EXERCISE 12.1

5. (i) Define an operation $*$ on $\mathbb{Q}$ as follows: $a*b=\left(\frac{a+b}{2}\right); a,b\in\mathbb{Q}$. Examine the closure, commutative, associative properties satisfied by $*$ on $\mathbb{Q}$.

 (ii) Define an operation $*$ on $\mathbb{Q}$ as follows: $a*b=\left(\frac{a+b}{2}\right); a,b\in\mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.

9. Let $M=\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x\in R-\{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the closure, commutative, associative, existence of identity and inverse properties.

10. Let A be $Q-\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ a binary on A. If so, examine the closure, commutative, associative, the existence of identity and existence of inverse properties.

EXERCISE 12.2

7. Verify whether the following compound propositions are tautologies or contradictions or contingency

 (i) $(p\wedge q)\wedge\neg(p\vee q)$ (ii) $((p\vee q)\wedge\neg p)\rightarrow q$

 (iii) $(p\rightarrow q)\leftrightarrow(\neg p\rightarrow q)$ (iv) $((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$

8. Show that (i) $\neg(p\wedge q)\equiv\neg p\vee\neg q$ (ii) $\neg(p\rightarrow q)\equiv p\wedge\neg q$.

10. Show that $p\rightarrow q$ and $q\rightarrow p$ are not equivalent.

11. Show that $\neg(p\leftrightarrow q)\equiv p\leftrightarrow\neg q$.

12. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table.
13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.
14. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.
15. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

Example 12.2

Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}$.

Example 12.3

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation − on $\mathbb{Z}$.

Example 12.4

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}_e$ = the set of all even integers.

Example 12.5

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}_o$ = the set of all odd integers.

Example 12.6

Verify (i) closure property, (ii) commutative property, and (iii) associative property of the following operation on the given set.

$(a * b) = a^b; \forall a, b \in \mathbb{N}$ (exponentiation property).

Example 12.7

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$m * n = m + n - mn; m, n \in \mathbb{Z}$

Example 12.9

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using multiplication table corresponding to addition modulo 5.

Example 12.10

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Example 12.16

Construct the truth table for $(p \bar{\vee} q) \wedge (p \bar{\vee} \neg q)$.

Example 12.18

Establish the equivalence property connecting the bi-conditional with conditional:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Example 12.19

Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

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12. DISCRETE MATHEMATICS.

Exercise - 12.1.

5(i)

(i) closure :- ①m

clearly,

$\frac{a+b}{2} \in Q; \forall a,b \in Q$.

$\therefore *$ is binary on Q

(ii) Commutative :- ①m

$a*b = \frac{a+b}{2} = \frac{b+a}{2} = b*a$

$\therefore *$ is commutative.

(iii) Associative :- ①m

$a*(b*c) = a*\left(\frac{b+c}{2}\right)$

$= \frac{a+\left(\frac{b+c}{2}\right)}{2} = \frac{2a+b+c}{4}$

$(a*b)*c = \left(\frac{a+b}{2}\right)*c$

$= \frac{\left(\frac{a+b}{2}\right)+c}{2} = \frac{a+b+2c}{4}$

$\therefore a*(b*c) \neq (a*b)*c$

$*$ is not associative

5(ii)

(i) Identity :- ①m

Let e be the identity element.

$a*e = a$

$\frac{a+e}{2} = a$

$e = a$

It is not unique.

Identity doesn't exist.

(ii) Inverse :- ①m

Since identity doesn't exist.

$\therefore$ Inverse doesn't exist.

9.

(i) closure :- ①m

$\forall \begin{pmatrix} x & x \\ x & x \end{pmatrix}, \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in M$ $(x,y \neq 0)$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$ $(2xy \neq 0)$

$*$ is binary on M.

(ii) Commutative :- ①m

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$= \begin{pmatrix} 2yx & 2yx \\ 2yx & 2yx \end{pmatrix} = \begin{pmatrix} y & y \\ y & y \end{pmatrix} * \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\therefore *$ is Commutative

(iii) Associative :- ①m

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \left[\begin{pmatrix} y & y \\ y & y \end{pmatrix} * \begin{pmatrix} z & z \\ z & z \end{pmatrix}\right]$

$= \begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} 2yz & 2yz \\ 2yz & 2yz \end{pmatrix}$

$= \begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix}$

$\left[\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix}\right] * \begin{pmatrix} z & z \\ z & z \end{pmatrix}$

$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} * \begin{pmatrix} z & z \\ z & z \end{pmatrix}$

$= \begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix}$

$\therefore *$ is associative

(iv) Identity :- ①m

Let $\begin{pmatrix} e & e \\ e & e \end{pmatrix}$ be the identity element.

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$2xe = x \Rightarrow e = \frac{1}{2}$

$\therefore$ Identity element $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in M$

(v) Inverse :- ①m

Let $\begin{pmatrix} y & y \\ y & y \end{pmatrix}$ be the inverse of $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$2xy = \frac{1}{2} \Rightarrow y = \frac{1}{4x}$

$\begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix}$ be the inverse of $\begin{pmatrix} x & x \\ x & x \end{pmatrix} \in M$.

10. Let $A = Q - \{1\}$.

(i) closure :- ①m

$\forall x \neq 1, y \neq 1 \in A$.

Assume that,

$x*y = 1$

$x+y-xy = 1$

$x-1+y-xy = 0$

$(x-1)(1-y) = 0$

$x = 1$ (or) $y = 1$.

which is a contradiction

$x*y \neq 1 \in A$.

$\therefore *$ is binary on A.

(ii) commutative :- ①m

$x*y = x+y-xy$

$= y+x-yx = y*x$

$\therefore *$ is commutative.

(iii) Associative:- ①m

$x*(y*z) = x*(y+z-yz)$

$= x+y+z-yz-x(y+z-yz)$

$= x+y+z-xy-yz-zx+xyz$

$(x*y)*z = (x+y-xy)*z$

$= x+y-xy+z-(x+y-xy)z$

$= x+y+z-xy-yz-zx+xyz$

$\therefore *$ is associative.

(iv) Identity:- ①m

Let e be the identity element.

$x*e = x$

$x+e-xe = x$

$e-xe = 0$

$e(1-x) = 0$

$e = 0 \in A \; (\because x \neq 1)$

(v) Inverse:- ①m

Let x^{-1} be the inverse of x.

$x*x^{-1} = e$

$x+x^{-1}-xx^{-1} = 0$

$x^{-1}-xx^{-1} = -x$

$x^{-1}(1-x) = -x$

$x^{-1} = \frac{-x}{1-x} \in A \; (\because x \neq 1)$

Example-12.2

(i) Closure:- ①m

Clearly, $a+b \in Z$; $\forall a,b \in Z$.

(ii) Commutative:- ①m

$a+b = b+a$; $\forall a,b \in Z$.

(iii) Associative:- ①m

$a+(b+c) = (a+b)+c$; $\forall a,b,c \in Z$.

(iv) Identity:- ①m

Identity element $= 0 \in Z$

$\ni a+0 = 0+a = a$.

(v) Inverse:- ①m

Inverse of a is $-a \in Z$

$\ni a+(-a) = (-a)+a = 0$.

Example-12.3

(i) Closure:- ①m

Clearly, $a-b \in Z$; $\forall a,b \in Z$

(ii) Commutative:- ①m

Commutative is not true.

For example,

$a=1, b=2$

$1-2=-1$ & $2-1=1$

$a-b \neq b-a$.

(iii) Associative:- ①m

Associative is not true.

For example,

$a=1, b=2, c=3$

$a-(b-c) = 1-(2-3) = 1+1 = 2$

$(a-b)-c = (1-2)-3 = -4$

$a-(b-c) \neq (a-b)-c$

(iv) Identity:- ①m

Since '$-$' is not commutative. Identity doesn't exist.

(v) Inverse:- ①m

Since identity doesn't exist. Inverse doesn't exist.

Example-12.4

Z_e = the set of all even integers.

$Z_e = \{\ldots -4, -2, 0, 2, 4, \ldots\}$

(i) Closure:- ①m

Clearly, $a+b \in Z_e$; $\forall a,b \in Z_e$

(ii) Commutative:- ①m

$a+b = b+a$; $\forall a,b \in Z_e$

(iii) Associative:- ①m

$a+(b+c) = (a+b)+c$; $\forall a,b,c \in Z_e$

(iv) Identity:- ①m

Identity element $= 0 \in Z_e$

$\ni a+0 = 0+a = a$.

(v) Inverse:- ①m

Inverse of a is $-a \in Z_e$

$\ni a+(-a) = (-a)+a = 0$.

Example-12.5

Z_o = the set of all odd integers.

$Z_o = \{\ldots -3, -1, 1, 3, \ldots\}$

(i) closure:- ②m

$1, 3 \in Z_o$

$1+3 = 4 \notin Z_o$

$\therefore$ closure property is not true.

we need not to verify other properties.

$\therefore$ other properties are not true. ③m

Example-12.6.

(i) Closure:- ①m

Clearly, $a*b = a^b \in N$; $\forall a,b \in N$.

(ii) Commutative:- ②m

Commutative is not true.

For example,

$a=1, b=2$.

$a*b = 1^2 = 1$

$b*a = 2^1 = 2$

$a*b \neq b*a$

(iii) Associative :- ②m

Associative is not true.

For example,

$a=4, b=2, c=3$

$a*(b*c) = 4*2^3$

$= 4*8$

$= 4^8$

$(a*b)*c = 4^2*3$

$= 16*3$

$= 16^3$

$a*(b*c) \neq (a*b)*c$.

Example-12.7

(i) closure:- ①m

clearly, $m+n-mn \in Z$; $\forall m,n \in Z$

$*$ is binary on Z.

(ii) Commutative:- ①m

$m*n = m+n-mn = n+m-nm$

$= n*m$; $\forall m,n \in Z$.

$*$ is commutative.

(iii) Associative:- ①m

$m*(n*p) = m*(n+p-np)$

$= m+n+p-np-m(n+p-np)$

$= m+n+p-mn-np-pm+mnp$

$(m*n)*p = (m+n-mn)*p$

$= m+n-mn+p-(m+n-mn)p$

$= m+n+p-mn-np-pm+mnp$

$*$ is associative.

(iv) Identity:- ①m

Let e be the identity element.

$m*e = m$

$m+e-me = m$

$e-me = 0$

$e(1-m) = 0$

$e = 0 \in Z$ $(\because m \neq 1)$

(v) Inverse :- ①m

Let m^{-1} be the inverse of m.

$m*m^{-1} = 0$

$m+m^{-1}-mm^{-1} = 0$

$m^{-1}-mm^{-1} = -m$

$m^{-1}(1-m) = -m$

$m^{-1} = \frac{-m}{1-m}$

* when $m=2$, $m^{-1} \in Z$

* when $m=1$, m^{-1} is not defined.

* when $\forall m \neq 2$, m^{-1} doesn't exist.

Example-12.9.

$Z_5 = \{[0],[1],[2],[3],[4]\}$

we take reminders $\{0,1,2,3,4\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

②m

From the table,

(i) Since all the elements in the table $\in Z_5$.

$+_5$ is a binary.

(ii) The entries are symmetrical about the main diagonal.

$+_5$ has commutative. ①m

(iii) As usual, associative is true.

$+_5$ has associative.

(iv) Identity element $= 0 \in Z_5$ ①m

(v) Inverse of 0 is 0.

Inverse of 1 is 4.

Inverse of 2 is 3.

Inverse of 3 is 2.

Inverse of 4 is 1. ①m

Example-12.10

$A = \{1,3,4,5,9\}$

The set of reminders: $\{0,1,2,3,4,5,6,7,8,9,10\}$

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

From the table, ②m

(i) Since all the elements in the table $\in A$.

$\times_{11}$ is binary.

(ii) The entries are symmetrical about the main diagonal. ①m

$\times_{11}$ has commutative.

(iii) As usual, associative is true.

$\times_{11}$ has associative.

(iv) Identity element $= 1 \in A$ ①m

(v) Inverse of 1 is 1.

Inverse of 3 is 4.

Inverse of 4 is 3.

Inverse of 5 is 9.

Inverse of 9 is 5. ①m

Exercise-12.2

7.(i) $(p\wedge q)\wedge\neg(p\vee q)$

p	q	$s: p\wedge q$	$t: p\vee q$	$\neg t$	$s\wedge\neg t$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

(4m)

It is a contradiction. (1m)

7.(ii) $((p\vee q)\wedge\neg p)\to q$

p	q	$r: (p\vee q)$	$s: \neg p$	$t: r\wedge s$	$t\to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(4m)

It is a tautology. (1m)

7.(iii) $(p\to q)\leftrightarrow(\neg p\to q)$

p	q	$r: p\to q$	$\neg p$	$s: \neg p\to q$	$r\leftrightarrow s$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

(4m)

It is contingency. (1m)

7.(iv) $((p\to q)\wedge(q\to r))\to(p\to r)$

p	q	r	$s: (p\to q)$	$t: (q\to r)$	$u: (p\to r)$	$w: s\wedge t$	$w\to u$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(4m)

It is a tautology. (1m)

8.(i) $\neg(p\wedge q)\equiv\neg p\vee\neg q$

p	q	$r: p\wedge q$	$\neg r$	$\neg p$	$\neg q$	$s: \neg p\vee\neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

(4m)

From the table,
$\neg(p\wedge q)\equiv\neg p\vee\neg q$ (1m)

8.(ii) $\neg(p\to q)\equiv p\wedge\neg q$

p	q	$\neg q$	$p\to q$	$\neg(p\to q)$	$p\wedge\neg q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(4m)

From the table,
$\neg(p\to q)\equiv p\wedge\neg q$. (1m)

10. $p\to q\not\equiv q\to p$

p	q	$p\to q$	$q\to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

(2m)

From the table,
$p\to q\not\equiv q\to p$. (1m)

Example-12.16

$(p\veebar q)\wedge(p\veebar\neg q)$

p	q	$\neg q$	$r: p\veebar q$	$s: p\veebar\neg q$	$r\wedge s$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

(3m)

Example-12.18

$p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$

p	q	$p\to q$	$q\to p$	$p\leftrightarrow q$	$(p\to q)\wedge(q\to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

(4m)

From the table,
$p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$ (1m)

Example-12.19

$p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$

$p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$

$\equiv(\neg p\vee q)\wedge(\neg q\vee p)$

$\equiv(\neg p\vee q)\wedge(p\vee\neg q)$ (by commutative law)

$\equiv(\neg p\wedge(p\vee\neg q))\vee(q\wedge(p\vee\neg q))$ (by distributive law)

$\equiv(\neg p\wedge p)\vee(\neg p\wedge\neg q)\vee(q\wedge p)\vee(q\wedge\neg q)$ (by distributive law)

$\equiv \mathbb{F}\vee(\neg p\wedge\neg q)\vee(q\wedge p)\vee\mathbb{F}$ (by complement law)

$\equiv(\neg p\wedge\neg q)\vee(q\wedge p)$ (by identity law)

$\equiv(p\wedge q)\vee(\neg p\vee\neg q)$ (by commutative law)

$\therefore p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$

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LIFE IS TAN 45°
BUT
VALUE IS TAN 90°
– P. NESAMANI.

Exercise-12.2

11. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

p	q	$\neg q$	$r: p\leftrightarrow q$	$\neg r$	$s: p \leftrightarrow \neg q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

(2)m

From the table, (1)m

$\neg(p\leftrightarrow q) \equiv p \leftrightarrow \neg q$

13. $\neg(p\vee q) \vee (\neg p \wedge q) \equiv \neg p$

p	q	$r: p\vee q$	$\neg p$	$s: \neg p \wedge q$	$\neg r$	$\neg r \vee s$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	T	T	F	T
F	F	F	T	F	T	T

From the table,

$\neg(p\vee q) \vee (\neg p \vee q) \equiv \neg p$.

12. $p \to (q \to p)$

$p \to (q\to p) \equiv p \to (\neg q \vee p)$

$\equiv \neg p \vee (\neg q \vee p)$

$\equiv \neg p \vee (p \vee \neg q)$

(by commutative Law)

$\equiv (\neg p \vee p) \vee \neg q$

(by associative Law)

$\equiv \mathbb{T} \vee \neg q$

(by complement Law)

$\equiv \mathbb{T}$ (by identity Law)

$\therefore p \to (q \to p)$ is a tautology.

14. $p \to (q \to r) \equiv (p \wedge q) \to r$

$p \to (q \to r) \equiv p \to (\neg q \vee r)$

$\equiv \neg p \vee (\neg q \vee r)$

$\equiv (\neg p \vee \neg q) \vee r$

(By associative Law)

$\equiv \neg(p \wedge q) \vee r$

(By de Morgan's Law)

$\equiv (p\wedge q) \to r.$

$\therefore p \to (q \to r) \equiv (p \wedge q) \to r$

15. $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$

p	q	r	$\neg p$	$\neg q$	$s: \neg q \vee r$	$t: p \to s$	$v: \neg p \vee s$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the table,

$p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$

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