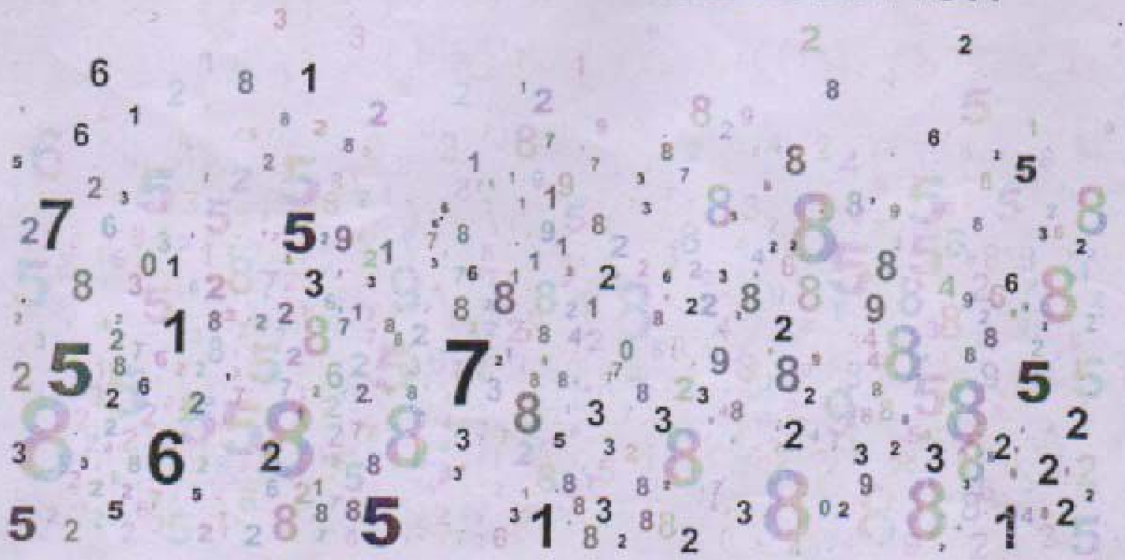
TUITION

Where Learning Never Ends

10TH

MATHEMATICS
UNIT-1

ALL EXERCISE SOLUTIONS,
UNIT EXERCISE SOLUTIONS,
EXAMPLE SOLUTIONS,
BOOK BACK ONE- MARKS SOLUTION



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CHAPTER - 1

RELATIONS AND FUNCTIONS

EXERCISE 1.1

1. Find $A\times B$, $A\times A$ and $B\times A$

(i) $A=\{2,-2,3\}$ and $B=\{1,-4\}$

Solution

$A\times B=\{2,-2,3\}\times\{1,-4\}=\{(2,1),(2,-4),(-2,1),(-2,-4),(3,1),(3,-4)\}$

$A\times A=\{2,-2,3\}\times\{2,-2,3\}=\{(2,2),(2,-2),(2,3),(-2,2),(-2,-2),(-2,3),(3,2),(3,-2),(3,3)\}$

$B\times A=\{1,-4\}\times\{2,-2,3\}=\{(1,2),(1,-2),(1,3),(-4,2),(-4,-2),(-4,3)\}$

(ii) $A=\{p,q\}=B$

Solution

$A\times A=\{p,q\}\times\{p,q\}=\{(p,p),(p,q),(q,p),(q,q)\}$

$A\times B=\{p,q\}\times\{p,q\}=\{(p,p),(p,q),(q,p),(q,q)\}$

$B\times A=\{p,q\}\times\{p,q\}=\{(p,p),(p,q),(q,p),(q,q)\}$

(iii) $A=\{m,n\}$, $B=\phi$

Solution.

$A\times B=\{m,n\}\times\{\ \}=\{\ \}$

$B\times A=\{\ \}\times\{m,n\}=\{\ \}$

$A\times A=\{m,n\}\times\{m,n\}=\{(m,m),(m,n),(n,m),(n,n)\}$

EXAMPLE 1.1 :- If $A=\{1,3,5\}$ and $B=\{2,3\}$, then

(i) find $A\times B$ & $B\times A$

$A\times B=\{1,3,5\}\times\{2,3\}$

$=\{(1,2),(1,3),(3,2),(3,3),(5,2),(5,3)\}$

$B\times A=\{2,3\}\times\{1,3,5\}$

$=\{(2,1),(2,3),(2,5),(3,1),(3,3),(3,5)\}$

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(ii) Is $A \times B = B \times A$? If not why?

from (i) Solution, $(1,2) \neq (2,1)$

$\therefore A \times B \neq B \times A$

(iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$

$n(A) = 3$, $n(B) = 2$, $n(A) \times n(B) = 3 \times 2 = 6$

from (i)

$n(A \times B) = 6$

$n(B \times A) = 6$

Example : 1.2

If $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$ then find A and B.

Solution

$A = \{$ Set of all first Co-ordinates of elements of $A \times B\}$

$A = \{3,5\}$

$B = \{$ Set of all Second Co-ordinates of elements of $A \times B\}$

$B = \{2,4\}$

EXERCISE 1.1

2. Let $A = \{1,2,3\}$ and $B = \{x \mid x$ is a prime number less than $10\}$

Find $A \times B$ and $B \times A$.

Solution

$A = \{1,2,3\}$, $B = \{2,3,5,7\}$

$A \times B = \{1,2,3\} \times \{2,3,5,7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7)$

$(3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{2,3,5,7\} \times \{1,2,3\}$

$= \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (5,1),$

$(5,2), (5,3), (7,1), (7,2), (7,3)\}$

3. If $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$ find A and B.

Solution

B = Set of all first Co-ordinates of element of $B \times A$

$B = \{-2, 0, 3\}$

A = Set of all Second co-ordinates of elements of $B \times A$

$A = \{3, 4\}$

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4. If $A = \{5,6\}$, $B = \{4,5,6\}$, $C = \{5,6,7\}$, show that $A \times A = (B \times B) \cap (C \times C)$

Solution

$A \times A = \{5,6\} \times \{5,6\} = \{(5,5), (5,6), (6,5), (6,6)\}$ — (1)

$B \times B = \{4,5,6\} \times \{4,5,6\}$

$= \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$C \times C = \{5,6,7\} \times \{5,6,7\}$

$= \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\}$

$(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\}$ — (2)

from (1) & (2) $A \times A = (B \times B) \cap (C \times C)$

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5. Given $A = \{1,2,3\}$, $B = \{2,3,5\}$, $C = \{3,4\}$, $D = \{1,3,5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution

$A \cap C = \{3\}$

$B \cap D = \{3,5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3,5\}$

$= \{(3,3), (3,5)\}$ — (1)

$A \times B = \{1,2,3\} \times \{2,3,5\} = \{(1,2), (1,3), (1,5), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5)\}$

$C \times D = \{3,4\} \times \{1,3,5\} = \{(3,1), (3,3), (3,5), (4,1), (4,3), (4,5)\}$

$(A \times B) \cap (C \times D) = \{(3,3), (3,5)\}$ — (2)

from (1) & (2), $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true.

6. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ & $C = \{3,5\}$

Verify

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$A = \{0,1\}$

$B = \{2,3,4\}$

$C = \{3,5\}$

$B \cup C = \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$= \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}$ - ①

$A \times B = \{\cancel{2,3,4}\ 0,1\} \times \{2,3,4\}$

$= \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$= \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cup (A \times C) = \{(0,3),(1,3),(0,2),(0,4),(1,2),(1,4),(0,5),(1,5)\}$ - ②

from ① & ② $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{3\}$

$A \times (B \cap C) = \{0,1\} \times \{3\}$

$= \{(0,3),(1,3)\}$ - ①

$A \times B = \{0,1\} \times \{2,3,4\}$

$= \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$= \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,3),(1,3)\}$ - ②

from ① & ②, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{0,1\} \cup \{2,3,4\} = \{0,1,2,3,4\}$

$(A \cup B) \times C = \{0,1,2,3,4\} \times \{3,5\}$

$= \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$ - ①

$A \times C = \{0,1\} \times \{3,5\} = \{(0,3),(0,5),(1,3),(1,5)\}$

$B \times C = \{2,3,4\} \times \{3,5\} = \{(2,3),(3,3),(4,3),(2,5),(3,5),(4,5)\}$

$(A\times C)\cup(B\times C) =$

$= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$ - ②

from ① & ②, $(A\cup B)\times C = (A\times C)\cup(B\times C)$

7. Let A = The Set of all natural number less than 8,
B = The Set of all prime number less than 8,
C = The Set of even prime number, Verify that

Solution

$A = \{1,2,3,4,5,6,7\}$

$B = \{2,3,5,7\}$

$C = \{2\}$

i) $(A\cap B)\times C = (A\times C)\cap(B\times C)$

$A\cap B = \{2,3,5,7\}$

$(A\cap B)\times C = \{2,3,5,7\}\times\{(2)\} = \{(2,2),(3,2),(5,2),(7,2)\}$ - ①

$A\times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B\times C = \{(2,2), (3,2), (5,2), (7,2)\}$

$(A\times C)\cap(B\times C) = \{(2,2), (3,2), (5,2), (7,2)\}$ - ②

from ① & ②, $(A\cap B)\times C = (A\times C)\cap(B\times C)$

ii) $A\times(B-C) = (A\times B)-(A\times C)$

$B-C = \{2,3,5,7\}-\{2\} = \{3,5,7\}$

$A\times(B-C) = \{1,2,3,4,5,6,7\}\times\{3,5,7\}$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ - ①

$A\times B = \{(2,2), (3,2), (1,2), (1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5), (6,7), (7,2), (7,3), (7,5), (7,7)\}$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A\times B) - (A\times C) = \{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),(3,5),(3,7),(4,3),(4,5),(4,7),(5,3),(5,5),(5,7),(6,3),(6,5),(6,7),(7,3),(7,5),(7,7)\}$ – ②

from ① & ②, $A \times (B - C) = (A\times B) - (A \times C)$

EXAMPLE 1.3

Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \le x < 2\}$ and $C = \{x \in N \mid x < 3\}$. Then verify that

(i) $A \times (B \cup C) = (A\times B) \cup (A\times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution

$A = \{2,3\}$, $B = \{0,1\}$, $C = \{1,2\}$

(i) LHS ⇒ $B \cup C = \{0,1,2\}$

$A \times (B\cup C) = \{2,3\} \times \{0,1,2\}$

$= \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ – ①

RHS ⇒

$A \times B = \{2,3\} \times \{0,1\}$

$= \{(2,0),(2,1),(3,0),(3,1)\}$

$A \times C = \{2,3\} \times \{1,2\}$

$= \{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cup(A\times C) = \{(2,0),(2,1),(2,2),(3,0),(3,1),(3,2)\}$ – ②

from ① & ② $A \times (B \cup C) = (A\times B) \cup (A \times C)$

(ii) LHS ⇒ $B \cap C = \{1\}$

$A \times (B\cap C) = \{2,3\} \times \{1\} = \{(2,1),(3,1)\}$ – ①

$A \times B = \{2,3\} \times \{0,1\} = \{(2,0),(2,1),(3,0),(3,1)\}$

$A \times C = \{2,3\} \times \{1,2\} = \{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B)\cap(A\times C) = \{(2,1),(3,1)\}$ – ②

from ① & ② $A\times(B\cap C) = (A\times B)\cap(A\times C)$

EXERCISE 1.2

1. Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relations from A to B?

Solution

$$A \times B = \{1,2,3,7\} \times \{3,0,-1,7\}$$

$$= \{(1,3), (1,0), (1,-1), (1,7), (2,3), (2,0), (2,-1), (2,7), (3,3), (3,0), (3,-1), (3,7), (7,3), (7,0), (7,-1), (7,7)\}$$

(i) $R_1 = \{(2,1), (7,1)\}$

$(2,1), (7,1) \notin A \times B$

$\therefore R_1$ is not relation to $A \times B$

(ii) $R_2 = \{(-1,1)\}$

$(-1,1) \in A \times B$

$\therefore R_2$ is relation to $A \times B$

(iii) $R_3 = \{(2,-1), (7,7), (1,3)\}$

$(2,-1) \in A \times B$, $(7,7) \in A \times B$, $(1,3) \in A \times B$

$\therefore R_3$ is relation to $A \times B$

(iv) $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

$(0,3) \notin A \times B$, $(0,7) \notin A \times B$

$\therefore R_4$ is not relation to $A \times B$

2. Let $A = \{1, 2, 3, 4, \ldots 45\}$ and R be the relation defined as "is square of number" on A. write R as a subset of $A \times A$. Also, find the domain and range of R.

$$A = \{1, 2, 3, \ldots 45\}$$

$$A \times A = \{(1,1), (1,2), \ldots, (45,45)\}$$

$$R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$$

$$R \subseteq A \times A$$

Domain of $R = \{1, 2, 3, 4, 5, 6\}$

Range ~~Co-Domain~~ of $R = \{1, 4, 9, 16, 25, 36\}$

3. A Relation R is given by the set $\{(x,y) / y = x+3, x \in \{0,1,2,3,4,5\}\}$. Determine its Domain and range.

domain $(x) = \{0,1,2,3,4,5\}$

Co-domain $(y) = \{x+3$

Put $x=0$	Put $x=1$	Put $x=2$	Put $x=3$	Put $x=4$
$y=0+3$	$y=1+3$	$y=2+3$	$y=3+3$	$y=4+3$
$y=3$	$y=4$	$y=5$	$y=6$	$y=7$

Put $x=5$
$y=5+3$
$y=8$

$R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

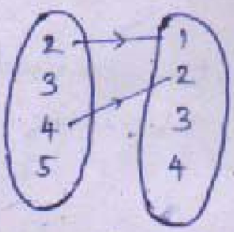
Domain $= \{0,1,2,3,4,5\}$

Range $= \{3,4,5,6,7,8\}$

4. Represent each of the given relation by
a) an arrow diagram (b) a graph (c) set in roster form, wherever possible

(i) $\{(x,y) \mid x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$

a) an arrow diagram

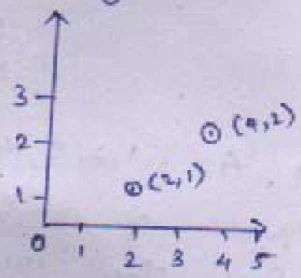


Given, $x = 2y$

If $y=1$, $x = 2(1) = 2$

If $y=2$, $x = 2(2) = 4$

(b) a graph



(c) a Set in roster form

$R = \{(2,1), (4,2)\}$

(ii) $\{(x,y) \mid y = x+3, x, y$ are natural numbers $< 10\}$

$x = y = \{1,2,3,4,5,6,7,8,9\}$

$y = x+3$

when $x=1$, $y=4$

when $x=2$, $y=5$

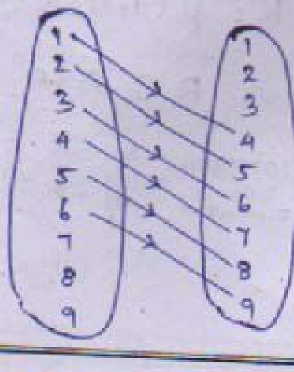
when $x=3$, $y=6$

when $x=4$, $y=7$

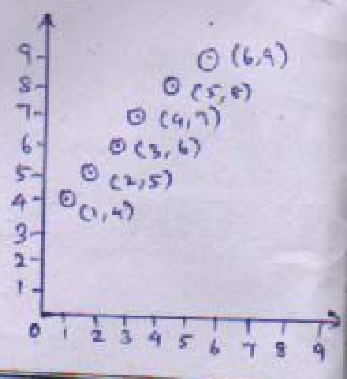
when $x=5$, $y=8$

when $x=6$, $y=9$

a) arrow diagram



(b) graph



(c) Roster form

$$R = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$$

5. A Company has four Categories of employee given by Assistants (A), Clerks (C), Managers (M) and Executive Officer (E). The Company provide ₹10,000, ₹25,000, ₹50,000, ₹1,00,000 as Salaries to the people who work in the Categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 where Assistants; C_1, C_2, C_3, C_4 were Clerks, M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy. where x is the Salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Solution

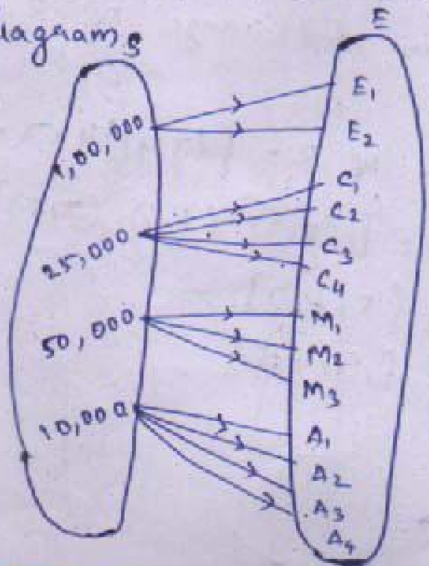
$$\text{Salaries } (S) = \{10{,}000,\ 25{,}000,\ 50{,}000,\ 1{,}00{,}000\}$$

$$\text{Employees } (E) = \{A_1, A_2, A_3, A_4, A_5, C_1, C_2, C_3, C_4, M_1, M_2, M_3, E_1, E_2\}$$

(i) ordered pairs

$R_1 = \{(10{,}000, A_1), (10{,}000, A_2), (10{,}000, A_3), (10{,}000, A_4),$
$(10{,}000, A_5), (25{,}000, C_1), (25{,}000, C_2), (25{,}000, C_3),$
$(25{,}000, C_4), (50{,}000, M_1), (50{,}000, M_2), (50{,}000, M_3),$
$(1{,}00{,}000, E_1), (1{,}00{,}000, E_2)\}$

(ii) An arrow diagram



EXAMPLE 1.4 : Let $A = \{3, 4, 7, 8\}$ and $B = \{1, 7, 10\}$. Which of the following sets are relations from A to B?

Solution

$$A \times B = \{3, 4, 7, 8\} \times \{1, 7, 10\}$$

$$= \{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)\}$$

(i) $R_1 = \{(3,7), (4,7), (7,10), (8,1)\}$

$R_1 \subseteq A \times B$

$\therefore R_1$ is relation from A to B

(ii) $R_2 = \{(3,1), (4,12)\}$

$R_2 \nsubseteq A \times B$, $(4,12) \notin A \times B$

$\therefore R_2$ is not a relation from A to B.

(iii) $R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$

$R_3 \nsubseteq A \times B$, $(7,8) \notin A \times B$

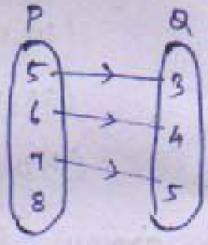
$\therefore R_3$ is not a relation from A to B.

EXAMPLE 1.5 :- The arrow diagram show figure a relationship between the sets P and Q. Write the relation in

(i) Set builder form

(ii) Roster form

(iii) What is the domain and range of R.



Solution

(i) Set builder form of $R = \{(x,y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5,3), (6,4), (7,5)\}$

(iii) Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

EXERCISE 1.3

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1) Let $f = \{(x,y) \mid x, y \in N, y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?

Solution

Given, $f = \{(x,y) \mid x, y \in N, y = 2x\}$

when, $x = 1,\ y = 2$

$x = 2,\ y = 4$

$x = 3,\ y = 6$ and so on

Domain $= \{1, 2, 3, \dots\}$

Co-domain $= \{1, 2, 3, 4, \dots\}$

Range $= \{2, 4, 6, 8, \dots\}$

All the elements in the domain have only one Image in the co-domain.

$\therefore$ The given function is a relation.

2) Let $X = \{3, 4, 6, 8\}$. Determine whether the Relation $R = \{(x, f(x)) \mid x \in X, f(x) = x^2+1\}$ is a function from X to N?

Solution

Given, $f(x) = x^2+1$, where $x \in \{3, 4, 6, 8\}$

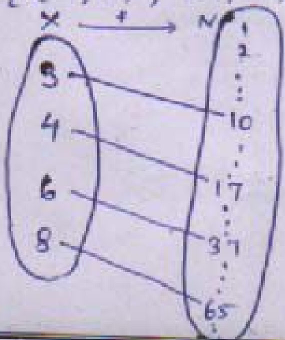
when, $x = 3$, $f(3) = (3)^2+1 = 9+1 = 10$

when, $x = 4$, $f(4) = (4)^2+1 = 16+1 = 17$

when, $x = 6$, $f(6) = (6)^2+1 = 36+1 = 37$

when, $x = 8$, $f(8) = (8)^2+1 = 64+1 = 65$

$\therefore R = \{(3,10), (4,17), (6,37), (8,65)\}$



$\therefore$ All elements of X have unique image in N.

$\therefore$ The given relation is a function

3) Given the function $f: x \to x^2 - 5x + 6$. evaluate

$f(x) = x^2 - 5x + 6$

(i) $f(-1)$

$f(-1) = (-1)^2 - 5(-1) + 6$
$= 1 + 5 + 6$
$= 12$

(ii) $f(2a)$

$f(2a) = (2a)^2 - 5(2a) + 6$
$= 4a^2 - 10a + 6$

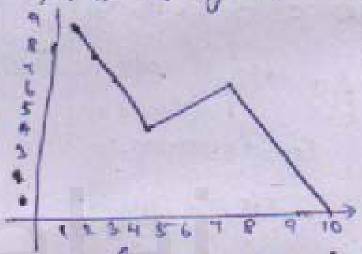
(iii) $f(2)$

$f(2) = (2)^2 - 5(2) + 6$
$= 4 - 10 + 6$
$= 0$

(iv) $f(x-1)$

$f(x-1) = (x-1)^2 - 5(x-1) + 6$
$= x^2 + 1 - 2x - 5x + 5 + 6$
$= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in Graph. It is clear that $f(9) = 2$



(Refer book for clear figure)

(i) Find the following values of the function

(a) $f(0) = 9$ (c) $f(2) = 6$

(b) $f(7) = 6$ (d) $f(10) = 0$

(ii) For what values of x is $f(x) = 1$?

Domain

$x = 9.5$

(iii) Describe : i) Domain $= \{x \mid 0 \le x \le 10, x \in R\}$

ii) Range $= \{x \mid 0 \le x \le 9, x \in R\}$

(iv) what is the image of 6 under f?

$f(6) = 5$

5) Let $f(x)=2x+5$. If $x\neq 0$, then find $\frac{f(x+2)-f(2)}{x}$

Solution

$f(x+2) = 2(x+2)+5$ $\quad$ $f(2) = 2(2)+5$

$= 2x+4+5$ $\quad$ $= 4+5$

$= 2x+9$ $\quad$ $= 9$

$$\frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x} = \frac{2x}{x} = 2$$

6) A function f is defined by $f(x) = 2x-3$

(i) find $\frac{f(0)+f(1)}{2}$

$f(0) = 2(0)-3 = -3$

$f(1) = 2(1)-3 = -1$

$$\frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-\cancel{4}}{\cancel{2}} = -2$$

(ii) find x such that $f(x)=0$

$2x-3=0$

$2x=3$

$\boxed{x=3/2}$

(iii) find x such that $f(x)=x$

$2x-3=x$

$2x-x=3$

$\boxed{x=3}$

(iv) find x such that $f(x) = f(1-x)$

$2x-3 = 2(1-x)-3$

$2x-\cancel{3}+\cancel{3} = 2-2x$

$2x+2x = 2$

$4x = 2$

$x = 2/4 \Rightarrow \boxed{x = 1/2}$

7(8) A open box is to be made from a square piece of material 24cm on a side, by cutting equal square from the corners and turning up the sides as shown in figure (refer book). Express volume V of the box as a function of x.

Solution

length = breadth = $24-2x$, height $h = x$ cm

$\therefore$ Volume of cuboid $= l \times b \times h$

$= (24-2x)\times(24-2x)\times x$

$= (24-2x)^2 x$

$= [576 + 4x^2 - 96x]\, x$

$= 576x + 4x^3 - 96x^2$

Volume of box $= 4x^3 - 96x^2 + 576x$

8) A function f is defined by $f(x) = 3-2x$. find x such that $f(x^2) = (f(x))^2$

Solution

$f(x^2) = 3-2x^2$

$(f(x))^2 = (3-2x)^2 = 9+4x^2-12x$

$f(x^2) = (f(x))^2$

$3-2x^2 = 9+4x^2-12x$

$0 = 4x^2+2x^2-12x+9-3$

$0 = 6x^2-12x+6$

$\div 6$

$0 = x^2-2x+1$

$0 = (x-1)(x-1)$

$(x-1)^2 = 0$

$x-1 = 0$

$\boxed{x=1}$

9) A plane is flying at speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

[$\because$ formula SdT]

Solution

$$\text{Speed} = \frac{\text{distance covered}}{\text{time taken}}$$

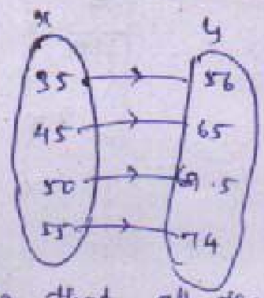
$$500 = \frac{d}{t}$$

$$\boxed{d = 500t}$$

10) The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants.

(i) Check if this relation is a function.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74



It's clear that all the elements in domain has only one image in the co-domain.

∴ The given relation is a function.

(ii) find a and b

Given, $f(x) = y = ax + b$

when $x = 35$, $56 = 35a + b$ — ①

when $x = 45$, $65 = 45a + b$ — ②

Solve ① & ②

$35a + b = 56$

$45a + b = 65$

$(-) \quad (-) \quad (-)$

$-10a = -9$

$\boxed{a = 9/10}$

~~Solve ① & ②~~

~~$y = 45a + b$~~

Sub a in ①

$35a + b = 56$

$35\left(\frac{9}{10}\right) + b = 56$

$\frac{63}{2} + b = 56$

$b = 56 - \frac{63}{2}$

$b = \frac{112 - 63}{2}$

$b = \frac{49}{2}$

$\boxed{b = 24.5}$

iii) Find the height of a person whose forehand length is 40 cm.

Solution

$y = \frac{9}{10}(40) + 24.5$

$= 36 + 24.5$

$= 60.5$ inches

(iv) Find the length of forehand of a person if the hight is 53.3 inches.

$$y = 53.3$$

$$\frac{9}{10}x + 24.5 = 53.3$$

$$\frac{9}{10}x = 53.3 - 24.5$$

$$9x = 28.8 \times 10$$

$$x = \frac{288}{9}$$

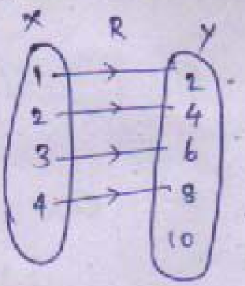
$$\boxed{x = 32 \text{ cm}}$$

$53.3 - 24.5 = 28.8$

$288 \div 9 = 32$

EXAMPLE 1.6

Let $X = \{1, 2, 3, 4\}$ and $Y = \{2, 4, 6, 8, 10\}$ and $R = \{(1,2), (2,4), (3,6), (4,8)\}$. Show that R is a function find its domain, co-domain and range?



$\therefore$ R is a function

Domain $X = \{1, 2, 3, 4\}$

Co-domain $Y = \{2, 4, 6, 8, 10\}$

Range of $f = \{2, 4, 6, 8\}$

Example 1.7

A relation $f : X \to Y$ is defined by $f(x) = x^2 - 2$, where $X = \{-2, -1, 0, 3\}$ and $Y = R$. (i) list the elements of f.

$$f(x) = x^2 - 2$$

$x = \{-2, -1, 0, 3\}$ $f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

$$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$$

$$f(0) = 0 - 2 = -2$$

$$f(3) = (3)^2 - 2 = 9 - 2 = 7$$

$$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$$

EXERCISE 1.4

TUITION
Where Learning Never Ends
PREPARED BY
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M.Sc., B.Ed
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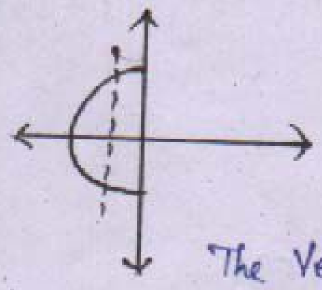
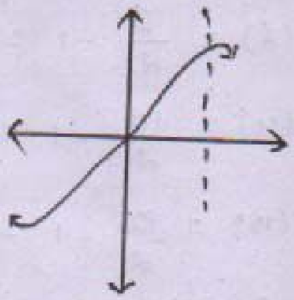
1) Determine whether the graph given below represent function. Give reason for your answer concerning each graph.

i)



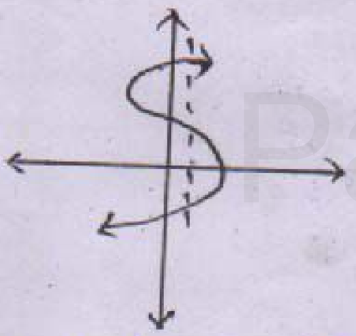
The Vertical line cuts the graph A and B. The given graph does not represent a function

ii)



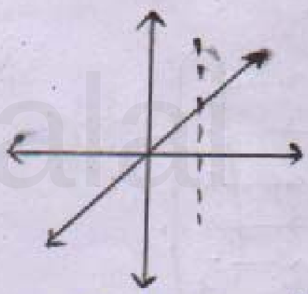
The vertical line cuts the graph atmost one point. The given graph represent a function.

iii)



The vertical line cuts the graph at three point, S, T and U. The given graph does not represent a function

iv)



The vertical line cuts the graph atmost one point D. The given graph represent a function.

2) Let $f: A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$ Represent f by

(i) Set of ordered pairs. (ii) a table

(iii) an arrow diagram (iv) a graph

$$f(x) = \frac{x}{2} - 1$$

$$A = \{2, 4, 6, 10, 12\}$$

$$B = \{0, 1, 2, 4, 5, 9\}$$

Put $x = 2$, $f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$

Put $x = 4$, $f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$

put $x = 6$, $f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$

put $x = 10$, $f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$

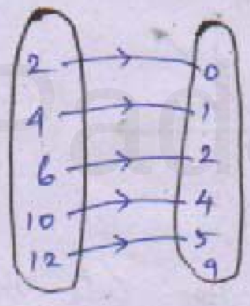
Put $x = 12$, $f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$

(i) $\{(2,0), (4,1), (6,2), (10,4), (12,5)\}$

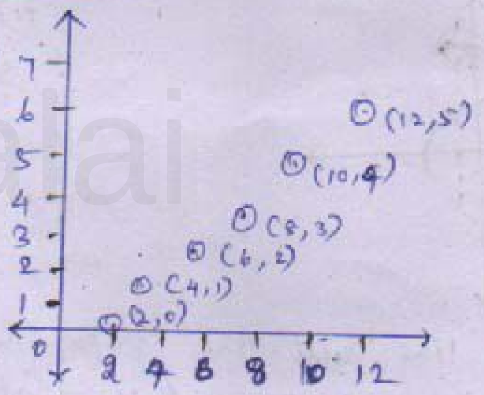
(ii)

A	2	4	6	10	12
B	0	1	2	4	5

(iii)

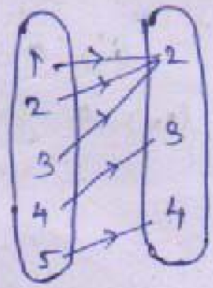


(iv)



3) Represent the function $f = \{(1,2), (2,2), (3,2), (4,3), ($
through (i) an arrow function (ii) a table (iii) a graph

(i) an arrow function (ii) a table

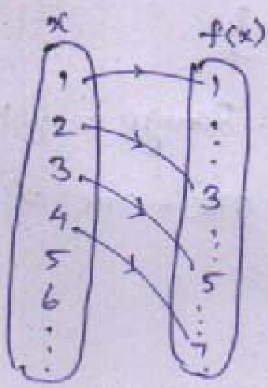


x	1	2	3	4	5	-
$f(x)$	2	2	2	3	4	-

4) Show that the function $f: N \to N$ defined by $f(x) = 2x-1$ is one-one but not onto.

$N = 1, 2, 3, \dots$

$x \in N, x = 1, 2, 3, 4, \dots$



$f(x) = 2x-1$

$f(1) = 2(1)-1 = 2-1 = 1$

$f(2) = 2(2)-1 = 4-1 = 3$

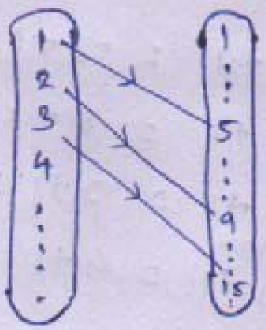
$f(3) = 2(3)-1 = 6-1 = 5$

$f(4) = 2(4)-1 = 8-1 = 7$

* Different element has a different images. These is one-one function.
* here Range $\neq$ co-domain
* These function is an not onto function.

5) Show that the function $f: N \to N$ defined by $f(m) = m^2+m+3$ is one-one function.

$N = \{1, 2, 3, \dots\}$



$f(m) = m^2+m+3$

$f(1) = (1)^2+1+3 = 5$

$f(2) = (2)^2+2+3 = 4+6 = 10$

$f(3) = (3)^2+3+3 = 9+6 = 15$

From diagram, we can understand different image in domain has a different image in co-domain. $\therefore$ This function is one-one function

6) Let $A = \{1,2,3,4\}$ and $B = N$. Let $f: A \to B$ be defined by $f(x) = x^3$ then, (i) find the range of f (ii) Identify the type of function

$f(x) = x^3$

$x \in N$, $A = \{1,2,3,4\}$

$f(1) = 1$

$f(2) = 8$

$f(3) = 27$

$f(4) = 64$

(i) Range $= \{1, 8, 27, 64\}$

(ii) one-one & into function

7) In each of the following cases state whether the function is bijective or not. Justify your answer

(i) $f: R \to R$ defined by $f(x) = 2x+1$

$x = \{-1, 0, 1, 2\}$

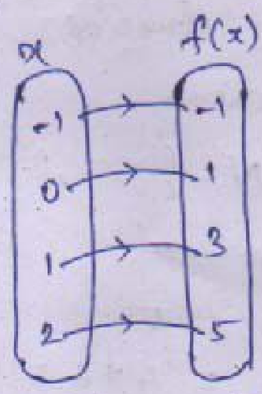
$f(x) = 2x+1$

$f(-1) = 2(-1)+1 = -2+1 = -1$

$f(0) = 2(0)+1 = 0+1 = 1$

$f(1) = 2(1)+1 = 2+1 = 3$

$f(2) = 2(2)+1 = 4+1 = 5$



It is bijective function, Distinct elements of A have distinct images in B and every element in B has a pre-image

iii) $f : R \to R$ defined by $f(x) = 3 - 4x^2$

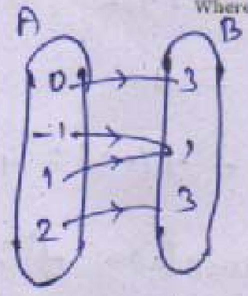
$x = \{0, -1, 1, 2\}$

$f(0) = 3 - 4(0) = 3 - 0 = 3$

$f(-1) = 3 - 4(-1)^2 = 3 - 4 = -1$

$f(1) = 3 - 4(1)^2 = 3 - 4 = -1$

$f(2) = 3 - 4(2)^2 = 3 - 4(4) = 3 - 16 = -13$



$\therefore$ It is not a bijective and one - one function

8) Let $A = \{-1, 1\}$, $B = \{0, 2\}$. If the function $f : A \to B$ is an onto function? defined by $f(x) = ax + b$ find a and b.

$A = \{-1, 1\}$, $B = \{0, 2\}$

$f(x) = ax + b$

$f(-1) = a(-1) + b$

$0 = -a + b$ — (1)

$\boxed{a = b}$

$f(1) = a(1) + b$

$2 = a + b$ — (2)

from (1)

$2 = a + a$

$2 = 2a$

$\boxed{a = 1}$ & $\boxed{b = 1}$

9) If the function f is defined by f(x)

$$= \begin{cases} x + 2, & \text{if } x > 1 \quad (2, 3, 4, \ldots) \\ 2, & \text{if } -1 \le x \le 1 \quad (-1, 0, 1) \\ x - 1, & \text{if } -3 < x < -1 \quad (-2) \end{cases}$$

Find the Value of

(i) $f(3)$

$f(x) = x + 2$

$f(3) = 3 + 2$

$= 5$

(ii) $f(0)$

$f(x) = 2$

$f(0) = 2$

(iii) $f(-1.5)$

$f(x) = x - 1$

$f(-1.5) = -1.5 - 1$

$= -2.5$

(iv) $f(2)+f(-2)$

$f(x)=x+2$ | $f(x)=x-1$

$f(2)=2+2$ | $f(-2)=-2-1$

$=4$ | $=-3$

$f(2)+f(-2)=4-3=1$

(10) A function $f: [-5,9] \to R$ defined as follows

$$f(x)=\begin{cases} 6x+1 & \text{if } -5 \le x < 2 \\ 5x^2-1 & \text{if } 2 \le x < 6 \\ 3x-4 & \text{if } 6 \le x \le 9 \end{cases}$$

$x=-5,-4,-3,-2,-1$
$x=2,3,4,5$
$x=6,7,8,9$

(i) $f(-3)+f(2)$

$f(x)=6x+1$ | $f(x)=5x^2-1$ | $f(-3)+f(2)$

$f(-3)=6(-3)+1$ | $f(2)=5(2)^2-1$ | $=-17+19$

$=-18+1$ | $=5(4)-1$ | $=2$

$=-17$ | $=20-1$

$=19$

(ii) $f(7)-f(1)$

$f(x)=3x-4$ | $f(x)=6x+1$ | $f(7)-f(1)=17-7$

$f(7)=3(7)-4$ | $f(1)=6(1)+1$ | $=10$

$=21-4$ | $=6+1$

$=17$ | $=7$

(iii) $2f(4)+f(8)$

$f(x)=5x^2-1$ | $f(x)=3x-4$

$f(4)=5(4)^2-1$ | $f(8)=3(8)-4$

$=5(16)-1$ | $=24-4$

$=80-1$ | $=20$

$=79$

$2f(4)=2\times 79$

$=158$

$2f(4)+f(8)=158+20$

$=178$

11) The distance S an object travels under the influence of gravity in time 't' seconds is given by $S(t) = \frac{1}{2} g t^2 + at + b$ where [g is acceleration due to gravity], a, b are constants. Check if the function $S(t)$ is one-one.

$$S(t) = \frac{1}{2} g t^2 + at + b$$

Let t be a time, $t = 1, 2, 3, \ldots$

$S(1) = \frac{1}{2} g (1)^2 + a(1) + b$
$= \frac{g}{2} + a + b$

$S(2) = \frac{1}{2} g (2)^2 + 2a + b$
$= 2g + 2a + b$

$S(3) = \frac{1}{2} g (3)^2 + a(3) + b$
$= \frac{9g}{2} + 3a + b$

$\therefore$ Every different values of 't' there will be different distance.

$\therefore$ It is one-one function.

12) The function 't' which maps temperature in Celsius (c) into temperature in fahrenheit (f) is defined by $t(c) = f$ where $F = \frac{9}{5} c + 32$, find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$
(iv) the values of when $t(c) = 212$
(v) the temperature when the Celsius value is equal to the fahrenheit value.

$t(c) = F, \; F = \frac{9}{5} c + 32$

(i) $t(0)$

$t(0) = \frac{9}{5}(0) + 32$
$= 32$.

(ii) $t(28)$

$t(28) = \frac{9}{5}(28) + 32$
$= 9 \times 5.6 + 32$
$= 50.4 + 32$
$= 82.4$

$5 \overline{)28}$ = 5.6; 25, 30, 30, 0

$5.6 \times 9 = 50.4$

(iii) $f(-10) = \frac{9}{5}(-10) + 32$

$= 9 \times -2 + 32$

$= -18 + 32$

$= 14$

(iv) $t(c) = 212$

$\frac{9}{5}c + 32 = 212$

$\frac{9}{5}c = 212 - 32$

$\frac{9}{5}c = \cancel{180}\ 20$

$\boxed{c = 100}$

(v) when $t = c$

$\frac{9}{5}c + 32 = c$

$32 = c - \frac{9}{5}c$

$32 = \frac{5c - 9c}{5}$

$32 = \frac{-4c}{5}$

$\cancel{160}$ (40) $= -4c$

$\boxed{c = -40}$

EXAMPLE 1.11 Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f: A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical form.

Solution: $A = \{1, 2, 3, 4\}$; $B = \{2, 5, 8, 11, 14\}$; $f(x) = 3x - 1$

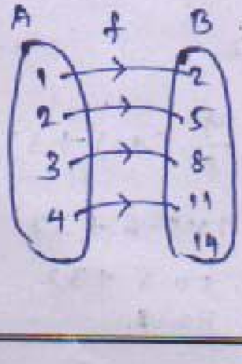
$f(1) = 3(1) - 1 = 3 - 1 = 2$

$f(2) = 3(2) - 1 = 6 - 1 = 5$

$f(3) = 3(3) - 1 = 9 - 1 = 8$

$f(4) = 3(4) - 1 = 12 - 1 = 11$

(i)

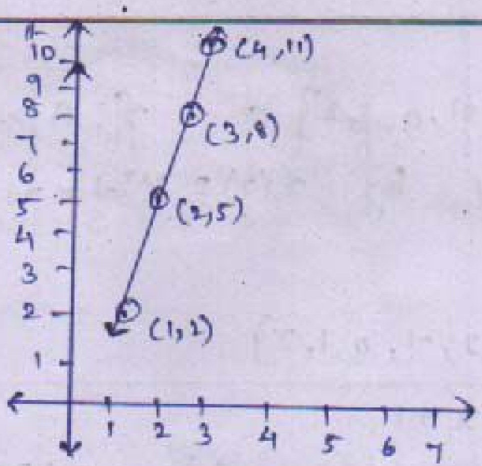


(ii)

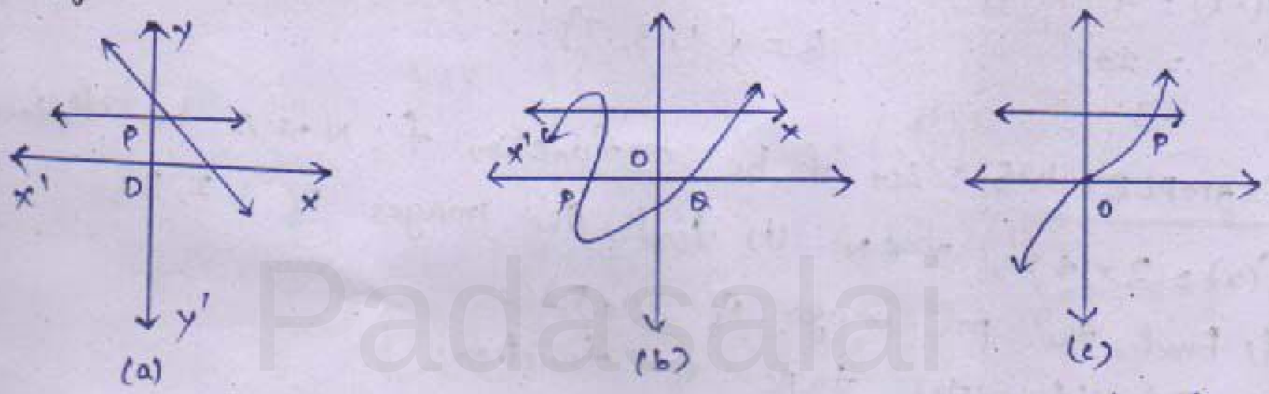
x	1	2	3	4
$f(x)$	2	5	8	11

(iii) $f = \{(1,2), (2,5), (3,8), (4$

(v) Graphical form



EXAMPLE 1.12 Using horizontal line test (three graph), determine which of the following functions are one - one.



The curves in (a) & (c) represent a one - one function as the horizontal lines meet the curves in only one point P.

The curves in (b) does not represent a one - one function the horizontal line intersects the curve at two points P and Q.

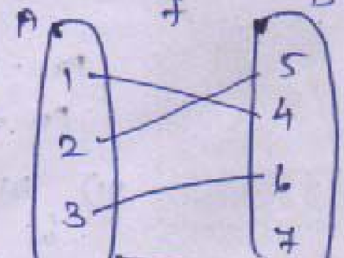
EXAMPLE 1.13 Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B. Show that f is one - one but not onto function.

Solution

Distinct elements of A have distinct elements of B

$\therefore$ one - one function

7 is co-domain does not have pre-image. $\therefore$ not a onto function



$\therefore$ f is one - one but not an onto function

EXAMPLE 1.14

If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Solution

$A = \{-2, -1, 0, 1, 2\}$

$f(x) = x^2 + x + 1$

$f(1) = 1 + 1 + 1 = 3$; $f(2) = 4 + 2 + 1 = 7$; $f(0) = 0 + 0 + 1 = 1$; $f(-1) = 1 + (-1) + 1 = 1$

$f(-2) = 4 - 2 + 1 = 2 + 1 = 3$

$B = \{1, 3, 7\}$

EXAMPLE 1.15 : Let f be a function $f: N \to N$ be defined $f(x) = 3x + 2$, $x \in N$ (i) find the images of 1, 2, 3

(ii) Find the pre-image of 29, 53

(iii) Identify the type of function.

Solution

(i) Put $x = 1$, $f(1) = 3 + 2 = 5$ (the image of 1 is 5)

Put $x = 2$, $f(2) = 6 + 2 = 8$ (image of 2 is 8)

Put $x = 3$, $f(3) = 9 + 2 = 11$ (image of 3 is 11)

(ii) $29 = 3x + 2$; $53 = 3x + 2$

$29 - 2 = 3x$; $53 - 2 = 3x$

$27 = 3x$; $51 = 3x$

$\boxed{x = 9}$; $\boxed{17 = x}$

$51 \div 3 = 17$

Pre-image of 29 is 9

Pre-image of 53 is 17

(iii) Different elements of N have different images in co-domain $\therefore$ f is one-one function

$\therefore$ f is not onto function

$\therefore$ f is into function (atleast one element in B not in ...)

EXERCISE 1.5

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1. Using the function f and g given below find fog and gof. check whether fog = gof

(i) $f(x) = x-6$, $g(x) = x^2$

$fog = f(g(x))$
$= f(x^2)$
$= x^2-6$

$gof = g(f(x))$
$= g(x-6)$
$= (x-6)^2$

$fog \neq gof$

(ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2-1$

$fog = f(g(x))$
$= f(2x^2-1)$
$= \frac{2}{2x^2-1}$

$gof = g(\frac{2}{x})$
$= 2(\frac{2}{x})^2-1$
$= 2(\frac{4}{x^2})-1$
$= \frac{8}{x^2}-1$

$gof \neq fog$

(iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3-x$

$fog = f(g(x))$
$= f(3-x)$
$= \frac{3-x+6}{3}$
$= \frac{9-x}{3}$

$gof = g(f(x))$
$= g(\frac{x+6}{3})$
$= 3-\frac{x+6}{3}$
$= \frac{9-x-6}{3} = \frac{3-x}{3}$

$fog \neq gof$

(iv) $f(x) = 3+x$, $g(x) = x-4$

$fog = f(g(x))$
$= f(x-4)$
$= 3+x-4$
$= x-1$

$gof = g(3+x)$
$= 3+x-4$
$= x-1$

$fog = gof$

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$fog = f(g(x))$
$= f(1+x)$
$= 4(1+x)^2 - 1$
$= 4(1+x^2+2x) - 1$
$= 4 + 4x^2 + 8x - 1$
$= 4x^2 + 8x + 3$

$gof = g(f(x))$
$= g(4x^2 - 1)$
$= 1 + 4x^2 - 1$
$= 4x^2$

$fog \neq gof$

2) Find the value of k, such that $fog = gof$

(i) $f(x) = 3x+2$, $g(x) = 6x - k$

$fog = f(g(x))$
$= f(6x - k)$
$= 3(6x - k) + 2$
$= 18x - 3k + 2$

$gof = g(f(x))$
$= g(3x+2)$
$= 6(3x+2) - k$
$= 18x + 12 - k$

$fog = gof$
$\cancel{18x} - 3k + 2 = \cancel{18x} + 12 - k$
$2 - 12 = -k + 3k$
$\boxed{-10 = 2k}$ $\boxed{k = -5}$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

$fog = f(g(x))$
$= f(4x+5)$
$= 2(4x+5) - k$
$= 8x + 10 - k$

$gof = g(f(x))$
$= g(2x - k)$
$= 4(2x - k) + 5$
$= 8x - 4k + 5$

$fog = gof$
$\cancel{8x} + 10 - k = \cancel{8x} - 4k + 5$
$-k + 4k = 5 - 10$
$3k = -5$
$\boxed{k = -5/3}$

3) If $f(x) = 2x-1$, $g(x) = \frac{x+1}{2}$, show that

$fog = gof = x$

$fog = f(g(x))$

$= f\left(\frac{x+1}{2}\right)$

$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$

$= x+1-1$

$= x$

$gof = g(f(x))$

$= g(2x-1)$

$= \frac{2x \cancel{-1} \cancel{+1}}{2}$

$= \frac{2x}{2}$

$= x$

$fog = gof = x$

4) If $f(x) = x^2-1$, $g(x) = x-2$ find a, if $gof(a) = 1$

$gof(a) = 1$

$f(x) = x^2-1$

$f(a) = a^2-1$

$g[f(a)] = 1$

$g(a^2-1) = 1$

$a^2-1-2 = 1$

$a^2-3 = 1$ $\Rightarrow$ $a^2 = 1+3$

$a^2 = 4$

$a^2 = 2^2$

$\boxed{a=2}$

5) Let $A, B, C \subseteq N$ and a function $f: A \to B$ be defined by $f(x) = 2x+1$ and $g: B \to C$ be defined by $g(x) = x^2$ find the range of fog and gof

$f(x) = 2x+1$

$g(x) = x^2$

$fog = f(g(x))$

$= f(x^2)$

$= 2x^2+1$

$gof = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2$

$fog \neq gof$

6) Let $f(x) = x^2 - 1$

(i) $f \circ f$

$$f \circ f = f(f(x))$$
$$= f(x^2 - 1)$$
$$= (x^2-1)^2 - 1$$
$$= (x^2)^2 + (1)^2 - 2x^2 - 1$$
$$= x^4 + \cancel{1} - 2x^2 - \cancel{1}$$
$$= x^4 - 2x^2$$

ii) $f \circ f \circ f$

$$f \circ f \circ f = f[f(f(x$$
$$= f[x^4 - 2x$$
$$= (x^4 - 2x^2)^2 -$$

7) If $f: R \to R$ and $g: R \to R$ are defined by $f(x) =$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?

$$f(g(x)) = f(x^4)$$
$$= (x^4)^5$$
$$= x^{20}$$

$\therefore$ one-one func

8) Consider the function $f(x), h(x)$ are given below. Show $(f \circ g) \circ h = f \circ (g \circ h)$ in each case

(i) $f(x) = x - 1$, $g(x) = 3x + 1$, $h(x) = x^2$

LHS $\Rightarrow (f \circ g) \circ h$

$$f \circ g = f(g(x))$$
$$= f(3x+1)$$
$$= 3x + 1 - 1$$
$$= 3x$$
$$(f \circ g) \circ h = 3x^2 \quad \text{—} (1)$$

RHS $\Rightarrow f \circ (g \circ h)$

$$g \circ h = g(h(x))$$
$$= g(x^2)$$
$$= 3x^2 + 1$$
$$f \circ (g \circ h) = 3x^2 + 1 -$$
$$= 3x^2 \quad \text{—} (2)$$

from (1) & (2), $(f \circ g) \circ h = f \circ (g$

(ii) $f(x) = x^2$, $g(x) = 2x$, $h(x) = x+4$

$f\circ(g\circ h) = (f\circ g)\circ h$

$g\circ h = g(h(x))$
$= g(x+4)$
$= 2(x+4)$

$f\circ g = f(g(x))$
$= f(2x)$
$= (2x)^2$
$= 4x^2$

$(f\circ g)\circ h = 4(x+4)^2$

$f\circ(g\circ h) = [2(x+4)]^2$
$= 4(x+4)^2$

$f\circ(g\circ h) = (f\circ g)\circ h$

(iii) $f(x) = x-4$, $g(x) = x^2$, $h(x) = 3x-5$

$f\circ(g\circ h) = (f\circ g)\circ h$

$g\circ h = g[h(x)]$
$= g(3x-5)$
$= (3x-5)^2$

$f\circ g = f(g(x))$
$= f(x^2)$
$= x^2-4$

$(f\circ g)\circ h = (3x-5)^2-4$

$f\circ(g\circ h) = (3x-5)^2-4$

$f\circ(g\circ h) = (f\circ g)\circ h$

9) Let $f = \{(-1,3), (0,-1), (2,-9)\}$ be a linear function from z into z. find $f(x)$

$\therefore f(0) = -1$, $f(-1) = 3$, $f(2) = -9$

$f(x) = ax+b$

$f(-1) = 03$
$-a+b = 3$ — ①

$f(0) = a(0)+b$
$\boxed{-1 = b}$

Sub b in ①
$-a-1 = 3$
$-a = 3+1 \Rightarrow -a = 4$
$\boxed{a = -4}$

$f(x) = -4x-1$
$= -(4x+1)$

10) In electrical circuit theory, a circuit $c(t)$ is called a linear circuit if it satisfies the superposition principle given by $c(at_1+bt_2) = ac(t_1)+bc(t_2)$. where a, b are constant. Show that the circuit $c(t) = 3t$ is linear.

$$c(t) = 3t$$
$$c(at_1) = 3at_1$$
$$c(bt_2) = 3bt_2$$
$$c(at_1+bt_2) = c(at_1) + c(bt_2)$$
$$= 3at_1 + 3bt_2$$
$$= a(3t_1) + b(3t_2)$$
$$= a(ct_1) + b(ct_2)$$

∴ Super position principle is satisfied.
Hence $c(t) = 3t$ is linear function.

EXAMPLE: 1.20
Represent the function $f(x) = \sqrt{2x^2-5x+3}$ as a comp[osition] of two function.

$$f_2(x) = 2x^2-5x+3$$
$$f_1(x) = \sqrt{x}$$
$$f(x) = \sqrt{2x^2-5x+3}$$
$$= \sqrt{f_2(x)}$$
$$= f_1 f_2(x)$$

EXAMPLE 1.22
Find k if $f \circ f(k) = 5$ w[here] $f(k) = 2k-1$

Solution

$$f \circ f(k) = 5$$
$$f[f(k)] = 5$$
$$f[2k-1] = 5$$
$$2[2k-1]-1 = 5$$
$$4k-2-1 = 5$$
$$4k-3 = 5$$
$$4k = 5+3$$
$$4k = 8$$
$$\boxed{k = 2}$$

EXAMPLE 1.16

Forensic scientist can determine the height of a person based on the length of the thigh bone. They usually do so using the function $h(b) = 2.47b + 54.10$ where b is the length of the thigh bone. (i) verify the function h is one-one or not. (ii) Also find the height of a person if the length of the thigh bone is 50cm. (iii) Find the length of the thigh bone if the height of a person is 147.96 cm.

Solution

(i) $h(b_1) = h(b_2)$

$$2.47b_1 + \cancel{54.10} = 2.47b_2 + \cancel{54.10}$$

$$\cancel{2.47}b_1 = \cancel{2.47}b_2$$

$$b_1 = b_2$$

$\therefore h$ is one-one function

(ii) $b = 50$

$$h(b) = 2.47b + 54.10$$

$$h(50) = 2.47(50) + 54.10$$

$$= 123.50 + 54.10$$

$$= 147.60 \text{ cm}$$

$2.47 \times 5 = 12.35$

$123.50 + 54.10 = 147.60$

(iii) $h(b) = 147.96$

$$147.96 = 2.47b + 54.10$$

$$147.96 - 54.10 = 2.47b$$

$$93.86 = 2.47b$$

$$b = \frac{93.86 \times 100}{2.47 \times 100}$$

$$b = \frac{9386}{247}$$

$$\boxed{b = 38}$$

$147.96 - 54.10 = 93.86$

$9386 \div 247 = 38$: 741, 1976, 1976, 0

$\therefore$ length of thigh bone $= 38$cm

EXAMPLE : 1.17

Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

Solution

$f(a) = 4 \qquad f(1) = b$

$f(x) = 3x - 5$

$f(a) = 3a - 5$

$4 = 3a - 5$

$4 + 5 = 3a$

$\cancel{9} = \cancel{3}a$

$\boxed{a = 3}$

$f(1) = 3(1) - 5$

$b = 3 - 5$

$\boxed{b = -2}$

EXAMPLE 1.18

If the function $f : R \to R$ is defined by

$$f(x) = \begin{cases} 2x + 7 ; & x < -2 \quad [-3, -4, -5, \ldots] \\ x^2 - 2 ; & -2 \le x < 3 \quad [-2, -1, 0, 1, 2] \\ 3x - 2 ; & x \ge 3 \quad [3, 4, 5, \ldots] \end{cases}$$

(i) $f(4)$ (ii) $f(-2)$ (iii) $f(4) + 2f(1)$ (iv) $\dfrac{f(1) - 3f(4)}{f(-3)}$

Solution

(i) $f(4)$

$f(x) = 3x - 2$

$f(4) = 3(4) - 2$

$= 12 - 2$

$= 10$

(ii) $f(-2)$

$f(x) = x^2 - 2$

$f(-2) = (-2)^2 - 2$

$= 4 - 2$

$= 2$

(iii) $f(4) = 10$

$f(1) = f(x) =$

$f(1) = (1)^2 - 2$

$= 1 - 2$

$= -1$

$f(4) + 2f(1)$

$= 10 + 2(-1)$

$= 10 - 2 = 8$

(iv) $f(-3) \Rightarrow f(x) = 2x + 7$

$f(-3) = 2(-3) + 7$

$= -6 + 7$

$= 1$

$\dfrac{f(1) - 3f(4)}{f(-3)} = \dfrac{-1 - 3(10)}{1} = -1 - 30 = -31$

EXAMPLE 1·8

If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$, then which of the following relations are functions from X to Y?

(i) $R_1 = \{(-5,a), (1,a), (3,b)\}$ (ii) $R_2 = \{(-5,b), (1,b), (3,a), (4,c)\}$

(iii) $R_3 = \{(-5,a), (1,a), (3,b), (4,c), (1,b)\}$

Solution

(i) $R_1 = \{(-5,a), (1,a), (3,b)\}$

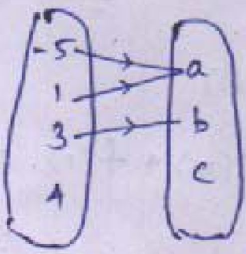


Fig (i)

We may represent the Relation R_1 in an arrow diagram from fig (i)

R_1 is a not function

$4 \in X$ does not have an image in Y.

(iii) $R_3 = \{(-5,a), (1,a), (3,b), (4,c), (1,b)\}$

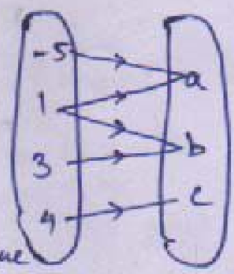
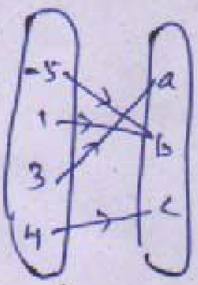


fig (iii)

R_3 is not a function as $1 \in X$ has two images $a \in Y$ and $b \in Y$

The image of an element should always an unique

(ii) $R_2 = \{(-5,b), (1,b), (3,a), (4,c)\}$



R_2 is ~~not~~ a function as each element of X has an unique image in Y.

EXAMPLE 1·9 Given, $f(x) = 2x - x^2$

(i) $f(1)$

$f(1) = 2(1) - (1)^2$

$= 2 - 1$

$= 1$

(ii) $f(x+1)$

$$f(x) = 2x - x^2$$

$$f(x+1) = 2(x+1) - (x+1)^2$$

$$= 2x + 2 - (x^2 + 2x + 1)$$

$$= \cancel{2x} + 2 - x^2 - \cancel{2x} - 1$$

$$= -x^2 + 1$$

(iii) $f(x) + f(1)$

$$f(x) + f(1) = 2x - x^2 + 1 \quad (\because f(1) = 1)$$

$$= -x^2 + 2x + 1$$

EXAMPLE 1.22

Find k if $f \circ f(k) = 5$, where $f(k) = 2$

$$f \circ f(k) = f(f(k))$$

$$5 = f(2k-1)$$

$$5 = 2(2k-1) - 1$$

$$5 = 4k - 2 - 1$$

$$5 + 3 = 4k$$

$$8 = 4k$$

$$\boxed{k = 2}$$

EXAMPLE 1.21

If $f(x) = 3x-2$, $g(x) = 2x+k$ and if $f\circ g = g\circ f$ then, find the value of k.

Solution

$$f\circ g = g\circ f$$
$$f(g(x)) = g(f(x))$$
$$f(2x+k) = g(3x-2)$$
$$3[2x+k]-2 = 2[3x-2]+k$$
$$\cancel{6x}+3k-2 = \cancel{6x}-4+k$$
$$3k-k = -4+2$$
$$2k = -2$$
$$\boxed{k=-1}$$

EXAMPLE 1.23

If $f(x) = 2x+3$, $g(x) = 1-2x$ and $h(x) = 3x$. Prove that $f\circ(g\circ h) = (f\circ g)\circ h$.

Prove solution

$f\circ(g\circ h) \Rightarrow g\circ h = g(h(x))$
$= g(3x)$
$= 1-2(3x)$
$= 1-6x$

$f\circ(g\circ h) = 2(1-6x)+3$
$= 2-12x+3$
$= 5-12x$ — (1)

$(f\circ g)\circ h \Rightarrow f\circ g = f(g(x))$
$= f(1-2x)$
$= 2(1-2x)+3$
$= 2-4x+3$
$= 5-4x$

$(f\circ g)\circ h = 5-4(3x)$
$= 5-12x$ — (2)

(1) = (2)

$\therefore f\circ(g\circ h) = (f\circ g)\circ h$

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EXAMPLE 1.24

Find x if $gff(x) = fgg(x)$, given $f(x) = 3x+1$, $g(x) = x+3$

$gff(x) = g \circ f[f(x)]$
$= g \circ f(3x+1)$
$= g \circ [3[3x+1]+1]$
$= g \circ [9x+3+1]$
$= g \circ [9x+4]$
$= 9x+4+3$
$= 9x+7$

$fgg(x) = f \circ g[g(x)]$
$= f \circ g(x+3)$
$= f \circ [x+3+3]$
$= f \circ [x+6]$
$= 3(x+6)+1$
$= 3x+18+1$
$= 3x+19$

Given,
$gff(x) = fgg(x)$
$9x+7 = 3x+19$
$9x - 3x = 19-7$
$6x = 12$
$\boxed{x = 2}$

UNIT-I

UNIT EXERCISE - I

1. If the ordered pairs (x^2-3x, y^2+4y) and $(-2, 5)$ equal, then find x and y.

Solution

$x^2 - 3x = -2$
$x^2 - 3x + 2 = 0$
$x^2 - 2x - x + 2 = 0$
$x(x-2) - 1(x-2) = 0$
$(x-2)(x-1) = 0$
$x-2 = 0$ | $x-1=0$
$\boxed{x=2}$ | $\boxed{x=1}$
$\boxed{x = 1, 2}$

$y^2+4y = 5$
$y^2+4y-5 = 0$
$y^2+5y-y-5=0$
$y(y+5)-1(y+5)=0$
$(y+5)(y-1)=0$
$y+5=0$ | $y-1=0$
$\boxed{y=-5}$ | $\boxed{y=1}$
$\boxed{y=-5, 1}$

2. The Cartesian product $A \times A$ has 9 elements among which $(-1,0)$ and $(0,+1)$ are found. Find the set A and the remaining elements of $A \times A$.

Solution

$n(A \times A) = 9$

$(-1,0), (0,1) \in A \times A$

$\therefore A = \{-1, 0, 1\}$

$A \times A = \{-1,0,1\} \times \{-1,0,1\}$

$= \{(-1,-1), (\underline{-1,0}), (-1,1), (0,-1), (0,0), (\underline{0,1}), (1,-1), (1,0), (1,1)\}$

$\therefore (-1,1), (-1,-1), (0,-1), (0,0), (1,-1), (1,0), (1,1)$ are the remaining elements of $A \times A$.

3. Given that $f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ 4, & x < 1 \end{cases}$. find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a+1)$ in the terms of a [Given $a \geq 0$]

Solution

(i) $f(0) = 4$

(ii) $f(3)$

$f(x) = \sqrt{x-1}$

$f(3) = \sqrt{3-1}$

$= \sqrt{2}$

(iii) $f(a+1)$ $(\because a \geq 0)$

$f(x) = \sqrt{x-1}$

$f(a+1) = \sqrt{a+1-1}$

$= \sqrt{a}$

4. Let $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$ and Let $f : A \to N$ be defined by $f(n) =$ the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.

Solution

Given, $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$

$f : A \to N$ defined by $f(n) =$ highest prime factor of

$f(9) = 3$ ($\because$ 3 is the highest prime factor of 9)

$f(10) = 5$ ($\because 5 \times 2 = 10$)

$f(11) = 11$

$f(12) = 3$ ($\because 12 = 3 \times 2 \times 2$)

$f(13) = 13$

$f(14) = 7$ ($\because 14 = 7 \times 2$)

$f(15) = 5$ ($\because 15 = 5 \times 3$)

$f(16) = 2$ ($\because 16 = 2 \times 2 \times 2 \times 2$)

$f(17) = 17$.

$\therefore f = \{(9,3), (10,5), (11,11), (12,3), (13,13), (14,7), (15,5), (16,2), (17,17)\}$

$\therefore$ Range of $f = \{2, 3, 5, 7, 11, 13, 17\}$

5. Find the domain of the function $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

Given, $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

If $x > +1$ and $x < -1$, $f(x)$ leads to unreal

If $x = -1$,

$f(-1) = \sqrt{1+\sqrt{1-\sqrt{1-(-1)^2}}}$

$= \sqrt{1+\sqrt{1-\sqrt{1-1}}}$

$= \sqrt{1+\sqrt{1-0}}$

$= \sqrt{1+1} = \sqrt{2}$

If $x = 0$,

$f(0) = \sqrt{1+\sqrt{1-\sqrt{1-0}}}$

$= \sqrt{1+\sqrt{1-1}}$

$= \sqrt{1} = 1$

If $x = 1$,

$f(1) = \sqrt{1+\sqrt{1-\sqrt{1-(1)^2}}}$

$= \sqrt{1+\sqrt{1-0}}$

$= \sqrt{1+1}$

$= \sqrt{2}$

$\therefore$ The domain of $f(x) = \{-1, 0, 1\}$

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x-2$, prove that $(f\circ g)\circ h = f\circ(g\circ h)$

Solution:

$f(x) = x^2$, $g(x) = 3x$, $h(x) = x-2$

$(f\circ g)\circ h \Rightarrow f\circ g = f(g(x))$

$= f(3x)$

$= (3x)^2$

$= 9x^2$

$(f\circ g)\circ h = 9(x-2)^2 \quad \text{--- (1)}$

$f\circ(g\circ h) \Rightarrow (g\circ h) = g(h(x))$

$= g(x-2)$

$= 3(x-2)$

$f\circ(g\circ h) = [3(x-2)]^2$

$= 9(x-2)^2 \quad \text{--- (2)}$

from (1) & (2)

$(f\circ g)\circ h = f\circ(g\circ h)$

7) Let $A = \{1,2\}$ and $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5,6,7,8\}$. Verify whether $A \times C$ is a subset of $B \times D$.

Solution

Given, $A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{5,6\}$

$D = \{5,6,7,8\}$

$A \times C = \{1,2\} \times \{5,6\}$

$= \{(1,5), (1,6), (2,5), (2,6)\}$

$B \times D = \{1,2,3,4\} \times \{5,6,7,8\}$

$= \{(1,5), (1,6), (1,7), (1,8), (2,5), (2,6), (2,7), (3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7),$

$\therefore$ Clearly $A \times C$ is a subset of $B \times D$.

8) If $f(x) = \frac{x-1}{x+1}$, $x \neq 1$ Show that $f(f(x)) = -\frac{1}{x}$ provided $x = 0$.

Given, $f(x) = \frac{x-1}{x+1}$, $f(f(x)) = \frac{-1}{x}$

$f[f(x)] = f\left(\frac{x-1}{x+1}\right)$

$\frac{-1}{x} = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$

$\frac{-1}{x} = \frac{\frac{\cancel{x} - 1 - \cancel{x} - 1}{\cancel{x+1}}}{\frac{x - \cancel{1} + x + \cancel{1}}{\cancel{x+1}}}$

$\frac{-1}{x} = \frac{-\cancel{2}}{\cancel{2}x}$

$\frac{-1}{x} = \frac{-1}{x}$

9) The function f and g are defined by

$f(x) = 6x+8$, $g(x) = \frac{x-2}{3}$.

(i) Calculate the value of $gg\left(\frac{1}{2}\right)$

(ii) write an expression for $gf(x)$ in its simplest form.

Solution $f(x) = 6x+8$; $g(x) = \frac{x-2}{3}$

(i) $gg(1/2)$

$g \circ g\left(\frac{1}{2}\right) \Rightarrow g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-2}{3}$

$= \frac{\frac{1-4}{2}}{3}$

$= \frac{-3}{6}$

$= -1/2$

$g\left[g\left(\frac{1}{2}\right)\right] = g\left(\frac{-1}{2}\right)$

$= \frac{\frac{-1}{2}-2}{3}$

$= \frac{\frac{-1-4}{2}}{3}$

$= \frac{-5}{6}$

(ii) $gf(x) = g[f(x)]$

$= g[6x+8]$

$= \frac{6x+8-2}{3}$

$= \frac{6x+6}{3} = \frac{\cancel{6}^{2}(x+1)}{\cancel{3}} = 2(x+1)$

10. write the domain of the following real functions.

(i) $f(x) = \frac{2x+1}{x-9}$

$f(x) = \frac{2x+1}{x-9}$

If $x-9=0$

$\boxed{x=+9}$

$f(x) \to \infty$

∴ The domain is $R - \{9\}$

(ii) $p(x) = \frac{-5}{4x^2+1}$

If $x \to c$, $4x^2+1$ does not tend to ∞

∴ The domain is R

(iii) $g(x) = \sqrt{x-2}$

The function exists only if $x \geq 2$

∴ The domain is $[2, \infty)$

(iv) $h(x) = x+6$

$h(x)$ exists for all real numbers

$\therefore$ The domain is $\mathbb{R}$

EXERCISE 1.6 (ONE MARK SOLUTIONS)

1. If $n(A \times B) = 6$, $A = \{1, 3\}$ then $n(B)$ is

(a) 1 (b) 2 (c) 3 (d) 6

Solution $n(A \times B) = n(A) \times n(B)$

$\cancel{6} = \cancel{2} \times n(B)$ $\boxed{n(B) = 3}$

2. $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$, then $n[(A \cup C) \times B]$

(a) 8 (b) 20 (c) 12 (d) 16

Solution $A \cup C = \{a, b, p, q, r, s\}$

$n(A \cup C) = 6$, $n(B) = 2$

$n[(A \cup C) \times B] = 6 \times 2 = 12$

$\boxed{\therefore n[(A \cup C) \times B] = 12}$

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then State which of the following statement is true.

(a) $(A \times C) \subset (B \times D)$ (b) $(B \times D) \subset (A \times C)$

(c) $(A \times B) \subset (A \times D)$ (d) $(D \times A) \subset (B \times A)$

Solution $A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$B \times D = \{(1,5), (1,6), (2,5), (2,6), (3,5), (3,6), (4,5), (4,6), (1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$

ANSWER: (A) $(A \times C) \subset (B \times D)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a Set B, then the number of elements in B is

(a) 3 (b) 2 (c) 4 (d) 8

Solution

$n(A \times B) = 10$ (1024 Consider as 10)

$n(A) \times n(B) = 10$

$5 \times n(B) = 10$

$\boxed{n(B) = 2}$

$\boxed{\text{Ans : (b) } 2}$

5. The range of the relation $R = \{(x, x^2) \mid x$ is prime number less than $13\}$ is

(a) $\{2, 3, 5, 7\}$

(b) $\{2, 3, 5, 7, 11\}$

(c) $\{4, 9, 25, 49, 121\}$

(d) $\{1, 4, 9, 25, 49, 121\}$

Solution

The Squares of 2, 3, 5, 7, 11 are 4, 9, 25, 49, 121

ANS: (c) $\{4, 9, 25, 49, 121\}$

6. If the ordered pairs $(a+2, 4)$ and $(5, 2a+b)$ are equal then (a, b) is ___.

(a) $(2, -2)$

(b) $(5, 1)$

(c) $(2, 3)$

(d) $(3, -2)$

Soln

$a + 2 = 5$

$a = 5 - 2$

$\boxed{a = 3}$

$4 = 2a + b$

$4 = 2(3) + b$

$4 - 6 = b$

$\boxed{b = -2}$

ANS : (d) $(3, -2)$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relation that can be defined from A to B is

(a) m^n

(b) n^m

(c) $2^{mn} - 1$

(d) 2^{mn}

Solution

$n(A) = m$, $n(B) = n$

$n(A \times B) = 2^{m \times n}$

Answer : (d) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively (a) (8, 6) (b) (8, 8) (c) (6, 8) (d) (8, 6)

Ans: (a) (8, 6)

Solution Given, Identity function, so, $a = 8$, $b = 6$

9) Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \to B$ is given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is
(a) Many - one function (b) Identity function
(c) One - one function (d) Into function

Solution Every distinct elements of A have distinct image in
Answer: (c) One - one function

10) If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then fog is

(a) $\frac{3}{2x^2}$ (b) $\frac{2}{3x^2}$ (c) $\frac{2}{9x^2}$ (d) $\frac{1}{6x^2}$

Solution $fog = f(g(x)) = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2 = 2\left(\frac{1}{9x^2}\right) = \frac{2}{9x^2}$

Answer: (c) $\frac{2}{9x^2}$

11) If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
(a) 7 (b) 49 (c) 1 (d) 14

Solution Given, function is bijective
So, the function is one - one & onto
$n(A) = n(B)$
$n(A) = 7$

ANSWER: (a) 7

12) Let f and g be two functions given by

$f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$

$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$

then the range of $f\circ g$ is

(A) $\{0,2,3,4,5\}$ (B) $\{-4,1,0,2,7\}$ (C) $\{1,2,3,4,5\}$

(D) $\{0,1,2\}$

Solution W.K.T, $f\circ g = g\circ f$

$g\circ f = g(f(x))$

$= \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$

$= \{0,1,2\}$ ANSWER : (D) $= \{0,1,2\}$

13. Let $f(x) = \sqrt{1+x^2}$ then

(a) $f(xy) = f(x)\cdot f(y)$ (B) $f(xy) \geq f(x)\cdot f(y)$

(c) $f(xy) \leq f(x)\cdot f(y)$ (D) None of these (c)

ANSWER : $f(xy) \leq f(x)\cdot f(y)$

14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are

(a) $(-1,2)$ (c) $(-1,-2)$

(b) $(2,-1)$ (d) $(1,2)$

ANSWER : (b) $(2,-1)$

$g(x) = \alpha x + \beta$

$\alpha = 2$

$\beta = -1$

$g(1) = 2(1) + (-1) = 2-1 = 1$

$g(2) = 2(2) + (-1) = 4-1 = 3$

$g(3) = 2(3) + (-1) = 6-1 = 5$

$g(4) = 2(4) + (-1) = 8-1 = 7$

15) $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is ____.

(a) linear (c) reciprocal

(b) Cubic (d) quadratic

Solution

$$f(x) = (x+1)^3 - (x-1)^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$f(x) = x^3 + 3x^2 + 3x + 1 - [x^3 - 3x^2 + 3x - 1]$$

$$= \cancel{x^3} + 3x^2 + \cancel{3x} + 1 - \cancel{x^3} + 3x^2 - \cancel{3x} + 1$$

$$= 6x^2 + 2 \quad \text{(Quadratic)}$$

ANSWER : (d) quadratic