)$Cos(\alpha-\beta)=Cos\alpha Cos\beta+Sin\alpha Sin\beta$ **Provebyvectormethod.**		$\hat{a}=cos\alpha\hat{i}+sin\alpha\hat{j}$ $\hat{b}=cos\beta\hat{i}+sin\beta\hat{j}$ $\hat{a}.\hat{b}=\cos(\alpha-\beta)$ $\hat{a}.\hat{b}=Cos\alpha Cos\beta+Sin\alpha Sin\beta$ **Cos(α−β)=CosαCosβ+ SinαSinβ**
13)$Cos(\alpha+\beta)=Cos\alpha Cos\beta-Sin\alpha Sin\beta$ **Provebyvectormethod.**		$\hat{a}=cos\alpha\hat{i}-sin\alpha\hat{j}$ $\hat{b}=cos\beta\hat{i}+sin\beta\hat{j}$ $\hat{a}.\hat{b}=\cos(\alpha+\beta)$ $\hat{a}.\hat{b}=Cos\alpha Cos\beta-Sin\alpha Sin\beta$ **Cos(α+ β)=CosαCosβ− SinαSinβ**
14)$Sin(\alpha-\beta)=Sin\alpha\ Cos\beta-Cos\alpha\ Sin\beta$ **Provebyvectormethod.**		$\hat{a}=cos\alpha\hat{i}+sin\alpha\hat{j}$ $\hat{b}=cos\beta\hat{i}+sin\beta\hat{j}$ $\hat{b}\times\hat{a}=\hat{k}Sin(\alpha-\beta)$ $\hat{b}\times\hat{a}=\hat{k}(Sin\alpha Cos\beta-Cos\alpha Sin\beta)$ **Sin(α−β)=SinαCosβ −CosαSinβ**

15)$Sin(\alpha+\beta)=Sin\alpha Cos\beta+Cos\alpha Sin\beta$ **Provebyvectormethod.**		$\hat{a}=cos\alpha\hat{i}-sin\alpha\hat{j}$ $\hat{b}=cos\beta\hat{i}+sin\beta\hat{j}$ $\hat{b}\times\hat{a}=\hat{k}Sin(\alpha+\beta)$ $\hat{b}\times\hat{a}=\hat{k}(Sin\alpha Cos\beta+Cos\alpha Sin\beta)$ $Sin(\alpha+\beta)=Sin\alpha Cos\beta+Cos\alpha Sin\beta$
16)Prove by vector method that the perpendiculars (attitudes)fromtheverticestotheoppositesidesofatriangle are concurrent.		$\vec{a}\cdot\vec{c}-\vec{a}\cdot\vec{b}=0$ $\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}=0$ **Adding** $\vec{a}\cdot\vec{c}-\vec{b}\cdot\vec{c}=0$ $\vec{CO}\cdot\vec{AB}=0$
17)Showthatthepoint $1,\ \frac{-1}{2}+\frac{i\sqrt{3}}{2}$ **and** $\frac{-1}{2}-\frac{i\sqrt{3}}{2}$ **arethe verticesofanequilateral triangle.**		$z_1=1;z_2=\frac{-1}{2}+\frac{i\sqrt{3}}{2};z_3=\frac{-1}{2}-\frac{i\sqrt{3}}{2}$ $\lvert z_1-z_2\rvert=\sqrt{3}$ $\lvert z_1-z_2\rvert=\sqrt{3}$ $\lvert z_1-z_2\rvert=\sqrt{3}$ Sidesareequal.Henceequilateral triangle.
18)Showthat $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ **ispurelyimaginary.**	$\frac{19+9i}{5-3i}=2+3i$ $\frac{8+i}{1+2i}=2-3i$	$z=(2+3i)^{15}-(2-3i)^{15}$ $\bar{z}=(2-3i)^{15}-(2+3i)^{15}$ $\bar{z}=-z$ purely imaginary

19)showthat $(\frac{19-7i}{9+i})^{12}+(\frac{20-5i}{7-6i})^{12}$ ispurelyreal.	$\frac{19-7i}{9+i}=2-i$ $\frac{20-5i}{7-6i}=2+i$	$z=(2-i)^{12}+(2+i)^{12}$ $\bar{z}=(2+i)^{12}+(2-i)^{12}$ $\bar{z}=z$ Purelyreal
20)If $z=x+iy$ and $arg(\frac{z-1}{z+1})=\frac{\pi}{2}$ thenshowthat $x^2+y^2=1$	$Re(\frac{z-1}{z+1})=0$ $Re(\frac{x+iy-1}{x+iy+1})=0$	$Re(\frac{x-1+iy}{x+1+iy})=0$ $x^2+y^2=1$
21)If $Im(\frac{2z+1}{iz+1})=0$ thenshowthatthelocusofz-is $2x^2+2y^2+x-2y=0$	$z=x+iy$ $Im(\frac{2(x+iy)+1}{i(x+iy)+1})=0$ $Im(\frac{2x+1+i2y}{1-y+ix})=0$	$2x^2+2y^2+x-2y=0$
22)*If* $z=x+iy$ and $arg(\frac{z-i}{z+2})=\frac{\pi}{4}$ thenshowthat thelocusis $x^2+y^2+3x-3y+2=0$	$z=x+iy$ $\arg(z-i)-\arg(z+2)=\frac{\pi}{4}$ $\arg(x+iy-i)-\arg(x+iy+2)=\frac{\pi}{4}$	$tan^{-1}(\frac{y-1}{x})-tan^{-1}(\frac{y}{x+2})=\frac{\pi}{4}$ $x^2+y^2+3x-3y+2=0$
23)If $\frac{1+z}{1-z}=cos2\theta+isin2\theta$ thenshowthat $z=itan$	$\frac{1+z}{1-z}=e^{i2\theta}=e^{i\theta}\times e^{i\theta}$ $\frac{1+z}{1-z}=\frac{e^{i\theta}}{e^{-i\theta}}$	$\frac{1+z}{1-z}=\frac{cos\theta+isin\theta}{cos\theta-isin\theta}$ $\frac{1+z}{1-z}=\frac{1+itan\theta}{1-itan\theta}$ $z=itan\theta$

24) If z_1, z_2, and z_3 are three complex numbers such that $\|z_1\| = 1, \|z_2\| = 2, \|z_3\| = 3$, and $\|z_1+z_2+z_3\|=1$ then show that $\|9z_1z_2+4z_1z_3+z_2z_3\|=6$	$z_1=\frac{1}{\bar{z}_1}; z_2=\frac{4}{\bar{z}_2}; z_3=\frac{9}{\bar{z}_3}$ $\left\|\frac{1}{\bar{z}_1}+\frac{4}{\bar{z}_2}+\frac{9}{\bar{z}_3}\right\|=1$	$\left\|\frac{\bar{z}_2\bar{z}_3+4\bar{z}_1\bar{z}_3+9\bar{z}_1\bar{z}_2}{\bar{z}_1\bar{z}_2\bar{z}_3}\right\|=1$ $\|9z_1z_2+4z_1z_3+z_2z_3\|=6$
25) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$ then show that (i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$ (ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$	$a=cis\alpha$ $b=cis\beta$ $c=cis\gamma$ $a+b+c=0$ $a^3+b^3+c^3=3abc$	$Cis3\alpha+Cis3\beta+Cis3\gamma=3Cis(\alpha+\beta+\gamma)$ Equating real and imaginary parts, we get (i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$ (ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$
26) If $z=\cos\theta+i\sin\theta$ then show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$	$z^n=\cos n\theta+i\sin n\theta \rightarrow (1)$ $\frac{1}{z^n}=\cos n\theta-i\sin n\theta \rightarrow (2)$	$(1)+(2) \Rightarrow z^n+\frac{1}{z^n}=2\cos n\theta$ $(1)-(2) \Rightarrow z^n-\frac{1}{z^n}=2i\sin n\theta$
27) If $2\cos\alpha=x+\frac{1}{x}$ and $2\cos\beta=y+\frac{1}{y}$, then prove the following (i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ (ii) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$ (iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$ (iv) $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$	$x=\cos\alpha+i\sin\alpha=cis\alpha$ $y=\cos\beta+i\sin\beta=cis\beta$ i) $\frac{x}{y}+\frac{y}{x}=cis(\alpha-\beta)+\frac{1}{cis(\alpha-\beta)}$ $=\cos(\alpha-\beta)$ ii) $xy-\frac{1}{xy}=cis(\alpha+\beta)-\frac{1}{cis(\alpha+\beta)}$ $=2i\sin(\alpha+\beta)$	(iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}$ $=cis(m\alpha-n\beta)-\frac{1}{cis(m\alpha-n\beta)}$ $=2i\sin(m\alpha-n\beta)$ (iv) $x^my^n+\frac{1}{x^my^n}$ $=cis(m\alpha+n\beta)+\frac{1}{cis(m\alpha+n\beta)}$ $=2\cos(m\alpha+n\beta)$

28) $If\ \omega\neq1$ isacuberootofunitythenshow thattherootsoftheequation $(z-1)^3+8=0$ are $-1,1-2\omega,1-2\omega^2$	$(z-1)^3=-8=(-2)^3$ $\left(\frac{z-1}{-2}\right)^3=1$ $\frac{z-1}{-2}=1^{1/3}$	$z-1=-2\ \{1,\omega,\omega^2\}$ Rootsare $=-1,1-2\omega,1-2\omega^2$
29)Showthatthevaluesof $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1\pm i)$	$z=\sqrt[4]{-1}$ $z^4=-1=i^2$ $z^2=\pm i=\pm\frac{2i}{2}$	$z^2=\frac{1+i^2\pm2i}{2}$ $z^2=\frac{(1\pm i)^2}{2}$ $z=\pm\frac{1}{\sqrt{2}}(1\pm i)$
30)Solvetheequation $6x^4-5x^3-38x^2-5x+6=0$ ifoneof therootis $\frac{1}{3}$	$\frac{1}{3}$ | 6 -5 -38 -5 6 0 2 -1 -13 -6 3 | 6 -3 -39 -18 0 0 18 45 18 -2 | 6 15 6 0 0 -12 -6 $\frac{-1}{2}$ | 6 3 0 0 -3 6 0	Rootsare $\frac{1}{3},3,\frac{-1}{2},-2$

31) Solve: $6x^4-35x^3+62x^2-35x+6=0$	$\begin{array}{r\|rrrrr} & 6 & -35 & 62 & -35 & 6 \\ 2 & 0 & 12 & -46 & 32 & -6 \\ \hline & 6 & -23 & 16 & -3 & 0 \\ 1/2 & 0 & 3 & -10 & 3 & \\ \hline & 6 & -20 & 6 & 0 & \\ 3 & 0 & 18 & -6 & & \\ \hline & 6 & -2 & 0 & & \\ 1/3 & 0 & 2 & & & \\ \hline & 6 & 0 & & & \end{array}$	**Roots are** $2, \frac{1}{2}, 3, \frac{1}{3}$
32) If the equations $x^2+px+q=0$ **and** $x^2+p'x+q'=0$ **have a common root, show that it is equal to** $\frac{pq'-p'q}{q-q'}$ **or** $\frac{q-q'}{p'-p}$	Common root is α then $\alpha^2+p\alpha+q=0$ $\alpha^2+p'\alpha+q'=0$ $\begin{bmatrix} p & q & 1 & p \\ p' & q' & 1 & p' \end{bmatrix}$	$\frac{\alpha^2}{pq'-qp'}=\frac{\alpha}{q-q'}=\frac{1}{p'-p}$ $\alpha=\frac{pq'-p'q}{q-q'}$ or $\alpha=\frac{q-q'}{p'-p}$
33) If $2+i$ **and** $3-\sqrt{2}$ **are roots of the equation** $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ **Find all roots.**	Other roots are $2-i, 3+\sqrt{2}$ $\sum_1=13$ and $\sum_6=-140$ $\alpha+\beta=3$ and $\alpha\beta=-4$	**Roots are** $-1, 4, \quad 2\pm i, \quad 3\pm\sqrt{2}$
34) **Find all the zeroes of the polynomial** $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$**, if it is known that** $1+2i$ **and** $\sqrt{3}$ **are two of its zeroes.**	Other roots $1-2i, -\sqrt{3}$ $\sum_1=3$ and $\sum_6=135$ $\alpha+\beta=1$ and $\alpha\beta=-9$	**Roots are** $1\pm 2i, \quad \pm\sqrt{3}, \quad \frac{1\pm\sqrt{37}}{2}$
35) Solve: $8x^{\frac{3}{2n}}-8x^{\frac{-3}{2n}}=63$	$x^{\frac{3}{2n}}-x^{\frac{-3}{2n}}=\frac{63}{8}$ $x^{\frac{3}{2n}}-\frac{1}{x^{\frac{3}{2n}}}=\frac{63}{8}$	$x^{\frac{3}{2n}}-\frac{1}{x^{\frac{3}{2n}}}=8-\frac{1}{8}$ $x^{\frac{3}{2n}}=8 \Rightarrow x=4^n$

36)Solve: $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$	$2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$ $\div\sqrt{6}\quad\Rightarrow\sqrt{\frac{2x}{3a}}+\sqrt{\frac{3a}{2x}}=\frac{b}{\sqrt{6a}}+\frac{\sqrt{6a}}{b}$	$\sqrt{\frac{2x}{3a}}=\frac{b}{\sqrt{6a}}$ and $\sqrt{\frac{3a}{2x}}=\frac{\sqrt{6a}}{b}$ $x=\frac{b^2}{4a}$ and $x=\frac{9a^3}{b^2}$
37)Find the non-parametric form of vector equation and Cartesian equation of the plane passingthroughthepoint(0,1,-5)–andparallelto the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$	$\vec{a}=0\vec{i}+\vec{j}-5\vec{k}$ $\vec{u}=2\vec{i}+3\vec{j}+6\vec{k}$ $\vec{v}=\vec{i}+\vec{j}-\vec{k}$ Non-parametricvectorequation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(9\vec{i}-8\vec{j}+\vec{k})=-13$	Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ $\begin{vmatrix} x-0 & y-1 & z+5 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}=0$ $9x-8y+z=-13$
38) Find the non-parametric form of vector equtiion, and cartesaian equation of the plane passingthroughthepoint(2,3,6)andparallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ **and** $\frac{x+1}{2}=\frac{y-1}{-5}=\frac{z+1}{-3}$	$\vec{a}=2\vec{i}+3\vec{j}+6\vec{k}$ $\vec{u}=2\vec{i}+3\vec{j}+1\vec{k}$ $\vec{v}=2\vec{i}-5\vec{j}-3\vec{k}$ Non-parametricvectorequation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(\vec{i}-2\vec{j}+4\vec{k})=20$	Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-3 & z-6 \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix}=0$ $x-2y+4z=20$
39) Find the non-parametric form of vector equation of the plane passing through the point(1,-2,4)andperpendiculartotheplane $x+2y-3z=11$ and parallelto theline $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$	$\vec{a}=\vec{i}-2\vec{j}+4\vec{k}$ $\vec{u}=1\vec{i}+2\vec{j}-3\vec{k}$ $\vec{v}=3\vec{i}-\vec{j}+\vec{k}$ **Non-parametricvectorequation** $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(\vec{i}+10\vec{j}+7\vec{k})=9$	**Cartesianequation** $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ $\begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{vmatrix}=0$ $x+10y+7z=9$

40)Findtheparametricformofvectorequation andCartesainequationsoftheplanecontaining the line $\vec{r}= (\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ **and perpendicular to the plane** $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=8$	$\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ $\vec{u}=2\hat{i}-\hat{j}+4\hat{k}$ $\vec{v}=\hat{i}+2\hat{j}+\hat{k}$ **Parametricvectorequation** $\vec{r}=\vec{a}+s\vec{u}+t\vec{v}$ $\vec{r}=\hat{i}-\hat{j}+3\hat{k}+s(2\hat{i}-\hat{j}+4\hat{k})$ $+t(\hat{i}+2\hat{j}+\hat{k})$	**Cartesianequation** $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ $\begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$ $9x-2y-5z+4=0$
41)Findtheparametricandnon-parametric Vector and cartesian equation of the plane passing through points $(-1,2,0),(2,2,-1)$ **andparallel to theline** $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$	$\vec{a}=-\hat{i}+2\hat{j}+0\hat{k}$ $\vec{b}=2\hat{i}+2\hat{j}-\hat{k}$ $\vec{v}=\hat{i}+1\hat{j}-1\hat{k}$ **Parametricvectorequation** $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\vec{v}$ $\vec{r}=-\hat{i}+2\hat{j}+0\hat{k}+s(3\hat{i}+0\hat{j}-\hat{k})$ $+t(\hat{i}+\hat{j}-\hat{k})$	**Non-parametricvectorequation** $(\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times\vec{c})=0$ $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=3$ **Cartesianequation** $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix}=0$ $\begin{vmatrix} x+1 & y-2 & z-0 \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix}=0$ $x+2y+3z=3$
42) Find the parametric form of vector equation,andCartesianequationsoftheplane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x+6y+6z=9$	$\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$ $\vec{b}=9\hat{i}+3\hat{j}+6\hat{k}$ $\vec{v}=2\hat{i}+6\hat{j}+6\hat{k}$ Parametricvectorequation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\vec{v}$ $\vec{r}=(2\hat{i}+2\hat{j}+\hat{k})+s(7\hat{i}+\hat{j}+5\hat{k})+t(2\hat{i}+6\hat{j}+6\hat{k})$	Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix}=0$ $3x+4y-5z-9=0$

43) Find parametric form of vector equation and Cartesian equations of the palne passing though the points $(2,2,1),(1,-2,3)$ and paralleltothestraightlinepassingthroughthe points $(2,1,-3)$ and $(-1,5,-8)$	$\vec{a}=2\vec{i}+2\vec{j}+\vec{k}$ $\vec{b}=\vec{i}-2\vec{j}+3\vec{k}$ $\vec{v}=3\vec{i}-4\vec{j}+5\vec{k}$ Parametricvectorequation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\vec{v}$ $\vec{r}=(2\vec{i}+2\vec{j}+\vec{k})+s(-\vec{i}-4\vec{j}+2\vec{k})+t(3\vec{i}-4\vec{j}+5\vec{k})$	Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix}=0$ $\begin{vmatrix} x-2 & y-2 & z-1 \\ -1 & -4 & 2 \\ 3 & -4 & 5 \end{vmatrix}=0$ $12x-11y-16z+14=0$
44) Findtheparametricandnon-parametricVector and cartesian equation of the plane passing through three non-collinear points $(3,6,-2),(-1,-2,6)$and$(6,-4,-2)$	$\vec{a}=3\vec{i}+6\vec{j}-2\vec{k}$ $\vec{b}=-\vec{i}-2\vec{j}+6\vec{k}$ $\vec{c}=6\vec{i}+4\vec{j}-2\vec{k}$ Parametricvectorequation $\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t(\vec{c}-\vec{a})$ $\vec{r}=(2\vec{i}+6\vec{j}-2\vec{k})+s(-4\vec{i}-8\vec{j}++8\vec{k})+t(3\vec{i}-2\vec{j})$	Non-parametricvectorequation $(\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times(\vec{c}-\vec{a}))=0$ $\vec{r}\cdot(2\vec{i}+3\vec{j}+4\vec{k})=16$ Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$ $\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$ $2x+3y+4z=16$
45) Find the parametric form of vector equation,andCartesianequationsofthe plane $\vec{r}=(6\hat{i}-\hat{j}+\hat{k})+s(-\hat{i}+2\hat{j}+\hat{k})+t(-5\hat{i}-4\hat{j}-5\hat{k})$	$\vec{a}=6\vec{i}-\vec{j}+\vec{k}$ $\vec{u}=-\vec{i}+2\vec{j}+\vec{k}$ $\vec{v}=-5\vec{i}-4\vec{j}-5\vec{k}$ Non-parametricvectorequation $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ $\vec{r}\cdot(3\vec{i}+5\vec{j}-7\vec{k})=6$	Cartesianequation $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$ $\begin{vmatrix} x-6 & y+1 & z-1 \\ -1 & 2 & 1 \\ -5 & -4 & -5 \end{vmatrix}=0$ $3x+5y-7z=6$

46)SolvebyusingCaramer'srule: $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0;\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$ **and** $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$	$\frac{1}{x}=a;\ \frac{1}{y}=b;\ \frac{1}{z}=c$ $3a-4b-2c=1$ $a+2b+c=2$ $2a-5b-4c=-1$	$\Delta=\begin{vmatrix}3 & -4 & -2\\ 1 & 2 & 1\\ 2 & -5 & -4\end{vmatrix}=-15\neq0$ $\Delta_a=-15$; $\Delta_b=-5$; $\Delta_c=-5$ $x=1, y=3, z=3$
47)Investigatethevalueofλandμthesystem of linear equations $2x+3y+5z=9; 7x+3y-5z=8$ **and** $2x+3y+\lambda z=\mu$ **have** **i) no solution** **ii) aunique solution** **iii) aninfinitenumberofsolutions.**	$[A/B]=\begin{bmatrix}2 & 3 & 5 & 9\\ 7 & 3 & -5 & 8\\ 2 & 3 & \lambda & \mu\end{bmatrix}$ $[A/B]=\begin{bmatrix}2 & 3 & 5 & 9\\ 0 & -15 & -45 & -47\\ 0 & 0 & \lambda-5 & \mu-9\end{bmatrix}$	**i) Nosolution** $\lambda=5, \mu\neq9 \Rightarrow \rho(A)\neq\rho(A/B)$ **ii) Uniquesolution** $\lambda\neq5, \mu\neq9 \Rightarrow \rho(A)=\rho(A/B)=3$ **iii) Infinitenumberofsolution** $\lambda=5, \mu=9 \Rightarrow \rho(A)=\rho(\frac{A}{B})=2<3$
48)Investigateforwhatvaluesofλ and $\mu$the systemoflinearequations $x+2y+z=7, x+y+\lambda z=\mu,$ $x+3y-5z=5$ **has** **i) no solution** **ii) aunique solution** **iii) aninfinitenumberofsolutions.**	$[A/B]=\begin{bmatrix}1 & 2 & 1 & 7\\ 1 & 1 & \lambda & \mu\\ 1 & 3 & -5 & 5\end{bmatrix}$ $[A/B]=\begin{bmatrix}1 & 2 & 1 & 7\\ 0 & 1 & -6 & -2\\ 0 & 0 & \lambda-7 & \mu-9\end{bmatrix}$	**i) Nosolution** $\lambda=7, \mu\neq9 \Rightarrow \rho(A)\neq\rho(A/B)$ **ii) Uniquesolution** $\lambda\neq7, \mu\neq9 \Rightarrow \rho(A)=\rho(A/B)=3$ **iii) Infinitenumberofsolutions** $\lambda=7, \mu=9 \Rightarrow \rho(A)=\rho(\frac{A}{B})=2<3$

49)Findthevalueof kforwhichtheequations $kx-2y+z=1$; $x-2ky+z=-2$; $x-2y+kz=1$ **Havei)nosolution** **ii) uniquesolution** **iii) infinitelymanysolution**	$[A/B]=\begin{bmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{bmatrix}$ $[A/B]=\begin{bmatrix} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{bmatrix}$ $[A/B]=\begin{bmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & (k-1)(k+2) & k+2 \end{bmatrix}$	i) **Nosolution** $k=1\Rightarrow\rho(A)\neq\rho(A/B)$ **ii) Uniquesolution** $k\neq1,k\neq-2\Rightarrow\rho(A)=\rho(A/B)=3$ **iii) Infinitelymanysolution** $k=-2\Rightarrow\rho(A)=\rho(A/B)=2<3$
50)Determinethevaluesofλforwhichthe following system of equations $x+y+3z=0;4x+3y+\lambda z=0$**and** $2x+y+2z=0$**has** **i) uniquesolution** **ii) non-trivialsolutionusingGaussian elimination method.**	$[A/B]=\begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$ $[A/B]=\begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & \lambda & 0 \end{bmatrix}$ $[A/B]=\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & \lambda-8 & 0 \end{bmatrix}$	(i) Trivialsolution $\lambda\neq8\Rightarrow\rho(A)=\rho(A/B)=3$ (ii) Non-trivialsolution $\lambda=8\Rightarrow\rho(A)=\rho(\frac{A}{B})=2<3$