Part - IV

IV. Answer all the questions. $7 \times 5 = 35$

41. a) Solve the system of equations $x - y + 2z = 2$, $2x + y + 4z = 7$, $4x - y + z = 4$ by Cramer's rule.

(OR)

b) Show that $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx = \frac{\pi}{2} - 1$

42. a) Prove that the local minimum values for the function $f(x) = 4x^6 - 6x^4$ attain at -1 and 1

(OR)

b) Show that the locus of $z = x + iy$ if $|z + i| = |z - 1|$ is $x + y = 0$

43. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

(OR)

b) Solve : $\left(1 + x + xy^2\right)\frac{dy}{dx} + \left(y + y^3\right) = 0$

44. a) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function of random variable X is $f(x) = \begin{cases} 0 & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres.

(OR)

b) Find the vector equation (any form) or Cartesian equation of a plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

45. a) Show that the area of the region bounded by $3x - 2y = 0$, $x = -3$ and $x = 1$ is $\frac{15}{2}$

(OR)

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Show that the angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$

46. a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

(OR)

b) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$, $(-2, -12)$ and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian Elimination method)

47. a) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

(OR)

b) Identify the type of conic and find centre, foci and vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

****

SECOND REVISION TEST - 2025

Standard XII

Reg.No.

MATHEMATICS

Time : 3.00 hrs | **Part - I** | *Marks : 90*

I. Choose the correct answer: $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj } AB| =$

a) -40 b) -80 c) -60 d) -20

2. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

a) 0 b) -2 c) -3 d) -1

3. The conjugate of a complex number is $\frac{1}{i-2}$. Then the complex number is

a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

4. z_1, z_2, z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

a) 3 b) 2 c) 1 d) 0

5. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

a) $-q/r$ b) $-p/r$ c) q/r d) $-p/q$

6. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$, then x is a root of the equation

a) $x^2 - x - 6 = 0$ b) $x^2 - x - 12 = 0$
c) $x^2 + x - 12 = 0$ d) $x^2 + x - 6 = 0$

7. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

a) $15 < m < 65$ b) $35 < m < 85$ c) $-85 < m < -35$ d) $-35 < m < 15$

8. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is

a) 8 b) 6 c) 10 d) 12

9. If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}} + \frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

a) 1 b) -1 c) 2 d) 3

10. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

11. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is
a) (4,11) b) (4,–11) c) (–4,11) d) (–4,–11)

12. The maximum value of the function x^2e^{-2x}, $x > 0$ is
a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{1}{e^4}$

13. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
a) $0.3x\,dx\,m^3$ b) $0.03x\,m^3$ c) $0.03x^2\,m^3$ d) $0.03x^3\,m^3$

14. For any value of $n \in Z$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]dx$ is
a) $\frac{\pi}{2}$ b) π c) 0 d) 2

15. If $f(x) = \int_1^x \frac{e^{\sin u}}{u} du$, $x > 1$ and $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of a is
a) 3 b) 6 c) 9 d) 5

16. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ is
a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1

17. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
a) $y + \sin^{-1}x = c$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = c$ d) $x^2 + 2\sin^{-1}y = 0$

18. A pair of dice numbered 1,2,3,4,5,6 of a six-sided die and 1,2,3,4 of a four-sided die is roed and the sum is determined. Let the random variable X denoted this sum. Then the number of elements in the inverse image of 7 is
a) 1 b) 2 c) 3 d) 4

19. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is
a) 1 b) 2 c) 3 d) 4

20. Which one of the following is incorrect? For any two propositions p and q, we have
a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$
c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ d) $\neg(\neg p) \equiv p$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $z = (2 + 3i)(1 - i)$, then prove that $z^{-1} = \frac{5}{26} - i\frac{1}{26}$

22. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

23. Find the principal value of $\tan^{-1}(\sqrt{3})$

24. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c

25. Show that the three vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

26. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

27. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

28. Show that the solution of $\frac{dy}{dex} = \sqrt{\frac{1-y^2}{1-x^2}}$ is $\sin^{-1}y = \sin^{-1}x + C$ or $\sin^{-1}x = \sin^{-1}y + C$

29. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$

30. X is the number of tails occurred when three fair coins are tossed simultaneously. Find the values of the random variable X and number of points in its reverse images.

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Show that the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ is 3

32. Which one of the points $10 - 8i$, $11 + 6i$ is closest to $1 + i$

33. If the sides of a cubic box are increased by 1, 2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Show that the volume of the cuboid is 60 cubic units.

34. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds?

35. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

36. An egg of a particular bird is spherical in shape. If the radius to the inside of the shell is 4 mm and radius to the outside of the shell is 4.2 mm, prove that the approximate volume of the shell is 12.8π mm^3.

37. Show that $\int_0^{\pi/3} \frac{\sec x \tan x}{1 + \sec^2 x} dx = \tan^{-1}(2) - \frac{\pi}{4}$

38. Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ₹30 for each black ball selected and the loss ₹20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

39. Show that $p \to q$ and $q \to p$ are not equivalent.

40. Solve : $x \cos y\,dy = e^x(x \log x + 1)\,dx$

XII - Maths 2nd Revision - Chengalpattu dist.

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RLT Academy Athur.

1. (b) -80
2. (d) -1
3. (b) $\frac{-1}{i+2}$
4. (d) 0
5. (a) $\frac{-q}{r}$
6. (b) $x^2-x-12=0$
7. (d) $-35<m<15$
8. (c) 10
9. (a) 1
10. (d) $\pi/2$
11. (a) $(4,11)$
12. (c) $1/e^2$
13. (d) $0.03x^3m^3$
14. (c) 0
15. (c) 9
16. (c) $(1,1)$
17. (a) $y+\sin^{-1}x=c$
18. (d) 4
19. (a) 1
20. (c) $\neg(p\vee q)\equiv\neg p\vee\neg q$

II. 21. $z=(2+3i)(1-i)$

$z=5+i$

$z^{-1}=\frac{1}{5+i}\times\frac{5-i}{5-i}$

$z^{-1}=\frac{5-i}{26}$

$\boxed{z^{-1}=\frac{5}{26}-i\frac{1}{26}}$

22. $\text{Root}_1=2-\sqrt{3}$, $\text{Root}_2=2+\sqrt{3}$

S.R $=4$

P.R $=1$

$\boxed{x^2-4x+1=0}$

23. $\tan^{-1}(\sqrt{3})$

$y=\tan^{-1}(\sqrt{3})$

$\tan y=\sqrt{3}$

$y=\pi/3\in[-\pi/2,\pi/2]$

24. $y=mx+c$, $x^2+y^2=a^2$,

$c^2=a^2(1+m^2)$.

$c=\pm\sqrt{9(1+16)}=\pm3\sqrt{17}$

$\boxed{c=\pm3\sqrt{17}}$

25. $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$

$\vec{c}=3\hat{i}+\hat{j}+3\hat{k}$.

$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}2&3&1\\1&-2&2\\3&1&3\end{vmatrix}=-16+9+7$
$=0.$

$\therefore$ It is coplanar.

26. $f(x)=x^2+3x$

$f'(x)=2x+3.$

$df=f'(x)dx=(2x+3)dx$

$x=2,\ dx=0.1$

$df=[2(2)+3]0.1$
$=0.7$

27. $A=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$

$A^T=\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$

$AA^T=\begin{bmatrix}\cos^2\theta+\sin^2\theta&\cos\theta\sin\theta-\sin\theta\cos\theta\\\sin\theta\cos\theta-\cos\theta\sin\theta&\cos^2\theta+\sin^2\theta\end{bmatrix}$

$AA^T=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I_2$.

$AA^T=I_2$. $\therefore A$ is orthogonal

(28) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1}y = \sin^{-1}x + C.$

(29) $\frac{dy}{dx} = 3x^2 - 6x + 1 = 1$

$3x^2 - 6x = 0$

at $x=0$, $x=2$.

points $(0,-2), (2,-4)$

Tangent $y = x$. is parallel

(30) $S = \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$n(x=0) = 1$

$n(x=1) = 3$

$n(x=2) = 3$

$n(x=3) = 1$

x	0	1	2	3
Random Variable	1	3	3	1

III (31) $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

$e(A) = \min(4 \times 3)$

$e(A) = 3.$

$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = -1(-32) + 2(-16)$

$= 16 \neq 0.$

$\boxed{\therefore e(A) = 3}$

(32) $1+i$ to $10-8i = |10-8i-1-i|$

$= 9\sqrt{2} = \sqrt{162}$

Distance From, $1+i$ to $11+6i$

$= |11+6i - 1-i| = 5\sqrt{5} = \sqrt{125}$

$11+6i$ is a closest to $1+i$.

(33) $(x+1)(x+2)(x+3) - x^3 = 52$

$6x^2 + 11x + 6 = 52$

$6(x+2)(x + \frac{23}{6}) = 0$

$\boxed{x=2}$. $\boxed{\text{Volume} = 60}$

(34) $\pi/2 < \cos^{-1}(3x-1) < \pi$

$-1 < 3x-1 < 0$

$0 < 3x < 1$

$0 < x < 1/3$ (or) $x \in (0, 1/3)$

(35) $AS = 94.5 \times 10^6$ km.

$SA' = 152 \times 10^6$ km

$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

$2c = 57.5 \times 10^6 = 575 \times 10^5$ km

$\boxed{SS' = 575 \times 10^5 \text{ km}}$

(36) $V(r) = \frac{4}{3}\pi r^3$

$V(5.3) - V(5) \simeq V'(r)\,dr$

$\Rightarrow 4\pi r^2 dr = 100\pi(0.3)$

$\Rightarrow 30\pi\, mm^3$

volume of shell $= 30\pi\, mm^3$

(37) $I = \int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\, dx$

$u = \sec x$, $du = \sec x \tan x\, dx$

$x=0$, $u=1$, $x=\pi/3$, $u=2$

$I = \int_1^2 \frac{du}{1+u^2} = (\tan^{-1}(u))_1^2$

$= \tan^{-1}(2) - \pi/4$

38. X (Both Black Balls)

$\Rightarrow ₹2(30) \Rightarrow ₹60$

X (1B & 1W) $= ₹30 - ₹20$

$= ₹10$

X (Both W) $= ₹2(-20)$

$= -₹40.$

X	60	10	-40	Total
Image	6	24	15	45

39.

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

From the table they are not equivalent.

40. $\cos y\, dy = e^x(x\log x + 1)dx$

$\cos y\, dy = e^x \frac{1}{x}(x\log x+1)dx$

$\Rightarrow e^x\left(\log x + \frac{1}{x}\right)dx$

$\cos y\, dy = e^x \log x\, dx + e^x \frac{1}{x} dx$

$\int \cos y\, dy = \int e^x \log x\, dx + \int e^x \frac{1}{x} dx$

$I_1 = e^x(\log x) - \int e^x \frac{1}{x} dx$

$\sin y = e^x \log x - \int e^x \frac{1}{x} dx + \int e^x \frac{1}{x} dx + C$

$\boxed{\sin y = e^x \log x + C}$

41 (a)

$x - y + 2z = 2,\ 2x + y + 4z = 7,\ 4x - y + z = 4.$

$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix} = -12$

$\Delta x = \begin{vmatrix} 2 & -1 & 2 \\ 7 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix} = -15$

$\Delta y = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 7 & 4 \\ 4 & 4 & 1 \end{vmatrix} = -33$

$\Delta z = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 7 \\ 4 & -1 & 4 \end{vmatrix} = -21$

$x = \frac{\Delta x}{\Delta} = 5/4,$

$y = \frac{\Delta y}{\Delta} = \frac{11}{4}$

$z = \frac{\Delta z}{\Delta} = 7/4$

(b) $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$

$\Rightarrow \int_0^1 \frac{1}{\sqrt{1-x^2}} dx + \int_0^1 \frac{-x}{\sqrt{1-x^2}} dx$

$= (\sin^{-1}x)_0^1 + \frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}} dx$

$\Rightarrow \sin^{-1}(1) - \sin^{-1}(0) + \frac{1}{2}\int (1-x^2)^{-1/2}(-2x)\, dx$

$\Rightarrow \pi/2 - 0 + \frac{1}{2}\left(\frac{(1-x^2)^{1/2}}{1/2}\right]_0^1$

$= \pi/2 + (0-1) = \pi/2 - 1$

Hence proved.

42 (a) $f(x) = 4x^6 - 6x^4$

$f'(x) = 24x^5 - 24x^3$

$= 24x^3(x^2-1)$

$= 24x^3(x+1)(x-1)$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$f'(x)$	$-$	$+$	$-$	$+$
Monotonicity	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

$f'(x) = 0$, $x = -1, 0, 1$,

$f''(x) = 120x^4 - 72x^2 = 24x^2(5x^2-3)$

$f''(-1) = 48$, $f''(0) = 0$, $f''(1) = 48$.

Local minimum $x=-1$, local Maximum $x=1$.

(b) $z = x+iy$

$|z+i| = |z-1|$

$|x+iy+i|^2 = |x+iy-1|^2$

$x^2 + (y+1)^2 = (y-1)^2 + y^2$

$2y = -2x$

$\boxed{x+y=0}$

(b) $(1+x+xy^2)\frac{dy}{dx} + (y+y^3) = 0.$

$\frac{dy}{dx} = \frac{-(y+y^3)}{1+x(1+y^2)}$

$\frac{dx}{dy} = \frac{-(1+x(1+y^2))}{y(1+y^2)}$

$\frac{dx}{dy} + \frac{1}{y}x = -\frac{1}{y(1+y^2)}$

$x e^{\int P dy} = \int Q e^{\int P dy} dy + c$

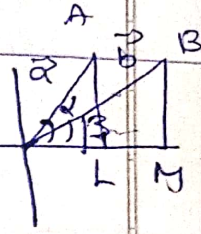
$xy = \int \frac{-1}{y(1+y^2)} y \cdot dy + c$

$xy = -\tan^{-1} y + c$

(or)

$\boxed{xy + \tan^{-1} y = c}$

43(a)



$\vec{a} = \vec{OA} = \vec{OL} + \vec{LA}$

$\Rightarrow \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

$\hat{a}\cdot\hat{b} = |\hat{a}||\hat{b}|\cos(\alpha-\beta) = \cos(\alpha-\beta)$ → (1)

$\hat{a}\cdot\hat{b} = (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j})\cdot(\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$

$\hat{a}\cdot\hat{b} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ → (2)

(1) = (2)

44(a)

(i) $\int_{200}^{600} k\,dx = 1$

$k(600-200) = 1$

$\boxed{k = 1/400}$

(ii) $F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$

$x < 200$,

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = 0$

$200 \le x \le 600$

$F(x) = \int_{-\infty}^{200} 0\,dx + \int_{200}^{x} \frac{1}{400}dx$

$= \frac{x}{400} - 1/2$ 600

$x > 600$, $F(x) = \int_{-\infty}^{200} 0\,dx + \int_{200}^{600} \frac{1}{400}dx$

$\Rightarrow \int_{200}^{600} \frac{1}{400}dx = 1$

$$F(x) = \begin{cases} 0 & , x < 200 \\ \frac{x}{400} - \frac{1}{2} & , 200 \le x \le 600 \\ 1 & , x > 600. \end{cases}$$

(ii) $P(300 \le x \le 500) = F(500) - F(300) = \left(\frac{500}{400} - \frac{1}{2}\right) - \left(\frac{300}{400} - \frac{1}{2}\right)$

$= 1/2$

(b) $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{v} = 2\hat{i} + 6\hat{j} + 6\hat{k}$

$\vec{r} = (1-s)(2\hat{i} + 2\hat{j} + \hat{k}) + s(9\hat{i} + 3\hat{j} + 6\hat{k}) + t(2\hat{i} + 6\hat{j} + 6\hat{k})$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0, \quad \begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$-24(x-2) - 32(y-2) + 40(z-1) = 0$

$\boxed{3x + 4y - 5z - 9 = 0}$

45(a) $= 3x - 2y + 6 = 0$, $x = -3$, $x = 1$.

$$\text{Area} = \int_{-3}^{-2} (-y)\,dx + \int_{-2}^{1} y\,dx \Rightarrow -\int_{-3}^{-2} \frac{3x+6}{2}\,dx + \int_{-2}^{1} \frac{3x+6}{2}\,dx$$

$$\Rightarrow -\frac{1}{2}\left\{\frac{3x^2}{2} + 6x\right\}_{-3}^{-2} + \frac{1}{2}\left\{\frac{3x^2}{2} + 6x\right\}_{-2}^{1}$$

$$= -\frac{1}{2}\left\{(6-12) - \left(\frac{27}{2} - 18\right)\right\} + \frac{1}{2}\left\{\left(\frac{3}{2} + 6\right) - (6 - 12)\right\} = 15/2$$

(b) $x^2 = -4ay$

$A(-6, -4)$

$36 = -4a(-4)$

$4a = 9$.

$\boxed{x^2 = -9y}$

$2x = -9\frac{dy}{dx}$

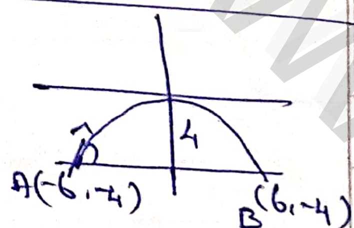
$\boxed{\frac{dy}{dx} = \frac{-2x}{9}}$

$\frac{dy}{dx} = \frac{+4}{3}$ at $(-6, -4)$

$\frac{dy}{dx} = \tan\theta$,

$\tan\theta = 4/3$

$\theta = \tan^{-1}(4/3)$



46(a) Let $\cos^{-1}x = \alpha$, $\cos^{-1}y = \beta$.

$x = \cos\alpha$, $y = \cos\beta$.

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\alpha + \beta = \pi - \cos^{-1}z$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$\cos(\pi - \cos^{-1} z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-\cos^{-1}(\cos^{-1}(z)) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-z = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-xy - z = -\sqrt{1-x^2}\sqrt{1-y^2}$

$\boxed{x^2+y^2+z^2+2xyz = 1}$

(b) $y = ax^2+bx+c$, $(-6,8)$ $(-2,-12)$ and $(3,8)$

$36a - 6b + c = 8$

$4a - 2b + c = -12$

$9a + 3b + c = 8$

$$[A\ B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right] \sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 36 & -6 & 1 & 8 \\ 9 & 3 & 1 & 8 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 12 & -8 & 116 \\ 0 & 30 & -5 & 140 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 9R_1 \\ R_3 \to 4R_3 - 9R_1 \end{array} \sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & -1 & 28 \end{bmatrix} \begin{array}{l} R_2 \to R_2/4 \\ R_3 \to \frac{R_3}{5} \end{array}$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 0 & 3 & -30 \end{bmatrix} R_3 \to R_3 - 2R_2$$

$\boxed{c = -10}$, $2b - 2c = 29$, $4a + c - 2b = -12$

$\boxed{b = 3}$ $\boxed{a = 1}$

$\boxed{y = x^2 + 3x - 10}$

47(a) $x^2 + 4y^2 = 8$.

$2x + 8y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{4y}$

$\boxed{m_1 = -\frac{a}{4b}}$

$x^2 - 2y^2 = 4$,

$2x - 4y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{2y}$

$\boxed{m_2 = \frac{a}{2b}}$

$m_1 \times m_2 = \left(\frac{-a}{4b}\right)\left(\frac{a}{2b}\right) = \frac{-a^2}{8b^2}$

$\frac{a^2}{-16-16} = \frac{b^2}{-8+4} = \frac{1}{-2-4}$

$\frac{a^2}{b^2} = \frac{32}{4} = 8$.

$\boxed{m_1 \times m_2 = -1}$

Hence curve is orthogonally

(b) $18x^2+12y^2-144x+48y+120=0$

$18(x^2-8x)+12(y^2+4y)+120=0$

$18(x-4)^2+12(y+2)^2 = 288+48-120$

$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$

$\boxed{\begin{aligned} X &= x-4 \\ Y &= y+2 \end{aligned}}$ $\frac{X^2}{12}+\frac{Y^2}{18}=1$

$a^2=18,\ b^2=12.$

$e=\sqrt{1-12/18} = 1/\sqrt{3}$

$ae = 3\sqrt{2}\times\frac{1}{\sqrt{3}} = \sqrt{6}$

$\frac{a}{e} = 3\sqrt{6}.$

Center	$(0,0)$	$(4,-2)$
vertices	$(0,\pm a)$ $(0,\pm 3\sqrt{2})$	$A(4,-2+3\sqrt{2})$ $A'(4,-2-3\sqrt{2})$
Foci,	$(0,\pm ae)$ $(0,\pm\sqrt{6})$	$S(4,-2+\sqrt{6})$, $S'(4,-2-\sqrt{6})$
Directrix, $\cancel{Y=\pm\frac{a}{e}}$ $\cancel{Y=\pm 3\sqrt{6}}$	$Y=\pm\frac{a}{e}$ $Y=\pm 3\sqrt{6}$	$Y=\pm 3\sqrt{6}$ $y=-2\pm 3\sqrt{6}$

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