12STDMATHEMATICS(VOLUME-II)

(OnlyforSlowLearners/5markquestionandanswer)

1)Verify (i) Closure property (ii) commutativeproperty(iii)Associative property(iv) Existence of Identity (v) Existence of Inverse for the operation$+_5$onZ_5usingtablecorrespondingto addition modulo 5.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

i) Closureproperty- true

ii) Commutativeproperty -true

iii) Associativeproperty-true

iv) ExistenceofIdentitye = 0

v) InverseProperty

Inverseof0=0

Inverseof1=4

Inverseof2=3

Inverseof3=2

Inverseof4=1

2)Verify (i) Closure property (ii) commutativeproperty(iii)Associative property(iv) Existence of Identity (v)ExistenceofInversefortheoperation$\times_{11}$ on asubsetA={1,3,4,5, 9} ofthesetof remainders{0, 1, 2,3,4, ,56,7,8,9,10}

$\times_{11}$	1	3	4	5	9
1	**1**	**3**	**4**	**5**	**9**
3	**3**	**9**	**1**	**4**	**5**
4	**4**	**1**	**5**	**9**	**3**
5	**5**	**4**	**9**	**3**	**1**
9	**9**	**5**	**3**	**1**	**4**

i) Closureproperty- true

ii) Commutativeproperty-true

iii) Associativeproperty-true

iv) Existenceofidentitye =1

v) Inverseproperty

inverseof 1 =1

inverseof 3 =4

inverseof 4 =3

inverseof 5 =9

inverseof 9 =5

3)Let$M=\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x\in R-\{0\}\right\}$ **andlet* be thematrixmultiplication.DeterminewhetherM is closed under *. If so examine the (i)Commutativeproperty(ii)Associative property (iii) Existence of Identity (iv)Existenceofinversepropertyforthe operation * on M.**

i) Closureproperty- true

ii) Commutativeproperty-true

iii) Associativeproperty-true

iv) Identityproperty-true

$$E=\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}\in M$$

v)Inverseproperty

inversematrix$=\begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix}\in M$

<table>
<tr>
<td>4)LetAbeQ\{1}.Define* onAbyx*y=x+y -xy. Is*binaryonA?Ifso, examinethethe (i)Commutativeproperty(ii)Associative property (iii) Existence of Identity (iv)Existenceofinversepropertyforthe operation * on A.</td>
<td>i) Closureproperty- true
ii) Commutativeproperty-true
iii) Associativeproperty-true</td>
<td>iv) Identityproperty -true
e=0
v) Inverseproperty
inverseelement $x^{-1}=\frac{-x}{1-x}\in A$</td>
</tr>
<tr>
<td>5)Verify (i) closure property (ii) commutativeproperty(iii)associative property (iv) existence of identity (v) existence of inverse for the following operation on the given set.
$m*n=m+n-mn;m,n\in Z$</td>
<td>i) Closureproperty- true
ii) Commutativeproperty-true
iii) Associativeproperty-true</td>
<td>iv) Identityproperty-true
e=0
v) Inverseproperty
inversedoesnotexist</td>
</tr>
<tr>
<td>6)ArandomvariableXhasthefollowing probability mass function
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$f(x)$</td><td>k</td><td>$2k$</td><td>$6k$</td><td>$5k$</td><td>$6k$</td><td>$10k$</td></tr>
</table>
Find(i) $P(2<X<6)$
(ii) $P(2\le X<5)$
(iii) $P(X\le 4)$
iv) $P(3<X)$</td>
<td>$\sum p_i=1$
$30k=1$
$k=\frac{1}{30}$
i)$P(2<X<6)=P(3)+P(4)+P(5)$
$=17k=\frac{17}{30}$</td>
<td>ii)$P(2\le X<5)=P(2)+P(3)+P(4)$
$=13k=\frac{13}{30}$
iii)$P(X\le 4)=P(1)+P(2)+P(3)+P(4)$
$=14k=\frac{14}{30}$
iv)$P(3<X)=P(4)+P(5)+P(6)$
$=21k=\frac{21}{30}$</td>
</tr>
<tr>
<td>7)ArandomvariableXhasthefollowing probability mass function
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$f(x)$</td><td>k^2</td><td>$2k^2$</td><td>$3k^2$</td><td>$2k$</td><td>$3k$</td></tr>
</table>
Find(i)valueof$k$ (ii)$P(2\le X<5)$
(iii)$P(3>X)$</td>
<td>i) $\sum p_i=1$
$6k^2+5k-1=0$
$k=\frac{1}{6}$</td>
<td>(ii)$P(2\le X<5)=P(2)+P(3)+P(4)$
$=5k^2+2k=\frac{5}{36}+\frac{2}{6}=\frac{17}{36}$
(iii)$P(3>X)=P(4)+P(5)$
$=5k=\frac{5}{6}$</td>
</tr>
</table>

8)Find the probability mass function f(x) of the discrete random variable X whose cumulativedistributionfunctionF(x)isgiven by

$$F(x)=\begin{cases} 0; & -\infty<x<-2 \\ 0.25; & -2\le x<-1 \\ 0.60; & -1\le x<0 \\ 0.90; & 0\le x<1 \\ 1; & 1\le x<\infty \end{cases}$$

Thenfindi)P(X<0)and ii)$P(X\geq-1)$

Probabilitymassfunction

X	-2	-1	0	1
f(x)	0.25	0.35	0.30	0.10

i) $P(X<0)=P(-2)+P(-1)$

$=0.25+0.35 \quad =0.60$

ii) $P(X\geq-1)=P(-1)+P(0)+P(1)$

$=0.35+0.30+0.10 \quad =0.75$

9)Thecumulativedistributionfunctionofa discrete random variable is given by

$$F(x)=\begin{cases} 0; & -\infty<x<-1 \\ 0.15; & -1\le x<0 \\ 0.35; & 0\le x<1 \\ 0.60; & 1\le x<2 \\ 0.85; & 2\le x<3 \\ 1; & 3\le x<\infty \end{cases}$$

Find(i)theprobabilitymassfunction (ii)P(X<1) (iii)$P(X\geq2)$

Probabilitymassfunction

X	-1	0	1	2	3
f(x)	0.15	0.20	0.25	0.25	0.15

i) $P(X<1)=P(-1)+P(0)$

$=0.15+0.20=0.35$

ii) $P(X\geq2)=P(2)+P(3)$

$=0.25+0.15=0.40$

10)Thecumulativedistributionfunctionofa discrete random variable is given by

$$F(x)=\begin{cases} 0; & -\infty<x<0 \\ 1/2; & 0\le x<1 \\ 3/5; & 1\le x<2 \\ 4/5; & 2\le x<3 \\ 9/10; & 3\le x<4 \\ 1; & 4\le x<\infty \end{cases}$$

Find(i)probabilitymassfunction (ii)P(X<3) (iii)$P(X\geq2)$

Probabilitymassfunction

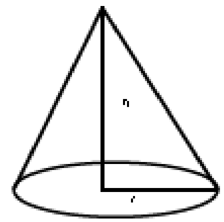
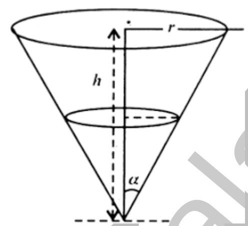
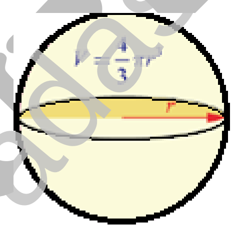
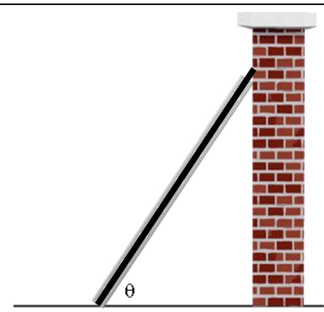
X	0	1	2	3	4
f(x)	$\frac{1}{2}=\frac{5}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

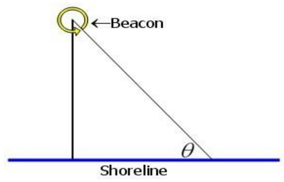
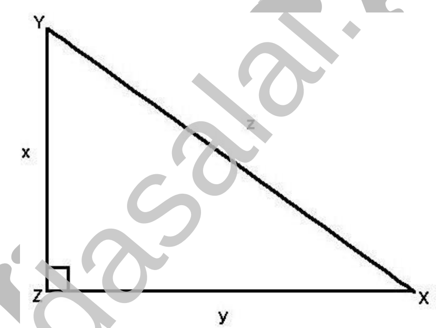
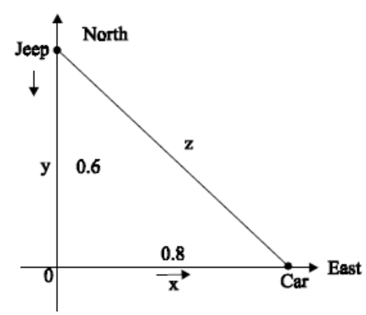
i) $P(X<3)=P(0)+P(1)+P(2)$

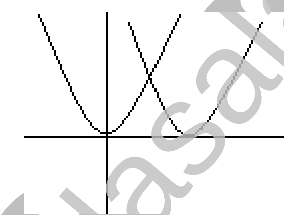
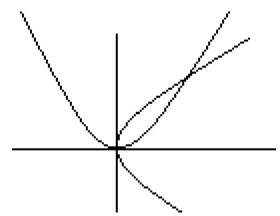
$$=\frac{5}{10}+\frac{1}{10}+\frac{2}{10}=\frac{8}{10}$$

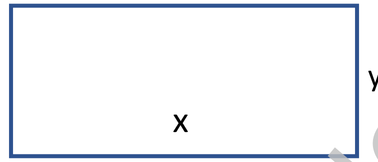
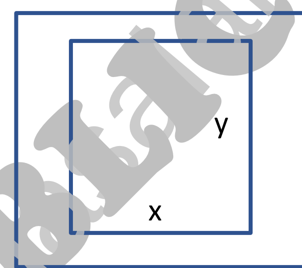
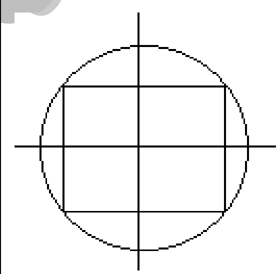
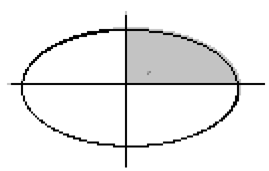
ii) $P(X\geq2)=P(2)+P(3)+P(4)$

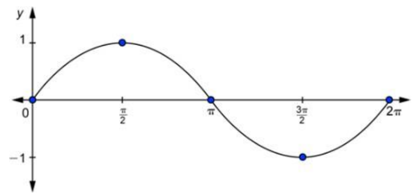
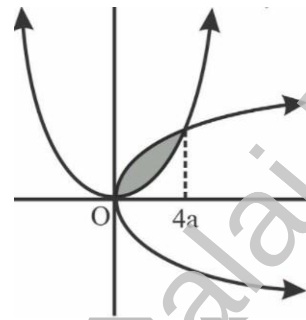
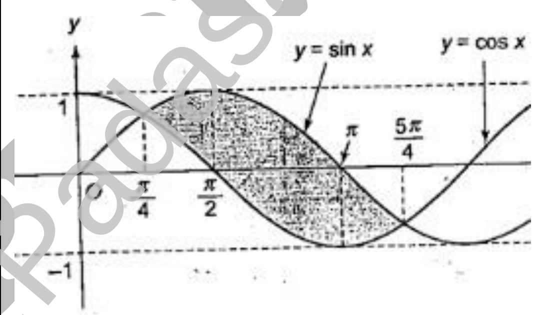
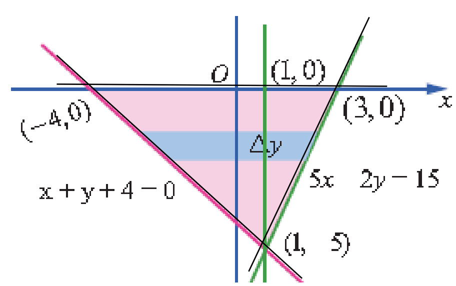
$$=\frac{2}{10}+\frac{1}{10}+\frac{1}{10}=\frac{4}{10}$$

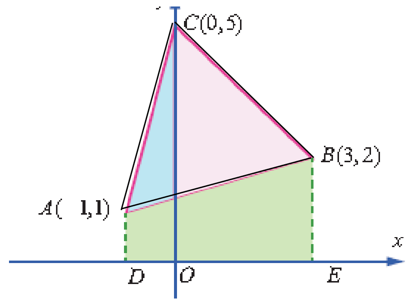
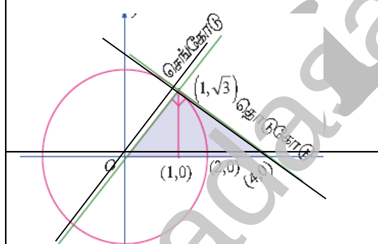
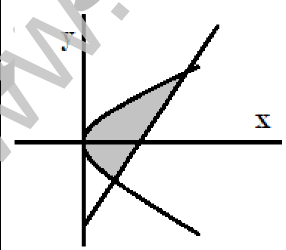
11)**Salt is poured from a conveyer belt at a rate of 30cubicmeterperminuteformingaconicalpilewitha circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10meter high?**		$\frac{dV}{dt}=30\&h=10$ radius$r=\frac{h}{2}$ rateofchangeofheight $\frac{dh}{dt}=\frac{6}{5\pi}$
12)**A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, howfastisthedepthofthewaterincreaseswhenthe water is 8 meters deep?**		$\frac{dV}{dt}=10\&h=8$ radius$r=\frac{5h}{12}$ rateofchangeofdepthofwaterlevel $\frac{dh}{dt}=\frac{9}{10\pi}$
13)**If we blow air into a balloon of spherical shape at a rate of 1000 cm^3per second, at what rate the radiusoftheballoonchangeswhentheradiusis7cm? Alsocomputetherateat whichthesurfacearea changes.**		$\frac{dV}{dt}=1000\&r=7$ changeinradius $\frac{dr}{dt}=\frac{250}{49\pi}$ changeinsurfacearea $\frac{dS}{dt}=\frac{2000}{7}$
14)**Aladder17meterlongisleaningagainstthewall. The base of the ladder is pulled away from the wall at arateof5m/swhenthebaseoftheladderis8meters from the wall. (i) How fast is the top of the ladder moving down the wall ?(ii) At what rate, the area of thetriangleformedbytheladder,wallandthefloor is changing.**		$\frac{dx}{dt}=5\&x=8$ $y=15,\frac{dy}{dt}=\frac{-8}{3}$ Changein area $\frac{dA}{dt}=\frac{1}{2}\left[x\frac{dy}{dt}+y\frac{dx}{dt}\right]=26.83$

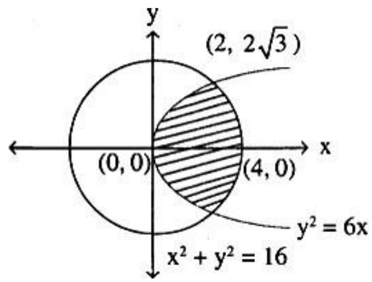
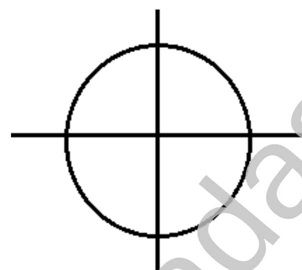
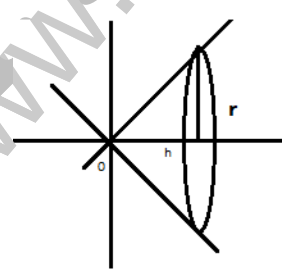
15)**A beacon makes one revolution every 10 seconds. It is locatedonashipwhichisanchored5kmfromastraightshore line.Howfastisthebeammovingalongtheshorelinewhen it makes an angle of 45° with the shore?**		$\frac{d\theta}{dt}=\frac{2\pi}{10}=\frac{\pi}{5}$ & $\theta=45°$ $\tan\theta=\frac{x}{5}\Rightarrow x=5\tan\theta$ rateofchangeofbeam $\frac{dx}{dt}=2\pi km/sec$
16)Aroadrunningnorth tosouthcrossesaroad goingeastto west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular timecarAis10kmtothenorthofPandtravelingat80km/hr, whilecarBis15kmtotheeastofPandtravelingat100km/hr. How fast is the distance between the two cars changing?		$x=10, \frac{dx}{dt}=80$**km/hr** $y=15, \frac{dy}{dt}=100$**km/hr** $z^2=x^2+y^2\Rightarrow z=5\sqrt{13}$ **Changeisdistance** $\frac{dz}{dt}=127.6$**km/hr**
17)Apolicejeep,approachinganorthogonalintersectionfrom the northern direction, is chasing a speeding car that has turnedandmovingstraighteast.Whenthejeepis0.6kmnorth of the intersection and car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hrattheinstantofmeasurement,whatisthespeedofthe car?		**x=0.6,** $y=0.8$ $\frac{dz}{dt}=20$**km/hr** $z^2=x^2+y^2\Rightarrow z=1$ **Speedofthecar** $\frac{dx}{dt}=70$**km/hr**

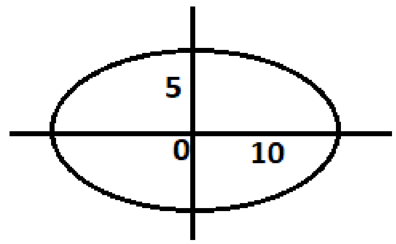
18) **Find the equation of the tangent and normal at any point to the Lissajous curve given by** $x=2\cos 3t$ **and** $y=3\sin 2t$ **, t∈R**	Slope $m=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{6\cos 2t}{-6\sin 3t}=\frac{-\cos 2t}{\sin 3t}$ **Tangent** $y-y_1=m(x-x_1)$ $y-3\sin 2t=\frac{-\cos 2t}{\sin 3t}(x-2\cos 3t)$	**Normal** $y-y_1=\frac{-1}{m}(x-x_1)$ $y-3\sin 2t=\frac{\sin 3t}{\cos 2t}(x-2\cos 3t)$
19) **Find the acute angle between** $y=x^2$ **and** $y=(x-3)^2$		**Intersecting point** $(x,y)=\left(\frac{3}{2},\frac{9}{4}\right)$ slopes $m_1=3; m_2=-3$ $\theta=\tan^{-1}\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\tan^{-1}\left(\frac{3}{4}\right)$
20) **Find the acute angle between the curves** $y=x^2$ **and** $x=y^2$ **at their points of intersection (0,0) and (1,1).**		$y=x^2$ $x=y^2$ $\frac{dy}{dx}=2x$ $\frac{dy}{dx}=\frac{1}{2y}$ At (0,0); $\theta=\frac{\pi}{2}$ At (1,1); $\theta=\tan^{-1}\left(\frac{3}{4}\right)$

21)**A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.**		$A=xy=x(20-x)=20x-x^2$ $\frac{dA}{dx}=20-2x$ & $\frac{d^2A}{dx^2}=-2<0$ $\frac{dA}{dx}=0\Rightarrow x=10 \& y=10$ Largest area $A=100m^2$
22)**A rectangular page is to contain 24 sq.cm of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the is 1cm. What should be the dimensions of the page so that the area of the paper used is minimum.**		$A=(x+2)(y+3)$ $A=3x+\frac{48}{x}+30$ $\frac{dA}{dx}=3-\frac{48}{x^2}$ & $\frac{d^2A}{dx^2}=\frac{96}{x^3}>0$ $\frac{dA}{dx}=0\Rightarrow x=4 \& y=6$ $\therefore x+2=6$ & $y+3=9$
23)**Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10cm.**		$x=20\cos\theta$ & $y=20\sin\theta$ $A=(2x)(2y)$ $A=200\sin 2\theta$ $\frac{dA}{d\theta}=0\Rightarrow\theta=\frac{\pi}{4}$ $\therefore L=2x=10\sqrt{2}$ & $B=2y=10\sqrt{2}$
24)Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		Area: $A=\int_a^b y\,dx$ $=4\int_0^a y\,dx$ $=\pi ab$

25)Findtheareaoftheregionboundedbyx-axis,thesine curve $y=\sin x$, the lines x=0 and $x=2\pi$		Area:$A=\int_a^b y\,dx$ $=\int_0^\pi \sin x\,dx-\int_\pi^{2\pi}\sin x\,dx$ $=4$
26)Findtheareaoftheregionbounded betweenthe parabolas $y^2=4x$ and $x^2=4y$		Intersectingpoints=(0,0),(4,4) **Area:** $A=\int_a^b [y_U-y_L]dx$ $=\frac{16}{3}$
27)Findtheareaoftheregionbounded by $y=\cos x, y=\sin x$,thelines$x=\frac{\pi}{4}$and $x=\frac{5\pi}{4}$		**Area** $A=\int_a^b [y_U-y_L]dx$ $=\int_{\pi/4}^{5\pi/4} [\sin x-\cos x]dx$ $=2\sqrt{2}$
28)Findbyintegrationtheareaoftheregionboundedby the lines$5x-2y=15, x+y+4=0$and the x-axis.		**Intersectingpoint =(1,-5)** **Pointswherethelinemeet thex-axis =(3,0),(-4,0)** **Area** $A=\left\|\int_{-4}^{1} y\,dx\right\|+\left\|\int_{1}^{3} y\,dx\right\|$ $=\frac{35}{2}$

29)Using integration find the area of the region boundedbytriangleABC,whoseverticesA,BandC are (-1,1), (3,2) and (0,5) respectively.		**Eqautionofstraightlines** $y=4x+5$ $y=-x+5$ $y=\frac{1}{4}(x+5)$ **Area** $A=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(-x+5)dx-\frac{1}{4}\int_{0}^{\pi}(x+5)dx$ $=\frac{15}{2}$
30)Using integration, find the area of the region whichisboundedbyx-axis,thetangentandnormal to the circle $x^2+y^2=4$ drawn at$(1,\sqrt{3})$		**Equationoftangent**$x+y\sqrt{3}=4$ **Equationofnormal**$y=\sqrt{3}x$ **Area** $A=\int_{0}^{1}y\,dx+\int_{1}^{4}y\,dx=2\sqrt{3}$
31)Findtheareaoftheregionboundedbythe parabola $y^2=x$ and the line $y=x-2$		**y-intercepts =-1,2** **Area** $A=\int_{c}^{d}[x_R-x_L]dy$ $=\int_{-1}^{2}[y+2-y^2]\,dy=\frac{9}{2}$

32)Findtheareaoftheregioncommontothe circle $x^2+y^2=16$and the parabola$y^2=6x$		Intersectingpoints=$(2,2\sqrt{3}),(2,-2\sqrt{3})$ Area $A=\int_c^d [x_R-x_L]dy$ $=\int_{-2\sqrt{3}}^{2\sqrt{3}}[\sqrt{16-y^2}-\frac{y^2}{6}]dy$ $=\frac{4}{3}(4\pi+\sqrt{3})$
33)Findthevolumeofasphereofradiusausing integration.		Limits: $x=-a to x=a$ Circle:$x^2+y^2=a^2$ $y^2=a^2-x^2$ Volume $V=\pi\int_{-a}^{a} y^2dx=\pi\int_{-a}^{a}(a^2-x^2)dx$ $V=\frac{4}{3}\pi a^3$
34)Findthevolumeofarightcircularconeof base radius r and height h using integration.		Limit: $x=0 to x=h$ Equationofstraightline $y=\frac{r}{h}x$ Volume $V=\pi\int_0^h y^2dx$ $=\pi\int_0^h \frac{r^2}{h^2}x^2dx=\frac{1}{3}\pi r^2h$

35)A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis10cmaboutitsmajor-axis.Finditsvolumeusing integration.		**Ellipse** $\frac{x^2}{10^2}+\frac{y^2}{5^2}=1$ $y^2=25\left(1-\frac{x^2}{100}\right)$ **Volume** $V=\pi\int_{-10}^{10} y^2dx = \frac{1000}{3}\pi$
36)Therateofincreaseinthenumberofbacteriaina certain bacteria culture is proportional to the number present.Giventhatthenumbertriplesin5 hours,findhow manybacteriawillbepresentafter10 hours?	$\frac{dA}{dt}=kA\ dt$ $A=Ce^{kt}$	$t=0; \Rightarrow C=A_0t$ $=5;\Rightarrow e^{5k}=3$ $t=10;\Rightarrow A=9A_0$
37)Find the population of a city at anytime t, given that the rate of increase of population is proportional to the populationatthatinstantandthatin aperiodof40years the population increased from 3,00,000 to 4,00,000.	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\Rightarrow C=3,00,000$ $t=40\Rightarrow k=\frac{1}{40}\log\left(\frac{4}{3}\right)$ $A=3,00,000\left(\frac{4}{3}\right)^{t/40}$
38)SupposeapersondepositsRs.10,000inabankaccount at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?	$\frac{dA}{dt}=kA\ dt$ $A=Ce^{0.05t}$	$t=0;\Rightarrow C=10,000$ $t=1.5;\Rightarrow A=10,000e^{0.075}$
39)Assumethattherateatwhichradioactivenucleidecay is proportional to the number of such nucleic that are present in each sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in aperiod of100 years.Whatpercentageof theoriginalradioactivenucleiwillremainafter1000years?	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0;\Rightarrow C=100$ $t=100; e^{100k}=\frac{9}{10}$ $t=1000;\Rightarrow A=\frac{9^{10}}{10^8}\ \%$

40)Aradioactiveisotopehasaninitialmass200mg,which twoyearslateris150mg. Findtheexpression fortheisotope remainingat anytime.Whatisitshalf-life?(half-life means thetimetakenfortheradioactivityofaspecifiedisotopeto falltohalfitsoriginal value)	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\Rightarrow C=200$ & $t=2\Rightarrow k=\frac{-1}{2}\log\left(\frac{4}{3}\right)$ $A(t)=200e^{-\frac{t}{2}\log\left(\frac{4}{3}\right)}$ & $t=\frac{2\log\left(\frac{1}{2}\right)}{\log\left(\frac{4}{3}\right)}$
41) Waterat temperature $100^{\circ}C$ coolsin10 minutes to$80^{\circ}C$ina room temperature of $25^{\circ}C$ Find(i)Thetemperatureofwaterafter20minutes (ii)Thetimewhenthetemperatureis$40^{\circ}C$	$\frac{dT}{dt}=k(T-25)$ $T=25+Ce^{kt}$	$t=0\Rightarrow C=75$ $t=20\text{min}\Rightarrow T=65.33\,C^{\circ}$ $T=40C^{\circ}\Rightarrow t=51.89\text{min}$
42)A pot of boiling water at$100C^{\circ}$is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, thewatertemperaturehasdecreased to$80C^{\circ}$, andanother5 minutes later it has dropped to$65\,C^{\circ}$. Determine the temperature of the kitchen.	$\frac{dT}{dt}=k(T-S)dt$ $T=S+Ce^{kt}$	$t=0\Rightarrow C=100-S$ $t=5\Rightarrow e^{5k}=\frac{80-S}{100-S}$ **Temperatureofthekitchen** $S=20C^{\circ}$
43)Inmurderinvestigation,acorpsewasfoundbydetective atexactly8p.m.Beingalert,thedetectivealsomeasuredthe bodytemperatureandfoundittobe$70F^{\circ}$.Twohourslater, the detective measured the body temperature again and found it to be$60F^{\circ}$.If the room temperature is$50F^{\circ}$and assuming that the body temperature of the person before deathwas$98.6F^{\circ}$,at whattimedidthemurder occur?	$\frac{dT}{dt}=k(T-50)$ $T=50+Ce^{kt}$	$t=0;\Rightarrow C=20$ $t=2\Rightarrow k=\frac{1}{2}\log\left(\frac{1}{2}\right)$ **Timeofdeathis= 5:30pm**
44)Atank contains1000litresofwaterinwhich 100 gramsof saltisdissolved.Brine(Brineisahighconcentrationsolution ofsalt-usuallysodiumchlorideinwater)runsinarateof10 litersperminute,andeachlitercontains5gramsofdissolved salt.Themixtureofthetank iskeptuniformbystirring. Brine runsoutat10litersperminute.findtheamountofsaltatany timet.	$\frac{dx}{dt}=IN-OUT\ dt$ $\frac{dx}{dt}=50-0.01x$ $x=5000+Ce^{-0.01t}$	$t=0;C=-4900$ **Amountofsaltattimet** $x=5000-4900e^{-0.01t}$

45)Atankinitiallycontains 50 litersofpurewater. Startingat time t = 0 abrine containingwith 2 grams of dissolved salt per liter flows into the tank at the rate of 3 liters per minute. The mixture is kept uniform by stirring and the well-stirredmixturesimultaneouslyflowsoutofthetankatthesamerate. Find the amount of salt present in the tank at any time t > 0.	$\frac{dx}{dt}=IN-OUT$ $\frac{dx}{dt}=6-\frac{3x}{50}$ $x=100+Ce^{\frac{-3t}{50}}$	$t=0;C=-100$ **Amountofsaltat timet** $x=100-100e^{-\frac{3t}{50}}$
46)If $u=sin^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$**,thenProve that** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}tanu$ **usingEulertheorem.**	$f(x,y)=\frac{x+y}{\sqrt{x}+\sqrt{y}}=sinu$ **fisahomogeneouswith degree** $n=\frac{1}{2}$	ByEuler'stheorem $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$ $x\frac{\partial(sinu)}{\partial x}+y\frac{\partial(sinu)}{\partial y}=\frac{1}{2}sinu$ $xcosu\frac{\partial u}{\partial x}+ycosu\frac{\partial u}{\partial x}=\frac{1}{2}sinu$ $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}tanu$
47)If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$ **then showthat** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$ **usingEulerstheorem.**	$u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$ **fisahomogeneouswith degree** $n=\frac{3}{2}$	ByEuler'stheorem $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$ $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$
48)If $v(x,y)=log\left[\frac{x^2+y^2}{x+y}\right]$ *thenshowthat* $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$ **usingEuler'stheorem.**	$f(x,y)=\frac{x^2+y^2}{x+y}=e^v$ **fisahomogeneouswithdegree** $n=1$	ByEuler'stheorem $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$ $x\frac{\partial(e^v)}{\partial x}+y\frac{\partial(e^v)}{\partial y}=1\times e^v$ $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$

49)Prove that $f(x,y)=x^3-2x^2y+3xy^2+y^3$ ishomogeneous;whatistheisthedegree? Verify Eulers theorem for f.	$f(\lambda x,\lambda y)=\lambda^3 f(x,y)$ fisahomogeneouswithdegree$n=3$	$\frac{\partial f}{\partial x}=3x^2-4xy+3y^2$ $\frac{\partial f}{\partial y}=-2x^2+6xy+3y^2$ $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=3f$
50)If $w(x,y,z)=\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$ thenfind $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$	$f=\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}=e^w$ fisahomogeneouswithdegree$n=5$	UsingEuler'stheorem $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+z\frac{\partial f}{\partial z}=nf$ $x\frac{\partial(e^w)}{\partial x}+y\frac{\partial(e^w)}{\partial y}+z\frac{\partial(e^w)}{\partial z}=5(e^w)$ $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}=5$
51)Forthe function $f(x,y)=tan^{-1}\left(\frac{x}{y}\right)$ find f_x,f_y Alsoverifythat $f_{xy}=f_{yx}$	$f_x=\frac{y}{x^2+y^2}$ $f_{xy}=\frac{x^2-y^2}{(x^2+y^2)^2}$ →(1)	$f_y=\frac{-x}{x^2+y^2}$ $f_{yx}=\frac{x^2-y^2}{(x^2+y^2)^2}$ →(2) $f_{xy}=f_{yx}$
52)If $u=sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ thenShowthat $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2cotu$ usingEuler's theorem.	$f(x,y)=\frac{x^3-y^3}{x+y}=secu$ fisahomogeneouswithdegree$n=2$	ByEuler'stheorem $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$ $x\frac{\partial(secu)}{\partial x}+y\frac{\partial(secu)}{\partial y}=2secu$ $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{2}{tanu}=2cotu$

