SECOND REVISION TEST - 2025

Standard XII Reg.No.

MATHEMATICS

Time : 3.00 hrs Part - I *Marks : 90*

I. Choose the correct answer: 20 x 1 = 20

1. If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj } AB| =$

   a) –40 b) –80 c) –60 d) –20

2. If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then the value of a_{23} is

   a) 0 b) –2 c) –3 d) –1

3. The conjugate of a complex number is $\frac{1}{i-2}$. Then the complex number is

   a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

4. z_1, z_2, z_3 are complex numbers such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^2$ is

   a) 3 b) 2 c) 1 d) 0

5. If α, β and γ are the zeros of x^3+px^2+qx+r, then $\Sigma\frac{1}{\alpha}$ is

   a) $-q/r$ b) $-p/r$ c) q/r d) $-p/q$

6. $\sin^{-1}\left(\tan\frac{\pi}{4}\right)-\sin^{-1}\left(\sqrt{\frac{3}{x}}\right)=\frac{\pi}{6}$, then x is a root of the equation

   a) $x^2-x-6=0$ b) $x^2-x-12=0$
   c) $x^2+x-12=0$ d) $x^2+x-6=0$

7. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

   a) $15<m<65$ b) $35<m<85$ c) $-85<m<-35$ d) $-35<m<15$

8. If P(x, y) be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is

   a) 8 b) 6 c) 10 d) 12

9. If $\left[\vec{a}, \vec{b}, \vec{c}\right]=1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}}+\frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}}+\frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

   a) 1 b) –1 c) 2 d) 3

10. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}$, $z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

    a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

11. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is
a) (4,11) b) (4,–11) c) (–4,11) d) (–4,–11)
12. The maximum value of the function x^2e^{-2x}, $x > 0$ is
a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{1}{e^4}$
13. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
a) $0.3x\,dx\,m^3$ b) $0.03x\,m^3$ c) $0.03x^2\,m^3$ d) $0.03x^3\,m^3$
14. For any value of $n \in Z$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]dx$ is
a) $\frac{\pi}{2}$ b) π c) 0 d) 2
15. If $f(x) = \int_1^x \frac{e^{\sin u}}{u} du$, $x > 1$ and $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of a is
a) 3 b) 6 c) 9 d) 5
16. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ is
a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1
17. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
a) $y + \sin^{-1}x = c$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = c$ d) $x^2 + 2\sin^{-1}y = 0$
18. A pair of dice numbered 1,2,3,4,5,6 of a six-sided die and 1,2,3,4 of a four-sided die is roed and the sum is determined. Let the random variable X denoted this sum. Then the number of elements in the inverse image of 7 is
a) 1 b) 2 c) 3 d) 4
19. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is
a) 1 b) 2 c) 3 d) 4
20. Which one of the following is incorrect? For any two propositions p and q, we have
a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$
c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ d) $\neg(\neg p) \equiv p$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If z = (2 + 3i) (1 – i), then prove that $z^{-1} = \frac{5}{26} - i\frac{1}{26}$
22. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

23. Find the principal value of $\tan^{-1}(\sqrt{3})$
24. If y = 4x + c is a tangent to the circle $x^2 + y^2 = 9$, find c
25. Show that the three vectors $2\hat{i}+3\hat{j}+\hat{k}$, $\hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.
26. Find df for $f(x) = x^2 + 3x$ and evaluate it for x = 2 and dx = 0.1
27. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal
28. Show that the solution of $\frac{dy}{dex} = \sqrt{\frac{1-y^2}{1-x^2}}$ is $\sin^{-1}y = \sin^{-1}x + C$ or $\sin^{-1}x = \sin^{-1}y + C$
29. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line y = x
30. X is the number of tails occurred when three fair coins are tossed simultaneously. Find the values of the random variable X and number of points in its reverse images.

Part - III

III. **Answer any 7 questions. (Q.No.40 is compulsory)** **7 x 3 = 21**

31. Show that the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ is 3
32. Which one of the points 10 – 8i, 11 + 6i is closest to 1 + i
33. If the sides of a cubic box are increased by 1, 2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Show that the volume of the cuboid is 60 cubic units.
34. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?
35. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
36. An egg of a particular bird is spherical in shape. If the radius to the inside of the shell is 4 mm and radius to the outside of the shell is 4.2 mm, prove that the approximate volume of the shell is 12.8π mm^3.
37. Show that $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x} dx = \tan^{-1}(2) - \frac{\pi}{4}$
38. Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ₹30 for each black ball selected and the loss ₹20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.
39. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.
40. Solve : $x \cos y\, dy = e^x(x \log x + 1)\, dx$

Part - IV

IV. Answer all the questions. $7 \times 5 = 35$

41. a) Solve the system of equations $x - y + 2z = 2$, $2x + y + 4z = 7$, $4x - y + z = 4$ by Cramer's rule.

(OR)

b) Show that $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx = \frac{\pi}{2} - 1$

42. a) Prove that the local minimum values for the function $f(x) = 4x^6 - 6x^4$ attain at -1 and 1

(OR)

b) Show that the locus of $z = x + iy$ if $|z + i| = |z - 1|$ is $x + y = 0$

43. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

(OR)

b) Solve : $\left(1 + x + xy^2\right)\frac{dy}{dx} + \left(y + y^3\right) = 0$

44. a) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function of random variable X is $f(x) = \begin{cases} 0 & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres.

(OR)

b) Find the vector equation (any form) or Cartesian equation of a plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

45. a) Show that the area of the region bounded by $3x - 2y = 0$, $x = -3$ and $x = 1$ is $\frac{15}{2}$

(OR)

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Show that the angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$

46. a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x,y,z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

(OR)

b) A boy is walking along the path $y = ax^2 + bx + c$ through the points (–6, 8), (–2,–12) and (3, 8). He wants to meet his friend at P(7, 60). Will he meet his friend? (Use Gaussian Elimination method)

47. a) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

(OR)

b) Identify the type of conic and find centre, foci and vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

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XII - Maths 2nd Revision - chengalpattu dist.

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I. 1. (b) -80

2. (d) -1

3. (b) $\frac{-1}{i+2}$

4. (d) 0

5. (a) $\frac{-q}{r}$

6. (b) $x^2-x-12=0$

7. (d) $-35<m<15$

8. (c) 10

9. (a) 1

10. (d) $\pi/2$

11. (a) $(4,11)$

12. (c) $1/e^2$

13. (d) $0.03x^3m^3$

14. (c) 0

15. (c) 9

16. (c) $(1,1)$

17. (a) $y+\sin^{-1}x=c$

18. (d) 4

19. (a) 1

20. (c) $\neg(p\vee q)\equiv\neg p\vee\neg q$

II. 21. $z=(2+3i)(1-i)$

$z=5+i$

$z^{-1}=\frac{1}{5+i}\times\frac{5-i}{5-i}$

$z^{-1}=\frac{5-i}{26}$

$\boxed{z^{-1}=\frac{5}{26}-i\frac{1}{26}}$

22. Root$_1$ $=2-\sqrt{3}$.

Root$_2$ $=2+\sqrt{3}$

S.R $=4$

P.R $=1$

$\boxed{x^2-4x+1=0}$

23. $\tan^{-1}(\sqrt{3})$

$y=\tan^{-1}(\sqrt{3})$

$\tan y=\sqrt{3}$

$y=\pi/3 \in[-\pi/2,\pi/2]$

24. $y=mx+c,\quad x^2+y^2=a^2,$

$c^2=a^2(1+m^2).$

$c=\pm\sqrt{9(1+16)}=\pm3\sqrt{17}$

$\boxed{c=\pm3\sqrt{17}}$

25. $\vec{a}=2\hat{i}+3\hat{j}+\hat{k},\ \vec{b}=\hat{i}-2\hat{j}+2\hat{k}$

$\vec{c}=3\hat{i}+\hat{j}+3\hat{k}.$

$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}2&3&1\\1&-2&2\\3&1&3\end{vmatrix}=-16+9+7$
$=0.$

$\therefore$ It is coplanar.

26. $f(x)=x^2+3x$

$f'(x)=2x+3.$

$df=f'(x)dx=(2x+3)dx$

$x=2,\ dx=0.1$

$df=[2(2)+3]0.1$

$=0.7$

27. $A=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$

$A^T=\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}$

$AA^T=\begin{bmatrix}\cos^2\theta+\sin^2\theta&\cos\theta\sin\theta-\sin\theta\cos\theta\\\sin\theta\cos\theta-\cos\theta\sin\theta&\cos^2\theta+\sin^2\theta\end{bmatrix}$

$AA^T=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I_2$

$AA^T=I_2.\ \therefore A$ is orthogonal

28) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1}y = \sin^{-1}x + c.$

29) $\frac{dy}{dx} = 3x^2 - 6x + 1 = 1$

$3x^2 - 6x = 0$

at $x=0$, $x=2$.

points $(0,-2), (2,-4)$

Tangent $y = x$. is parallel

30) $S = \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$n(x=0) = 1$

$n(x=1) = 3$

$n(x=2) = 3$

$n(x=3) = 1$

x	0	1	2	3
Random Variable	1	3	3	1

31) $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

$\rho(A) = \min(4 \times 3)$

$\rho(A) = 3.$

$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = -1(-32) + 2(-16)$

$= 16 + 0.$

$\boxed{\therefore \rho(A) = 3}$

32) $1+i$ to $10-8i = |10-8i-1-i|$

$= 9\sqrt{2} = \sqrt{162}$

Distance from, $1+i$ to $11+6i$

$= |11+6i-1-i| = 5\sqrt{5} = \sqrt{125}$

$11+6i$ is a closest to $1+i$.

33) $(x+1)(x+2)(x+3) - x^3 = 52$

$6x^2 + 11x + 6 = 52$

$6(x+2)(x+\frac{23}{6}) = 0$

$\boxed{x=2}$. $\boxed{\text{Volume} = 60}$

34) $\pi/2 < \cos^{-1}(3x-1) < \pi$

$-1 < 3x-1 < 0$

$0 < 3x < 1$

$0 < x < 1/3$ (or) $x \in (0, 1/3)$

35) $AS = 94.5 \times 10^6$ km.

$SA' = 152 \times 10^6$ km

$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

$2c = 57.5 \times 10^6 = 575 \times 10^5$ km

$\boxed{SS' = 575 \times 10^5 \text{ km}}$

36) $V(r) = \frac{4}{3}\pi r^3$

$V(5.3) - V(5) \simeq V'(r)\,dr$

$\Rightarrow 4\pi r^2 dr = 100\pi(0.3)$

$\Rightarrow 30\pi\,mm^3$

volume of shell $= 30\pi\,mm^3$

37) $I = \int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\,dx$

$u = \sec x$, $du = \sec x \tan x\,dx$

$x=0$, $u=1$, $x=\pi/3$, $u=2$

$I = \int_1^2 \frac{du}{1+u^2} = (\tan^{-1}(u))_1^2$

$= \tan^{-1}(2) - \pi/4$

38. X (Both Black Balls)
$\Rightarrow ₹2(30) \Rightarrow ₹60$
X(1B & 1W) = ₹30 − ₹20
= ₹10
X(Both W) = ₹2(−20)
= −₹40.

X	60	10	−40	Total
Image	6	24	15	45

39.

p	q	p→q	q→p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

From the table they are not equivalent.

40. $x\cos y\,dy = e^x(x\log x+1)dx$

$\cos y\,dy = e^x \frac{1}{x}(x\log x+1)dx$

$\Rightarrow e^x\left(\log x+\frac{1}{x}\right)dx$

$\cos y\,dy = e^x \log x\,dx + e^x \frac{1}{x}dx$

$\int \cos y\,dy = \int e^x \log x\,dx + \int e^x \frac{1}{x}dx$

$I_1 = e^x(\log x) - \int e^x \frac{1}{x}dx$

$\sin y = e^x \log x - \int e^x \frac{1}{x}dx + \int e^x \frac{1}{x}dx + C$

$\boxed{\sin y = e^x \log x + C}$

41(a)

$x-y+2z=2,\ 2x+y+4z=7,\ 4x-y+z=4.$

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix} = -12$$

$$\Delta_x = \begin{vmatrix} 2 & -1 & 2 \\ 7 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix} = -15$$

$$\Delta_y = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 7 & 4 \\ 4 & 4 & 1 \end{vmatrix} = -33$$

$$\Delta_z = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 7 \\ 4 & -1 & 4 \end{vmatrix} = -21$$

$x = \frac{\Delta_x}{\Delta} = 5/4,$

$y = \frac{\Delta_y}{\Delta} = \frac{11}{4}$

$z = \frac{\Delta_z}{\Delta} = 7/4$

(b) $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}}\,dx$

$\Rightarrow \int_0^1 \frac{1}{\sqrt{1-x^2}}dx + \int_0^1 \frac{-x}{\sqrt{1-x^2}}dx$

$= (\sin^{-1}x)_0^1 + \frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}}dx$

$\Rightarrow \sin^{-1}(1) - \sin^{-1}(0) + \frac{1}{2}\int_0^1 (1-x^2)^{-1/2}(-2x)\,dx$

$\Rightarrow \pi/2 - 0 + \frac{1}{2}\left[\frac{(1-x^2)^{1/2}}{1/2}\right]_0^1$

$= \pi/2 + (0-1) = \pi/2 - 1$

Hence proved.

42 (a) $f(x)=4x^6-6x^4$

$f'(x)=24x^5-24x^3$

$=24x^3(x^2-1)$

$=24x^3(x+1)(x-1)$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$f'(x)$	$-$	$+$	$-$	$+$
Monotonicity	↘	↗	↘	↗

$f'(x)=0,\ x=-1,0,1,$

$f''(x)=120x^4-72x^2=24x^2(5x^2-3)$

$f''(-1)=48,\ f''(0)=0,\ f''(1)=48.$

Local minimum $x=-1$, local Maximum $x=1$.

(b) $z=x+iy$

$|z+i|=|z-1|$

$|x+iy+i|^2=|x+iy-1|^2$

$x^2+(y+1)^2=(y-1)^2+y^2$

$2y=-2x$

$\boxed{x+y=0}$

(b) $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0.$

$\frac{dy}{dx}=\frac{-(y+y^3)}{1+x(1+y^2)}$

$\frac{dx}{dy}=\frac{-(1+x(1+y^2))}{y(1+y^2)}$

$\frac{dx}{dy}+\frac{1}{y}x=-\frac{1}{y(1+y^2)}$

$xe^{\int P\,dy}=\int Qe^{\int P\,dy}dy+c$

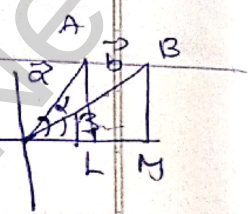
$xy=\int\frac{-1}{y(1+y^2)}y\cdot dy+c$

$xy=-\tan^{-1}y+c$

(or)

$\boxed{xy+\tan^{-1}y=c}$

43(a)



$\vec{a}=\vec{OA}=\vec{OL}+\vec{LA}$

$\Rightarrow\cos\alpha\hat{i}+\sin\alpha\hat{j}$

$\vec{b}=\cos\beta\hat{i}+\sin\beta\hat{j}$

$\hat{a}\cdot\hat{b}=|\hat{a}||\hat{b}|\cos(\alpha-\beta)=\cos(\alpha-\beta)$ — (1)

$\hat{a}\cdot\hat{b}=(\cos\alpha\hat{i}+\sin\alpha\hat{j})\cdot(\cos\beta\hat{i}+\sin\beta\hat{j})$

$\hat{a}\cdot\hat{b}=\cos\alpha\cos\beta+\sin\alpha\sin\beta.$ — (2)

(1) = (2)

44(a)

(i) $\int_{200}^{600}k\,dx=1$

$k(600-200)=1$

$\boxed{k=1/400}$

(ii) $F(x)=P(X\le x)=\int_{-\infty}^{x}f(u)du$

$x<200,$

$F(x)=P(X\le x)=\int_{-\infty}^{x}f(x)dx$

$=0$

$200\le x\le 600$

$F(x)=\int_{-\infty}^{200}0\,dx+\int_{200}^{x}\frac{1}{400}dx$

$=\frac{x}{400}-1/2$

$x>600,\ F(x)=\int_{-\infty}^{200}0\,dx+\int_{200}^{600}\frac{1}{400}dx$

$\Rightarrow\int_{200}^{600}\frac{1}{400}dx=1$

$$F(x) = \begin{cases} 0 & , x < 200 \\ \frac{x}{400} - \frac{1}{2} & , 200 \le x \le 600 \\ 1 & , x > 600. \end{cases}$$

(iii) $P(300 \le x \le 500) = F(500) - F(300) = \left(\frac{500}{400} - \frac{1}{2}\right) - \left(\frac{300}{400} - \frac{1}{2}\right)$
$= \frac{1}{2}$

(b) $\vec{\alpha} = 2\hat{i}+2\hat{j}+\hat{k}$, $\vec{\beta} = 9\hat{i}+3\hat{j}+6\hat{k}$, $\vec{v} = 2\hat{i}+6\hat{j}+6\hat{k}$

$\vec{r} = (1-s)(2\hat{i}+2\hat{j}+\hat{k}) + s(9\hat{i}+3\hat{j}+6\hat{k}) + t(2\hat{i}+6\hat{j}+6\hat{k})$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0, \quad \begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$-24(x-2) - 32(y-2) + 40(z-1) = 0$

$\boxed{3x+4y-5z-9=0}$

46(a) $\cancel{y} = 3x - 2y + 6 = 0$, $x = -3$, $x = 1$,

Area $= \int_{-3}^{-2} (-y)\,dx + \int_{-2}^{1} y\,dx \Rightarrow -\int_{-3}^{-2} \frac{3x+6}{2}\,dx + \int_{-2}^{1} \frac{3x+6}{2}\,dx$

$\Rightarrow -\frac{1}{2}\left\{\frac{3x^2}{2} + 6x\right\}_{-3}^{-2} + \frac{1}{2}\left\{\frac{3x^2}{2} + 6x\right\}_{-2}^{1}$

$= -\frac{1}{2}\left\{(6-12) - \left(\frac{27}{2} - 18\right)\right\} + \frac{1}{2}\left\{\left(\frac{3}{2}+6\right) - (6-12)\right\} = \frac{15}{2}$

(b) $x^2 = -4ay$

$A(-6,-4)$

$36 = -4a(-4)$

$4a = 9$.

$\boxed{x^2 = -9y}$

$2x = -9\frac{dy}{dx}$

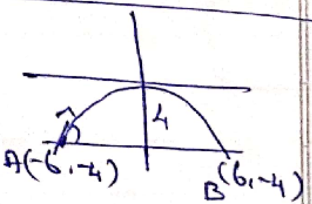
$\boxed{\frac{dy}{dx} = \frac{-2x}{9}}$

$\frac{dy}{dx} = +\frac{4}{3}$ at $(-6,-4)$

$\frac{dy}{dx} = \tan\theta$,

$\tan\theta = 4/3$

$\theta = \tan^{-1}(4/3)$



46(a) Let $\cos^{-1}x = \alpha$, $\cos^{-1}y = \beta$.

$x = \cos\alpha$, $y = \cos\beta$.

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\alpha + \beta = \pi - \cos^{-1}z$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$\cos(\pi - \cos^{-1} z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-\cos^{-1}(\cos^{-1}(z)) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-z = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-xy - z = -\sqrt{1-x^2}\sqrt{1-y^2}$

$\boxed{x^2+y^2+z^2+2xyz = 1}$

(b) $y = ax^2+bx+c$, $(-6,8)$ $(-2,-12)$ and $(3,8)$

$36a - 6b + c = 8$

$4a - 2b + c = -12$

$9a + 3b + c = 8$

$[A\ B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right] \sim \left[\begin{array}{cccc} 4 & -2 & 1 & -12 \\ 36 & -6 & 1 & 8 \\ 9 & 3 & 1 & 8 \end{array}\right] R_1 \leftrightarrow R_2$

$\sim \left[\begin{array}{cccc} 4 & -2 & 1 & -12 \\ 0 & 12 & -8 & 116 \\ 0 & 30 & -5 & 140 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 9R_1 \\ R_3 \to 4R_3 - 9R_1 \end{array} \sim \left[\begin{array}{cccc} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & -1 & 28 \end{array}\right] \begin{array}{l} R_2 \to R_2/4 \\ R_3 \to \frac{R_3}{5} \end{array}$

$\sim \left[\begin{array}{cccc} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 0 & 3 & -30 \end{array}\right] R_3 \to R_3 - 2R_2$

$\boxed{c = -10}$, $2b - 2c = 29$, $4a + c - 2b = -12$

$\boxed{b = 3}$ $\boxed{a = 1}$

$\boxed{y = x^2 + 3x - 10}$

47(a) $x^2 + 4y^2 = 8$.

$2x + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{4y}$

$\boxed{m_1 = -\frac{a}{4b}}$

$x^2 - 2y^2 = 4$,

$2x - 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{2y}$

$\boxed{m_2 = \frac{a}{2b}}$

$m_1 \times m_2 = \left(\frac{-a}{4b}\right)\left(\frac{a}{2b}\right) = \frac{-a^2}{8b^2}$

$\frac{a^2}{-16-16} = \frac{b^2}{-8+4} = \frac{1}{-2-4}$

$\frac{a^2}{b^2} = \frac{32}{4} = 8$.

$\boxed{m_1 \times m_2 = -1}$

Hence curve is orthogonally

(b) $18x^2+12y^2-144x+48y+120=0$

$18(x^2-8x)+12(y^2+4y)+120=0$

$18(x-4)^2+12(y+2)^2=288+48-120$

$$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$$

$$\frac{X^2}{12}+\frac{Y^2}{18}=1$$

$\boxed{X=x-4 \quad Y=y+2}$ $a^2=18,\ b^2=12$

$e=\sqrt{1-12/18}=1/\sqrt{3}$

$ae=3\sqrt{2}\times\frac{1}{\sqrt{3}}=\sqrt{6}$

$\frac{a}{e}=3\sqrt{6}$.

Center	$(0,0)$	$(4,-2)$
vertices	$(0,\pm a)$ $(0,\pm 3\sqrt{2})$	$A(4,-2+3\sqrt{2})$ $A'(4,-2-3\sqrt{2})$
Foci,	$(0,\pm ae)$ $(0,\pm\sqrt{6})$	$S(4,-2+\sqrt{6})$, $S'(4,-2-\sqrt{6})$
Directrix, ~~$Y=\pm\frac{a}{e}$~~ ~~$Y=\pm 3\sqrt{6}$~~	$Y=\pm\frac{a}{e}$ $Y=\pm 3\sqrt{6}$	$Y=\pm 3\sqrt{6}$ $y=-2\pm 3\sqrt{6}$

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