HIGHER SECONDARY FIRST YEAR FIRST REVISION EXAMINATION – JANUARY 2025

PHYSICS KEY ANSWER

Note:

1. Answers written with **Blue** or **Black** ink only to be evaluated.
2. Choose the most suitable answer in Part A, from the given alternatives and write the option code and the corresponding answer.
3. For answers in Part-II, Part-III and Part-IV like reasoning, explanation, narration, description and listing of points, students may write in their own words but without changing the concepts and without skipping any point.
4. In numerical problems, if formula is not written, marks should be given for the remaining correct steps.
5. In graphical representation, physical variables for X-axis and Y-axis should be marked.

PART – I

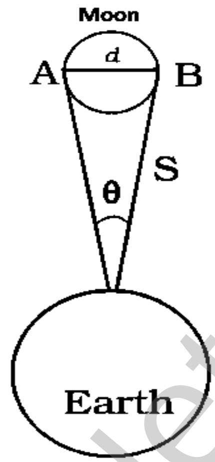
Answer all the questions. 15x1=15

Q. No.	OPTION	ANSWER	Q. No.	OPTION	ANSWER
1	(a)	increases	9	(a)	number of moles and T
2	(b)	need not be zero	10	(d)	a straight line
3	(a)	$[ML^2T^{-1}]$	11	(c)	1.0 m
4	(b)	zero	12	(b)	Vector, Scalar
5	(d)	$\frac{L}{\sqrt{2}}$	13	(b)	$[M^2 L^{-2} T^{-2}]$
6	(c)	can be positive or negative	14	(c)	Temperature
7	(d)	0.5	15	(a)	1
8	(c)	a straight line			

PART – II

Answer any **six** questions. Question number **21** is compulsory. 6x2=12

16	**Couple with examples:** Pair of forces which are equal in magnitude but **opposite in direction** and separated by a **perpendicular distance** so that **their lines of action do not coincide** that causes a turning effect is called a couple **Examples :** Steering wheel applied by the car driver Opening and closing of a water tap Winding the spring of an alarm clock Unlocking the locker by using a key Opening and closing of a cap of a water bottle, or jug. Turning of a screwdriver	2

17	**<u>Measure the diameter of the Moon using parallax method:</u>** 1. It is possible to determine the size of any planet or moon once we know the distance S of the planet. 2. The image of every heavenly body (moon) is a disc when viewed through an optical telescope. 3. The angle θ between two extreme points A and B on the disc with respect to a certain point on the Earth is determined with the help of a telescope. 4. The angle θ is called the angular diameter of the planet. The linear diameter d of the moon is then given by 5. d = distance × angular diameter 6. $d = s \times \theta$ 	2
18	Rotational velocity, $v_{ROT} = R\omega$ $v_{ROT} = 1.5 \times 3$; $v_{ROT} = 4.5\ ms^{-1}$ As $v_{CM} > R\omega$ (or) $v_{TRANS} > R\omega$, It is <u>not in pure rolling</u>, but sliding.	2
19	**<u>Water falls from the top of a hill to the ground:</u>** This is because the top of the hill is a point of **higher gravitational potential than the surface of the Earth.** i.e. **Vhill > Vground.**	2
20	From Hook's law $\frac{Strees}{Strain}$ = Constant or Coefficient of elasticity is the strees which produces for unit strain. SI Unit is Nm^{-2}	2
21	Frequency, f = 900 MHz; = 900×10^6 Hz The speed of wave is $c = 3 \times 10^8 ms^{-1}$ $\lambda = \frac{v}{f} = \frac{3 \times 10^8}{900 \times 10^6}$; = 0.33m	2
22	**<u>Second law of thermodynamics in terms of entropy:</u>** **"For all the processes that occur in nature** (irreversible process), **the entropy always increases.** For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.	2
23	**<u>Force constant of a spring:</u>** The displacement of the particle is measured **in terms of linear displacement $\vec{r}$. The restoring force is $\vec{F} = -k\vec{r}$**, where k is a spring constant or force constant. 1) Oscillations of a loaded spring 2) Vibrations of a turning force	2

24	**Law of equipartition of energy:** According to kinetic theory, the average kinetic energy of system of **molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom** (x or y or freedom will get ½ kT of energy. This is called law of equipartition of energy.	2

PART – II

Answer **any six** questions. Question number **30 is compulsory.** 6x3=18

25	**Cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion:** 1. If he stops his hands soon after catching the ball, the ball comes to rest very quickly. 2. It means that the momentum of the ball is brought to rest very quickly. 3. So the average force acting on the body will be very large. 4. Due to this large average force, the hands will get hurt. 5. To avoid getting hurt, the player brings the ball to rest slowly	3

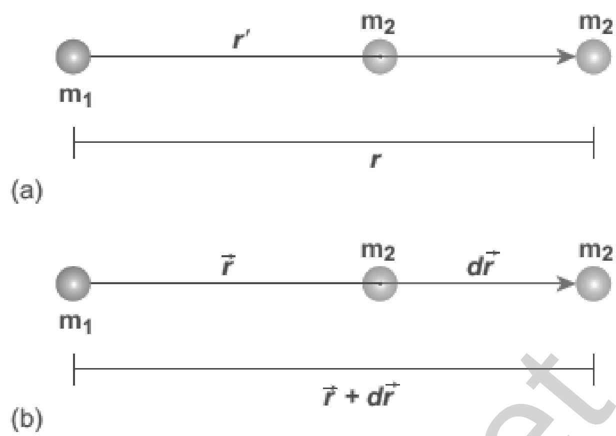
Question 26 (3 marks):

Conservative forces	Non-conservative forces
Work done is **independent of the path**	Work done **depends upon the path**
Work done in a **round trip is zero**	Work done in a **round trip is not zero**
Total energy **remains constant**	Energy is **dissipated as heat energy**
Work done is **completely recoverable**	Work done is **not completely recoverable**
Force is the negative gradient of potential energy	No such relation exists.
Examples : Elastic spring force, electrostatic force, magnetic force, magnetic force, gravitational force etc..	**Examples :** Frictional forces, Viscous force

27

Gravitational potential energy:

1) Two masses m_1 and m_2 are initially separated by a distance $r^{/}$. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from $r^{/}$ to r as shown in Figure (a)



2) To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from r to $\vec{r} + d\vec{r}$ (shown in the Figure (b), work has to be done externally.

This infinitesimal work is given by **$dW = \vec{F}_{ext} . d\vec{r}$** -------------- 1

The work is done against the gravitational force, therefore,

$$|\vec{F}_{ext}| = |\vec{F}_G| = \frac{Gm_1m_2}{r^2} \text{ ---------- } 2$$

Substituting equation (2) in (1), we get

$dW = \frac{Gm_1m_2}{r^2}\hat{r} . d\vec{r}$; $d\vec{r} = dr\,\hat{r}$

$\frac{Gm_1m_2}{r^2}\hat{r} . d\vec{r}$; $\hat{r}.\hat{r} = 1$ (Since both are unit vectors)

$dW = \frac{Gm_1m_2}{r^2} dr$

Thus the total work done for displacing the particle from

r is $W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2} dr$

$W = -\left(\frac{Gm_1m_2}{r^2}\right)_{r'}^{r}$

$-\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'}$

W = U (r) – U (r') Where $U(r) = \frac{Gm_1m_2}{r}$

5) This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r' respectively.

3

28	**Reversible and irreversible processes:** **Reversible process:** A thermodynamic process can be considered **reversible only if it possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states** as in the initial, direct process. Example: **A quasi–static isothermal expansion of gas, slow compression and expansion of a spring.** **Irreversible process: All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure**, temperature at every stage of the process.	3
29	**Expression for work done by torque:** i) Consider a rigid body rotating about a fixed axis. A point P on the body rotating about an axis perpendicular to the plane of the page. A tangential force F is applied on the body. ii) It produces a small displacement, ds on the body. The work done (dw) by the force is, dw = F ds iii) As the distance ds, the angle of rotation dθ and radius r, are related by the expression, ds = r dθ The expression for work done now becomes, dw = F ds; dw = F r dθ iv) The term (Fr) is the torque τ produced by the force on the body. dw = τdθ This expression gives the work done by the external torque τ, which acts on the body rotating about a fixed axis through an angle dθ.	3
30	(a) Absolute Temperature T = 27^0 C = 27+273 = 300 K Gas constant R = 8.32 J $mol^{-1}k^{-1}$ **For Oxygen molecule:** Molar mass M = 32 g = 32 x 10^{-3} kg mol^{-1} RMS speed of oxygen molecule, (v_{rms}) $= \sqrt{\frac{3RT}{M}}$; $= \sqrt{\frac{3 \times 8.32 \times (27+273)}{32 \times 10^{-3}}}$ $= \sqrt{\frac{3 \times 8.32 \times 300}{32 \times 10^{-3}}}$; = 483.73 ms^{-1} ; $(v_{rms}) \approx$ **484 ms^{-1}** **For Hydrogen molecule:** Molar mass M = 2 g ; = 2 x 10^{-3} kg mol^{-1} RMS speed of hydrogen molecule, (v_{rms}) $= \sqrt{\frac{3RT}{M}}$; $= \sqrt{\frac{3 \times 8.32 \times (27+273)}{2 \times 10^{-3}}}$ $= \sqrt{\frac{3 \times 8.32 \times 300}{2 \times 10^{-3}}}$; = 1934 ms^{-1} ; $(v_{rms}) =$ **1.93 k ms^{-1}** Note that the rms speed is inversely proportional to $\sqrt{M}$ and the molar mass of oxygen is 16 times higher than molar mass of hydrogen. It implies that the rms speed of hydrogen is 4 times greater than rms speed of oxygen at the same temperature. $\frac{1934}{484} \approx$ **4**	3

31	**Soldiers are not allowed to march on a bridge**. This is to avoid resonant vibration of the bridge. While crossing a bridge, if the period of stepping on the ground by marching soldiers equals the natural frequency of the bridge, it may result in resonance vibrations. This may be so large that the bridge may collapse.	3
32	**Error in the division or quotient of two quantities:** Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Consider the quotient, Z = $\frac{A}{B}$ The error ΔZ in Z is given by $Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$; $= \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$ or $Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$ [Using $(1+x)^n \approx 1+nx$, when x <<1] Dividing both sides by Z, we get, $1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$ $= 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} . \frac{\Delta B}{B}$ As the terms ΔA/A and ΔB/B are small, their product term can be neglected. The maximum fractional error in Z is given by $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$	3
33	**Applications of viscosity:** 1) Viscosity of liquids helps in choosing **the lubricants for various machinery parts. Low viscous lubricants** are used in light machinery parts and high viscous lubricants are used in heavy machinery parts. 2) As **high viscous liquids damp the motion**; they are used **in hydraulic brakes as brake oil.** 3) **Blood circulation** through arteries and veins depends upon the viscosity of fluids. 4) Viscosity is used in **Millikan's oil-drop method** to find the **charge of an electron.**	3

PART – IV

Answer **all** the questions. 5x5=25

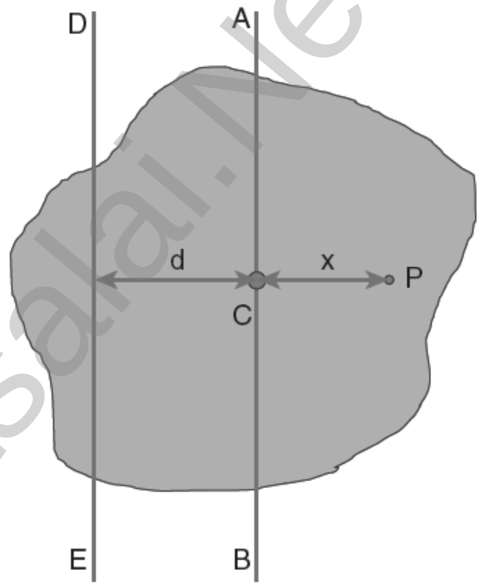
34 (a)

Parallel axis theorem:

i) Parallel axis theorem states that **the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.**

ii) If IC is the moment of inertia of the body of mass M about an axis passing through the center of mass, then the moment of inertia I about a parallel axis at a distance d from it is given by the relation, $I = I_C + Md^2$

iii) let us consider a rigid body as shown in Figure. Its **moment of inertia about an axis AB passing through the center of mass is I_C.** DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of I_C. For this, let us consider a point mass m on the body at position x from its center of mass.



iv) The moment of inertia of the point mass about the axis DE is, $m(x + d)^2$.The moment of inertia I of the whole body about DE is the summation of the above expression.

$I = \Sigma m(x + d)^2$ This equation could further be written as,

$I = \Sigma m(x^2 + d^2 + 2xd)$

$I = \Sigma(mx^2 + md^2 + 2dmx)$

$I = \Sigma mx^2 + \Sigma md^2 + 2d\Sigma mx$

v) Here, Σmx^2 is the moment of inertia of the body about thecenter of mass. Hence, $I_C = \Sigma mx^2$

The term, $\Sigma mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation (Σmx) will be zero

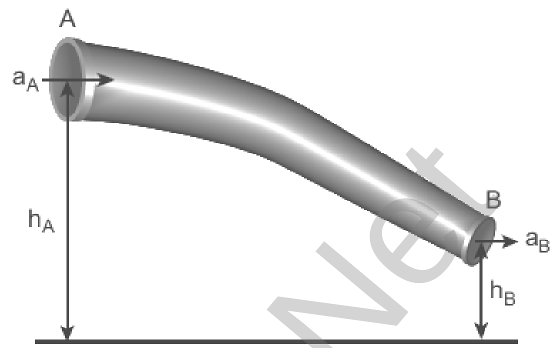
Thus, **$I = I_C + \Sigma md^2; I_C + (\Sigma m)d^2$**

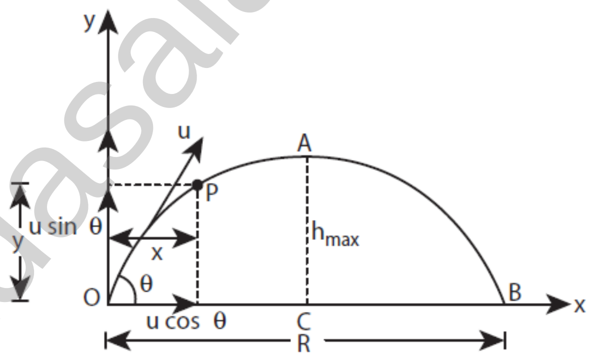
vi) Here, Σm is the entire mass M of the object ($\Sigma m = M$)

$I = I_C + Md^2$

Hence, the parallel axis theorem is proved.

5

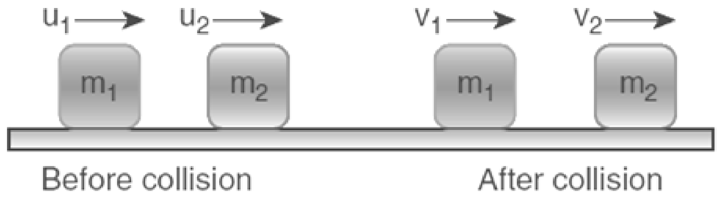
34 (b)	**Bernoulli's theorem:** According to Bernoulli's theorem, **the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant**. $\frac{P}{\rho} + \frac{1}{2}v^2 + gh =$ **Constant**, this is known as Bernoulli's equation. **Proof:**  Let us consider a flow of liquid through a pipe AB as shown in Figure. Let Vbe the volume of the liquid when it enters Ain a time *t* which is equal to the volume of the liquid leaving *B* in the same time. Let a_A, $v_A$and PAbe the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at *A* respectively. Let the **force exerted by the liquid at Ais $F_A = P_A a_A$** **Distance travelled by the liquid in time t is d = v_A t** Therefore, the work done is W = $F_A d = P_A a_A v_A$ t But **$a_A v_A t = a_A d$ =V,** volume of the liquid entering at A. Thus, the **work done is the pressure energy (at A), W = $F_A d = P_A V$** Pressure energy per unit volume at A = $\frac{\text{Pressure energy}}{\text{Volume}} = \frac{P_A V}{V} = P_A$ Pressure energy per unit mass at A = $\frac{\text{Pressure energy}}{\text{Mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$ Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is $E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$ Potential energy of the liquid at A, $P_{EA} = mg h_A$, Due to the flow of liquid, the **kinetic energy of the liquid at A, $KE_A = ½ mV_A^2$** Therefore, the total energy due to the flow of liquid at A, $E_A = EP_A + KE_A + PE_A$; **$E_A = m\frac{P_A}{\rho} + ½ mV_A^2 + mgh_A$** Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get **$E_B = m\frac{P_B}{\rho} + ½ mV_B^2 + mgh_B$** From the **law of conservation of energy, $E_A = E_B$** $E_A = m\frac{P_A}{\rho} + ½ mV_A^2 + mgh_A = E_B = m\frac{P_B}{\rho} + ½ mV_B^2 + mgh_B$ **$\frac{P_A}{\rho} + ½ V_A^2 + gh_A = \frac{P_B}{\rho} + ½ V_B^2 + gh_B$ = constant** Thus, the above equation can be written as $\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g}$ **+ h = constant**	5

35 (a)	T∝ $m^a l^b g^c$; T = k $m^a l^b g^c$ Here k is the dimensionless constant. Rewriting the above equation with dimensions $[T^1] = [M^a] [L^b] [LT^{-2}]^c$ $[M^0L^0T^1] = [M^a L^{b+c} T^{-2c}]$ Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1 Solving for a,b and c a = 0, b = 1/2, and c = −1/2 From the above equation T = k. $m^0 l^{1/2} g^{1/2}$ $T = k\left(\frac{l}{g}\right)^{1/2}$; $k\sqrt{l/g}$; **Experimentally k = 2π,** **hence** $T = 2\pi\sqrt{l/g}$	5
35 (b)	i) Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal. then, $\vec{u} = u_x\hat{i} + u_y\hat{j}$ where u_x= u Cos θ is the horizontal component and u_y= u sin θ the vertical component of velocity.  ii) u_x remains constant throughout the motion.uy changes with time under the effect of acceleration due to gravity. First it decreases, becomes zero at the maximum height, after which it again increases till the prose reach be ground. iii) Hence after the time t, the velocity along horizontal motion $v_x = u_x + a_x t = u_x$ = u cos θ The horizontal distance travelled by projectile in time t is $s_x = u_x t + ½ a_x t^2$. Here, s_x = x, u_x = u cos θ, a_x = 0 iv) Thus, x = u cos θ. t **or** $t = \frac{x}{u\,Cos\theta}$ ------------- (1) Next, for the vertical motion $v_y = u_y + a_y t$ Here $u_y = u\sin\theta$, $a_y = -g$ (Acceleration due to gravity acts opposite to the motion). Thus, v_y = u Sin θ – gt -----------------(2) The vertical distance travelled by the projectile in the same time *t* is $s_y = u_y t + ½ a_y t^2$ Here, s_y = y, u_y = u sin θ, $a_x = -g$ *Then*, y = u Sin θ t – ½ gt² --------------- (3) v) Substitute the value of *t* from equation (1) in equation (3), we have $y = u\,Sin\,\theta\frac{x}{u\,Cos\theta} - \frac{1}{2}g\frac{x^2}{u^2Cos^2\theta}$	

	$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$ -----------------(4) Thus, the path followed by the projectile is an inverted parabola. **Maximum height (h_{max})** The **maximum vertical distance travelled by the projectile** during its journey is called maximum height. This is determined as follows: For the vertical part of the motion, $v_y^2 = u_y^2 + 2a_y s$ Here, $u_y = u\sin\theta$, $a = -g$, $s = h_{max}$, and at the maximum height $v_y = 0$ Here, $(0)^2 = u^2\sin^2 = gh_{max}$ (or) $h_{max} = \frac{u^2\sin^2\theta}{2g}$ **Horizontal range (R)** The **maximum horizontal distance between the point of projection** and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R). This is found easily since the horizontal component of initial velocity remains thesame. We can write Range R = Horizontal component of velocity x time of flight $= u\cos\theta \times T_f$ $R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$ $R = \frac{u^2\sin 2\theta}{g}$	
36 (a)	<u>**Time period of the satellite:**</u> The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E + h)$ and time taken for it is the time period, T. Then $\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi(RE+h)}{T}$ From equation, $\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi(RE+h)}{T}$ --------------- 1 $T = \frac{2\pi}{\sqrt{GM_E}}(R_E+h)^{\frac{3}{2}}$ ----------------- 2 Squaring both sides of the equation (2), we get $T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3$ $\frac{4\pi^2}{GM_E}$ = Constant say c, $T^2 = c(R_E+h)^3$ --------------- 3 Equation (3) implies that a satellite orbiting the **Earth has the same relation between time and distance as that of Kepler's law of planetary motion.** For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E . Then, $T^2 = \frac{4\pi^2}{GM_E}R_E^3$; $T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}}$ $T^2 = \frac{4\pi^2}{g}R_E$ Since $\frac{GM_E}{R_E^2} = g$; $T = 2\pi\sqrt{\frac{R_E}{g}}$	5

36 (b)	i) The force on each particle (Newton's second law) can be written as $\vec{F}_{12}=\frac{d\vec{p}_1}{dt}$ and $\vec{F}_{21}=\frac{d\vec{p}_2}{dt}$ ii) Here $\vec{p}_1$ is the momentum of particle 1which changes due to the force $\vec{F}_{12}$ exertedby particle 2. Further $\vec{p}_2$ is the momentum of particle 2. These changes due to $\vec{F}_{21}$ exertedby particle 1. $\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$; $\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$; $\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$ iii) It implies that $\vec{p}_1 + \vec{p}_2$= constant vector (always). iv) $\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute thesystem. v) If there are no external forces acting on the system, then the total linear momentum ofthe system ($\vec{p}_{tot}$) is always a constant vector. **Examples:** Consider the firing of a gun. Here the system is Gun+bullet. Initially the gun and bullet are at rest, hence the total linear momentum of the system is zero. Let $\vec{p}_1$ be the momentum of the bullet and $\vec{p}_2$ the momentum of the gun before firing. Since initially both are at rest, $\vec{p}_1 = 0, \vec{p}_2 = 0$. Total momentum before firing the gun is zero, $\vec{p}_1 + \vec{p}_2 = 0$. According to the law of conservation of linear momentum total linear momentum has to be zero after the firing also. When the gun is fired, a force is exerted by the gun on the bullet in forward direction. Now the momentum of the bullet changes from $\vec{p}_1$ to $\vec{p}_1'$. To conserve the total linear momentum of the system, the momentum of the gun must also change from $\vec{p}_2$ to $\vec{p}_2'$. Due to the conservation of linear momentum, $\vec{p}_1' + \vec{p}_2' = 0$. It implies that $\vec{p}_1' = -\vec{p}_2'$, the momentum of the gun is exactly equal, but in the opposite direction to the momentum of the bullet. This is the reason after firing, the gun suddenly moves backward with the momentum $(-\vec{p}_2)$. It is called 'recoil momentum'. This is an example of conservation of total linear momentum.	5

37(a)



Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positivex direction) on a frictionless horizontal surface.

i) In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e.,$\mu_1 > \mu_2$ For elastic collision, the total linear momentum and kinetic energies of thetwo bodies before and after collision must remain the same.

From the law of conservation of linear momentum,

Total momentum before collision (*pi*) = Total momentum after collision (*pf*)

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2} \text{ ———— (1) (or)}$$

$$\mathbf{m_1(u_1 - v_1) = m_2(v_2 - u_2)} \text{ ---------------- (2)}$$

For elastic collision,

Total kinetic energy before collision KE_i= Total kinetic energy after collision KE_f

$$\mathbf{\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 = \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2} \text{ ———— (3)}$$

After simplifying and rearranging the terms,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Using the formula, $a^2 - b^2 = (a + b)(a - b)$, we can rewrite the above equation as

$$\mathbf{m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)} \text{ --------- (4)}$$

Dividing equation (4) by (2) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)} = \frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$\mathbf{u_1 + v_1 = v_2 + u_2}$, Re-arranging $\mathbf{u_1 - u_2 = v_2 - v_1}$ ------------- (5)

Equation (5) can be rewritten as $(u_1 - u_2 = -(v_1 - v_2)$

ii) This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2 ,

$v_1 = v_2 + u_2 - u_1$ ------------ (6) or $\mathbf{v_2 = u_1 + v_1 - u_2}$ ---------------(7)

To find the final velocities $v_1$and v_2:

Substituting equation (7) in equation (2) gives the velocity of m_1 as

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$$

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$$

5

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2)\, u_1 + 2m_2u_2 = (m_1 + m_2)v_1$ **(or)**

$$v_1 = \left(\frac{m_1 - m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2 \text{ ---------(8)}$$

Similarly, by substituting (6) in equation (2) or substituting equation (8) in equation (7), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)u_2 \text{ ---------- (9)}$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

Equation (8) $-\!\!\to$ $v_1 = (0)u_1 + \left(\frac{2m_2}{2m_2}\right)u_2$; $\mathbf{v_1 = u_2}$ ------------ (10)

Equation (9) $-\!\!\to$ $v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)\, u_2$; $\mathbf{v_2 = u_1}$ ------------ (11)

The equations (10) and (11) show that in **one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.**

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and second body (usually called target) is at rest **($u_2 = 0$)**,

By substituting $m_1 = m_2$ **and** $u_2 = 0$ in equations (8) and equations (9) we get,

From equation (8) $-\!\!\to$ $v_1 = 0$ -------------(12)

From equation (9) $-\!\!\to$ $v_2 = u_1$ ------------- (13)

Equations (12) and (13) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: The first body is very much lighter than the second body $\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ then the ratio $\frac{m_1}{m_2} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2}+1}\right)u_1 + \left(\frac{2}{\frac{m_1}{m_2}+1}\right)(0);\ v_1 = \left(\frac{0-1}{0+1}\right)u_1;\ \mathbf{v_1 = -u_1} \text{ ---------(14)}$$

Similarly, Dividing numerator and denominator of equation (9) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0);\ v_2 = (o)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0);\ \mathbf{v_2 = 0} \text{ ----- (15)}$$

The equation (14) implies that **the first body which is lighter returns back rebounds) in the opposite direction with the same initial velocity as it has a negative sign.**

The equation (15) implies that **the second body which is heavier in mass continues to remain at rest even after collision**. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4: The second body is very much lighter than the first body $\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ then the ratio $\frac{m_2}{m_1} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0);$$

$v_1 = \left(\frac{0-1}{0+1}\right)u_1 + \left(\frac{0}{1+0}\right)(0)$; **$v_1 = u_1$** -------------------(16)

Similarly, Dividing numerator and denominator of equation (14) by m_1, we get

$v_1 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}\right)(0)$; $v_2 = \left(\frac{2}{1+0}\right)u_1$; $v_2 = 2u_1$ ------(17)

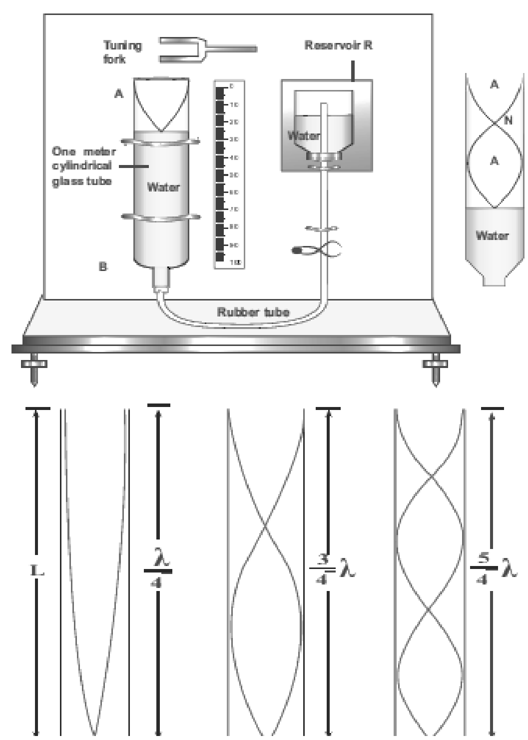
The equation (16) implies that **the first body which is heavier continues to move with the same initial velocity.**

The equation (17) suggests that **the second body which is lighter will move with twice the initial velocity of the first body.**

It means that **the lighter body is thrown away from the point of collision.**

37 (b)

<u>Resonance air column apparatus:</u>



1) The resonance air column apparatus is **one of the simplest techniques to measure the speed of sound in air at room temperature.**

2) It consists of a cylindrical glass tube of **one-meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube** as shown in Figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it.

3) The **tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R.** The surface of the water will act as a closed end and other as the open end.

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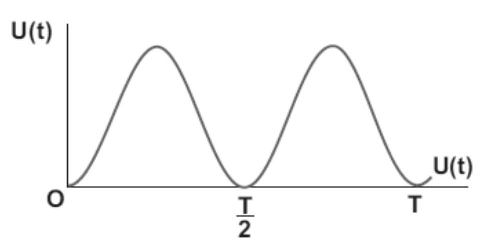
	4) Therefore, it **behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end.** 5) When a **vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column**. These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves. 6) The **length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork).** 7) At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced. Let the **first resonance occur at lengthL_1, then** $\frac{1}{4}\lambda$ = L_1 8) But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some **small distance above the open end. Including this end correction, the first resonance is$\frac{1}{4}\lambda = L_1 + e$** 9) Now the length of the air column is increased to get the second resonance. Let L_2be the length at which **the second resonance occurs. Again taking end correction into account, $\frac{3}{4}\lambda = L_2 + e$** In order to avoid end correction, let us take the difference of equation$\frac{1}{4}\lambda = L$, and equation $f_1: f_2: f_3: f_4:... = 1 : 2 : 3 : 4 : ...$ $\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e - L_1 + e)$ $\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L \Rightarrow \lambda = 2\,\Delta L$	
38 (a)	<u>**Translational equilibrium:**</u> 1) Linear momentum is constant 2) Net force is zero **Rotational equilibrium:** 1) Angular momentum is constant 2) Net torque is zero **Static equilibrium:** 1) Linear momentum and angular momentum are zero 2) Net force and net torque are zero **Dynamic equilibrium:** 1) Linear momentum and angular momentum are constant 2) Net force and net torque are zero **Stable equilibrium:** 1) Linear momentum and angular momentum are zero 2) The **body tries to come back to equilibrium if slightly disturbed** and released 3) The center of mass of the **body shifts slightly higher if disturbed** from equilibrium	5

	4) Potential energy of the **body is minimum and it increases** if disturbed **Unstable equilibrium:** 1) Linear momentum and angular momentum are zero 2) **The body cannot come back to equilibrium if slightly disturbed** and released 3) The **center of mass of the body shifts slightly lower if disturbed** from equilibrium 4) Potential energy of the body is **not minimum and it decreases** if disturbed **Neutral equilibrium:** 1) Linear momentum and angular momentum are zero 2) The **body remains at the same equilibrium if slightly disturbed** and released 3) The **center of mass of the body does not shift higher** or lower if disturbed from equilibrium 4) **Potential energy remains same even if disturbed**	
38 (b)	**a. <u>Expression for Potential Energy</u>** 1) **For the simple harmonic motion, the force and the displacement are related** by Hooke's law $\vec{F} = -k\vec{r}$ 2) Since force is a vector quantity, **in three dimensions it has three components.** Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case F = − kx ---------------(1) **The work done by the conservative force field is independent of path.** The potential energy U can be calculated from the following expression. $F = \frac{dU}{dx}$ ------------ 2 Comparing (1) and (2), we get $-\frac{dU}{dx} = -kx$; dU = kxdx 3) This work done by the force F during a small displacement dx stores as potential energy $U(x) = \int_0^x kx'dx = \frac{1}{2}(x')^2\Big\|_0^x = \frac{1}{2}kx^2$ ------ 3 From equation $\omega = \sqrt{\frac{k}{m}}$, we can substitute the value of force constant k = $m\omega^2$in equation (3),$U(x) = m\,\omega^2x^2$ 4) where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation x = A sin ωt $U(t) = \frac{1}{2} m\omega^2A^2Sin^2\omega t$ -----------4 This variation of U is shown below.  **b. Expression for Kinetic Energy**	5

Kinetic energy KE = ½ mv_x^2 = ½ m $\left(\frac{dx}{dt}\right)^2$

Since the particle is executing simple harmonic motion, from equation

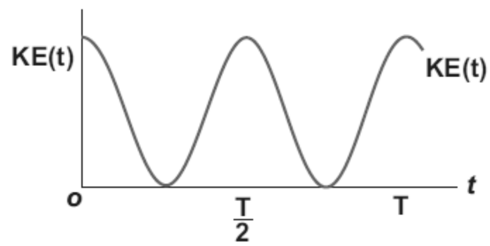
y = A sin ωt ; x = A sin ωt Therefore, velocity is $v_x = \frac{dx}{dt}$ A ωcosωt

= aω$\sqrt{1-\left(\frac{x}{A}\right)^2}$; v_x =ω $\sqrt{A^2-x^2}$ ------------- 5

Hence, KE = ½ mv_x^2 = ½$m\omega^2(A^2-x^2)$ ------------ 6

KE = ½$m\omega^2A^2\cos^2\omega t$ ------------- 7

This variation with time is shown below.



c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy

E = KE + U ------------8 ;**E = ½m $\omega^2(A^2-x^2)$ + ½m ω^2x^2**

Hence, cancelling x^2 term, E = ½m ω^2A^2 = Constant ---------9

Alternatively, from equation (4) and equation (7),

we get the total energy as **E = ½ $m\omega^2A^2\sin^2\omega t$ +½$m\omega^2A^2\cos^2\omega t$**

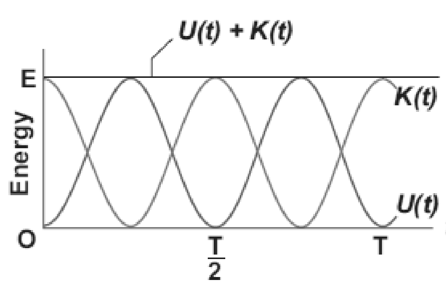
E = ½ $m\omega^2A^2(\sin^2\omega t + \cos^2\omega t)$

From trigonometry identity,

$(\sin^2\omega t + \cos^2\omega t) = 1$

E = ½ $m\omega^2A^2$ = Constant.

which gives the law of conservation of total energy. This is depicted in Figure. Thus the amplitude of simple harmonic oscillator,



can be expressed in terms of total energy. **A = $\sqrt{\frac{2E}{m\omega^2}}$ = $\sqrt{\frac{2E}{k}}$**