Common Second Revision Test - 2025

Standard 12

MATHEMATICS

Time: 3.00 Hours Marks: 90

PART - I

Choose the correct answer **20×1=20**

1) If $|adj\,(adj\,A)| = |A|^9$, then the order of the square matrix A is
   a) 3 b) 4 c) 2 d) 5
2) If α and β are the roots of $x^2+x+1=0$ then $\alpha^{2020}+\beta^{2020}$ is
   a) −2 b) −1 c) 1 d) 2
3) The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^{n}C_r(-1)^r x^r$ is
   a) 0 b) n c) < n d) r
4) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) + \sec^{-1}\left(\frac{5}{3}\right) - \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) =$
   a) 2π b) π c) 0 d) $\tan^{-1}\frac{12}{65}$
5) An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
   a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{\sqrt{3}}$
6) If the planes $\vec{r}.(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r}.(4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are
   a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$
7) The maximum value of the function x^2e^{-2x}, $x>0$ is
   a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$
8) The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangent is −0.25?
   a) −8 b) −4 c) −2 d) 0
9) If $v(x, y) = \log(e^x+e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} =$
   a) $e^x + e^y$ b) $\frac{1}{e^x + e^y}$ c) 2 d) 1
10) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is
   a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{2}{3}$
11) The value of $\int_0^1 x(1-x)^{99}\,dx$ is
   a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$
12) The population P in any year t is such that the rate of increase in the population is proportional to the population. Then
   a) $P = Ce^{kt}$ b) $p = Ce^{-kt}$ c) $P = Ckt$ d) $P = C$

13) The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

a) straight lines b) circles c) parabola d) ellipse

14) If the function $f(x) = \frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable x, then which of the following cannot be the value of a and b?

a) 0 and 12 b) 5 and 17 c) 7 and 19 d) 16 and 24

15) Which one of the following statements has much value F?

a) Chennai is in India or $\sqrt{2}$ is an integer

b) Chennai is in India or $\sqrt{2}$ is an irrational number

c) Chennai is in China or $\sqrt{2}$ is an integer

d) Chennai is in China or $\sqrt{2}$ is an irrational number

16) If A is 3 × 3 matrix such that $|5 \text{ adj } A|=5$ then $|A| =$

a) $\pm \frac{1}{5}$ b) ± 5 c) ± 1 d) $\pm \frac{1}{25}$

17) Sum of the roots of the equation $4^x-3(2^{x+3})+2^7=0$ is

a) 4 b) 5 c) 6 d) 7

18) If $z = \left(2+i\sqrt{3}\right)^{10} + \left(2-i\sqrt{3}\right)^{10}$ then

a) $Re(z)=0$ b) $Im(z)=0$

c) $Re(z)>0, Im(z)\angle 0$ d) $Re(z)<0, Im(z)>0$

19) The area of region enclosed by $y=x^2$ and $y = \sqrt{x}$ is

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{8}{3}$ d) $\frac{16}{3}$

20) A die is thrown 5 times, Getting an odd number is consider a success. The variance of the distribution is

a) $\frac{3}{4}$ b) $\frac{1}{4}$ c) $\frac{5}{4}$ d) $\frac{7}{4}$

PART - II

Answer any 7 questions. Q.No. 30 is compulsory. **7×2=14**

21) Find the inverse of $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

22) If $z_1 = 3-2i$ and $z_2=6+4i$, find $\frac{z_1}{z_2}$ in the rectangular form

23) If α, β and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\Sigma \frac{1}{\beta\gamma}$

24) Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

25) Prove that the length of the latus rectum of the parabola $y^2=4ax$ is 4a

26) If $2\hat{i} - \hat{j} + 3\hat{k}, 3\hat{i} + 2\hat{j} + k, \hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m

27) The probability density function of x is given by $f(x) = \begin{cases} k \times e^{-2x}, & x > 0 \\ 0 & x \le 0 \end{cases}$, find the value of k

28) Show that $q \to p \equiv \neg p \to \neg q$

29) Evaluate: $\lim\limits_{x \to 0} x \log x$

30) Evaluate: $\int_0^{\pi} \sin^4 x \, dx$

PART - III

Answer any 7 questions. Q.No.40 is compulsory. **7×3=21**

31) Solve by using Cramer's rule: $5x-2y = -16, x + 3y = 7$

32) If $z = x + iy$ is a complex number such that $\frac{z-4i}{z+4i} = 1$, show that the locus of z is real axis

33) Find the domain of $\sin^{-1}(2-3x^2)$

34) Find the distance between the parallel planes $x+2y-2z + 1 = 0$ and $2x+4y-4z+5 = 0$

35) Show that the value of mean value theorem for $f(x) = \frac{1}{x}$ on a closed interval of positive numbers [a, b] is $\sqrt{ab}$

36) If $u(x, y) = x^2y+3xy^4$, $x=e^4$, $y=\sin t$, find $\frac{du}{dt}$ and evaluate it at $t = 0$

37) Evaluate : $\int_{-5}^{5} x \cos\left[\frac{e^x - 1}{e^x + 1}\right] dx$

38) Show that $y=a \cos[\log x]+b \sin[\log x]$, $x > 0$ is a solution of the differential equation $x^2y^{11}+xy^1+y=0$.

39) Verify (i) closure property (ii) commutative property (iii) associative property on $m*n=m+n-mn$, $m, n \in z$.

40) The probability mass function of a random variable is defined as

x	−2	−1	0	1	2
f(x)	k	2k	3k	4k	5k

Prove that $E(x) = \frac{2}{3}$

PART - IV

Answer all the questions. **7×5=35**

41) a) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T.A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(OR)

b) Find the area of the region in the first quadrant bounded by the parabola $y^2=4x$, the line $x+y=3$ and y-axis

42) a) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary

(OR)

b) Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C.
Find (i) The temperature of water after 20 minutes. (ii) The time when the temperature is 40°C. $\left[\log_e \frac{11}{15} = -0.3101, \log_e 5 = 1.6094\right]$

43) a) Determine K and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots

(OR)

b) Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$, for $x>0$

44) a) Prove that by vector method, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(OR)

b) A tunnel through a mountain for a fourlane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

45) a) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at the rate of 5 m/s. When the base of the ladder is 8 metres from the wall

i) how fast is the top of the ladder moving down the wall?

ii) At what rate, the area of the triangle formed by the ladder, wall and the floor, is changing?

(OR)

b) On the average, 20% of the products manufactured by ABC company are found to be defective. If we select 6 of these products at random and x denotes the number of defective products find the probability that (i) two products are defective (ii) atleast two products are defective.

46) a) Evaluate: $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$

(OR)

b) Let A be Q1{1}. Define * on A by x*y=x+y−xy. It * binary on A? If so examine the existence of identity, existence of inverse properties for the operation * on A.

47) a) Solve: $\frac{dy}{dx}=\frac{x+1}{2-y},(y\neq 2)$

(OR)

b) Find the non-parametric vector equation and cartesian equation of the plane passing through the points (2, 5, −3), (−2, −3, −5) and (5, 3, −3)

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1) b) 4
2) b) −1
3) b) n
4) c) 0
5) a) $1/\sqrt{2}$
6) c) $-1/2, -2$
7) c) $1/e^2$
8) b) −4
9) d) 1
10) d) 2/3
11) b) $\frac{1}{10100}$
12) a) $P = Ce^{kt}$
13) c) Parabola
14) d) 16 and 24
15) c) Chennai is in China...
16) a) $\pm 1/5$
17) d) 7
18) b) $Im(z) = 0$
19) b) 1/3
20) c) 5/4.

21) $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{-7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

22) $\frac{z_1}{z_2} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i} = \frac{5-12i}{26}$

23) $S_1 = -p,\ S_2 = q,\ S_3 = -r$

$\Sigma \frac{1}{\beta\gamma} = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = p/r$

24) $\tan^{-1} 1/2 + \tan^{-1} 1/3 = \pi/4$

LHS $\tan^{-1}\left[\frac{1/2 + 1/3}{1 - 1/2(1/3)}\right] = \tan^{-1}(1) = \pi/4 = RHS$

25) $y^2 = 4ax$; $x = a \Rightarrow L.R$

$y^2 = 4a \cdot a \Rightarrow y^2 = 4a^2$

$y = \pm 2a$

Pt of intersection $(a, 2a)(a, -2a)$

$D = \sqrt{(a-a)^2 + (2a+2a)^2} = 4a$

26) $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0 \Rightarrow m = -3$

27) $\int_0^{\infty} kxe^{-2x} = 1$

$k\frac{1!}{2^2} = 1$

$\boxed{k = 4}$

28) $q \to p \equiv \neg p \to \neg q$.
Correct proof.

29) $\lim_{x \to 0} \frac{\log x}{1/x}$

$\lim_{x \to 0} \frac{1/x}{-1/x^2} = \frac{1}{x} \times \frac{-x^2}{1} = -x = 0$

30) $\int_0^{\pi} \sin^4 x\,dx = 2\int_0^{\pi/2} \sin^4 x\,dx$

$= \cancel{2}\frac{(4-1)\times(4-3)}{4-2} \times \frac{\pi}{\cancel{2}}$.

$= \frac{3 \times 1}{2} \times \pi = \frac{3\pi}{2}$.

3-Marks

31) $5x - 2y = -16$
$x + 3y = 7$
$\Delta = 17,\ \Delta x = -34,\ \Delta y = 51$
$x = -2,\ y = 3$

32) $\left|\frac{z-4i}{z+4i}\right| = 1$

$\left|\frac{x+iy-4i}{x+iy+4i}\right| = 1 \Rightarrow \left|\frac{x+i(y-4)}{x+i(y+4)}\right| = 1$

$|x + i(y-4)| = |x + i(y+4)|$
$y = 0$
locus of z is real.

33) $-1 \le 2 - 3x^2 \le 1$

$-1-2 \le -3x^2 \le 1-2$

$1 \le 3x^2 \le 3$

$\frac{1}{3} \le x^2 \le 1 \Rightarrow \pm\frac{1}{\sqrt{3}} \le x \le \pm 1$

$|x| \ge \frac{1}{\sqrt{3}},\ |x| \le 1$

$x \in \left[-1, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right]$

34) $d = \left|\frac{d_1 - d_2}{\sqrt{a^2+b^2+c^2}}\right| = \frac{2-5}{\sqrt{2^2+4^2+(-4)^2}}$

$d = 1/2$

35) $f(x) = 1/x$

$f'(x) = -1/x^2 \Rightarrow f'(c) = -1/c^2$

$-1/c^2 = \dfrac{\frac{1}{b} - \frac{1}{a}}{b-a}$

$c = \sqrt{ab} \Rightarrow$ Hence prove

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36) $u(x,y) = x^2y + 3xy^4$

$x = e^{4t}$, $y = \sin t$

Question wrong.

∴ mear attempt.

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37) $\int_{-5}^{5} x\cos\left(\dfrac{e^x-1}{e^x+1}\right)dx = f(x)$

$f(-x) = (-x)\cos\left(\dfrac{e^{-x}-1}{e^{-x}+1}\right)$

$= -x\cos\left(\dfrac{1/x - 1}{1/x + 1}\right) = -f(x)$

∴ This is odd function.

∴ Ans = 0

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38) $y = a\cos(\log x) + b\sin(\log x)$ — ①

$y' = a\left(-\sin\log x\right)\cdot\frac{1}{x} + b\cos(\log x)\frac{1}{x}$

$xy' = -a\sin(\log x) + b\cos(\log x)$ — ②

$y'' = -a\left[\frac{1}{x}\cos(\log x)\frac{1}{x} + \sin(\log x)(-1/x^2)\right] +$

$b\left[\frac{1}{x}\left(-\sin\log x\right)\frac{1}{x} + \cos(\log x)(-1/x^2)\right]$

$x^2y'' = -a\cos(\log x) + a\sin(\log x) -$
$b\sin(\log x) - b\cos(\log x)$ — ③

from ①, ② and ③

$x^2y'' + xy' + y = 0$.

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39) (i) closure is true

(ii) $m * n = n * m \Rightarrow$ commutative true

(iii) $(m * n) * x = m * (n * x)$

$m + n + x - mn - mx - nx + mnx$

associative also true

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40) $15k = 1 \Rightarrow k = 1/15$

$x f(x) = E(x)$

$= -\frac{2}{15} - \frac{2}{15} + 0 + \frac{4}{15} + \frac{10}{15}$

$= -\frac{4}{15} + \frac{14}{15} = \frac{10}{15} = 2/3$

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5 – Marks.

41) a) $A^TA^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\cos^2 x$

$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

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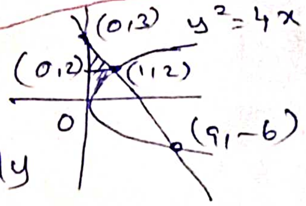
b)



$A = \int_0^2 x\,dy + \int_2^3 x\,dy$

$= \int_0^2 \frac{y^2}{4}dy + \int_2^3 (3-y)dy = 7/6.$

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42) a) $z = \left(\dfrac{19+9i}{5-3i}\right)^{15} - \left(\dfrac{8+i}{1+2i}\right)^{15}$

$\bar{z} = -z$

∴ z is Purely imaginary.

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b) $\dfrac{dT}{dt} \propto (T - 25°)$

$T = 25 + ce^{kt}$ — ①

$t = 0, T = 100°$

① ⇒ $c = 75° \Rightarrow T = 25 + 75e^{kt}$ — ②

$t = 10, T = 80°c$

$80 = 25 + 75e^{10k} \Rightarrow e^{10k} = \frac{11}{15}$

$k = \frac{1}{10}\log(11/15)$

(i) when $t = 20$ ② ⇒

$T = 25 + 75e^{20k} = 25 + 75(e^{10k})^2$

$T = 65.33°c$

(ii) when $T = 40.$

$40 = 25 + 75e^{kt} \Rightarrow e^{kt} = 1/5$

$kt = \log(1/5)$

$t = 51.899$

$= 51.9$ min

43) a) $2x^3 - 6x^2 + 3x + k = 0$

$S_1 = 3,\ S_2 = 3/2,\ S_3 = -k/2$

$\alpha + \beta = 1,\ \beta\gamma = -k/4$

$\alpha\beta + \beta\gamma + \alpha\gamma = 3/2$

$\boxed{k = 2}$

$\therefore 2x^3 - 6x^2 + 3x + 2 = 0$

$x = \frac{1 \pm \sqrt{3}}{2}$

b) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = 1/2 \tan^{-1} x$

$\tan^{-1}(1) - \tan^{-1} x = 1/2 \tan^{-1} x$

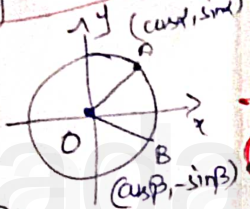
$\tan^{-1}(1) = 1/2 \tan^{-1} x + \tan^{-1} x$

$\pi/4 = 3/2 \tan^{-1} x$

$\tan^{-1} x = \pi/6$

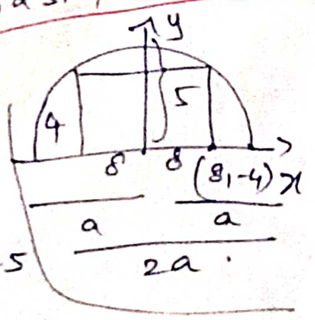
$\boxed{x = 1/\sqrt{3}}$

44) a)



$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

b) $\frac{x^2}{a^2} + \frac{y^2}{5^2} = 1$



at $(8,-4)$ and $b = 5$

$\frac{8^2}{a^2} + \frac{(-4)^2}{25} = 1$

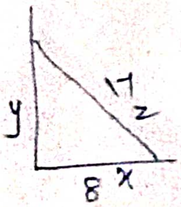
$a^2 = \frac{64 \times 25}{9} \Rightarrow \boxed{\frac{40}{3} = a}$

$\boxed{a = 26.66m}$

45) a) $x = 8m$

$\frac{dx}{dt} = 5 m/s$

$y = ?,\ dy/dt = ?$



$8^2 + y^2 = 17^2$

$y^2 = 225$

$\boxed{y = 15}$

(i) $x^2 + y^2 = 17^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$8(5) + (15)\,dy/dt = 0$

$dy/dt = -\frac{40}{15} = -\frac{8}{3}$ m/sec

(ii) $A = \frac{1}{2}xy = 1/2\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$

$= \frac{1}{2}\left[8\left(-\frac{8}{3}\right) + 15(5)\right]$

$= 26.83\ m^2/sec$

b) $n = 6,\ p = \frac{20}{100} = 0.2,\ q = 0.8$

(i) $P(X=2) = 6C_2 (0.2)^2 (0.8)^4$

$= 15 \times 1.63 \simeq 19.45$

(ii) $P(X \geq 2) = 1 - P[X < 2]$

$= 1 - P(X=0) + P(X=1)$

$= 1 - 2(0.8)^5 = 1 - 2\left(\frac{4}{5}\right)^5$

46) a) $I_1 = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$

$I_2 = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}}\,dx$

$= \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+1/a^x}\,dx = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{a^x+1}$

$I_1 + I_2 = \int_{-\pi}^{\pi} \frac{\cos^2 x + a^x\cos^2 x}{1+a^x}\,dx$

$2I = 2\int_0^{\pi} \cos^2 x\,dx = 4\int_0^{\pi/2} \cos^2 x\,dx$

$2I = 4 \times \frac{1}{2} \times \frac{\pi}{2} = \pi$

$\boxed{I = \pi/2}$

b) binary $x * y = y * x$

Identity

$a * e = a$

$a + e + ae = a$

$\boxed{e = 0}$

$a * a^{-1} = 0$

$a + a^{-1} + a^{-1}a = 0$

$a^{-1}(1+a) = -a$

$\boxed{a^{-1} = \frac{-a}{1+a}}$

47) a) $\frac{dy}{dx} = \frac{x+1}{2-y}$

$$\int (2-y)\,dy = \int (x+1)\,dx.$$

$$2y - \frac{y^2}{2} = \frac{x^2}{2} + x + c.$$

$$\frac{4y - y^2}{2} = \frac{x^2+2x}{2} + C$$

$$x^2 + y^2 + 2x - 4y + c = 0$$

b) VE

$$\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}.$$

cartesian form.

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & -2 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$2x + 3y - 16z - 67 = 0$$

Non-Parametric form

$$\vec{r}\cdot(2\hat{i} + 3\hat{j} - 16\hat{k}) = 67$$