1. Relations and Functions

Exercise 1.1

Concept corner

Definition:

- ✓ A **set** is a collection of well defined objects.
- ✓ If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A, b \in B$ is called **the Cartesian Product of A and B**, and is denoted by $A \times B$. Thus $A \times B = \{(a, b) | a \in A, b \in B\}$

Note:

- ➢ $A \times B$ is the set of all possible ordered pairs between the elements of A and B such that the first coordinate is an element of A and the second coordinate is an element of B.
- ➢ $B \times A$ is the set of all possible ordered pairs between the element of A and B such that the first coordinate is an element of B and the second coordinate is an element of A
- ➢ If $a = b$, then $(a, b) = (b, a)$.
- ➢ The "Cartesian product" is also referred as "cross product"
- ➢ In general $A \times B \neq B \times A$, but $n(A \times B) = n(B \times A)$
- ➢ $A \times B = \emptyset$ if and only if $A = \emptyset$ or $B = \emptyset$
- ➢ If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$
- ➢ The set of all points in the Cartesian plane can be viewed as the set of all ordered pairs (x, y) where x, y are real numbers. In fact $\mathbb{R} \times \mathbb{R}$ is the set of all points which we call as the Cartesian plane.
- ➢ Distributive property of Cartesian product:
  (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- ➢ $A \times B$ represent a shape in two dimensions and $A \times B \times C$ represent an object in three dimensions.

Exercise 1.2

Concept corner

Definition: Let A and B be any two non-empty sets. A **relation (R)** from A to B is a subset of $A \times B$ satisfying some specified conditions. If $x \in A$ is related to $y \in B$ through R, then we write it as xRy.

xRy if and only if $(x, y) \in R$

- ✓ The domain of the relation $R = \{x \in A | xRy$, for some $y \in B\}$
- ✓ The co-domain of the relation R is B
- ✓ The range of the relation $R = \{y \in B | xRy$, for some $x \in A\}$

Note:

- ➢ A relation may be represented algebraically either by the roster method or by the set builder method.

Exercise 1.3

Concept corner

Definition: A relations f between two non-empty sets X and Y is called a **function** from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.That is, $f = \{(x, y)/ \text{ for all } x \in X, \ y \in Y\}$

Note:

- ✓ If $f: X \to Y$ is a function then, the set X is called the domain, f and the set Y is called its co-domain.
- ✓ A function is also called as a mapping or transformation.
- ✓ $f: X \to Y$ is a function only if
  i) every element in the domain of f has an image.
  ii) the image is unique.
- ✓ If A and B are finite sets such that $n(A) = p, \ n(B) = q$ then the total number of functions that exist between A and B is q^p
- ✓ If $f(a) = b$, then b is called **image** of a under f and a is called a **pre-image** of b.
- ✓ The set of all images of the elements X under f is called the **range** of f.
- ✓ Describing domain of a function
  (i) Let $(x) = \frac{1}{1+x}$. If $x = -1$ then $f(-1)$ is not defined. Hence f is defined for all real numbers except at $x = -1$. So, domain of f is $\mathbb{R} - \{-1\}$
  (ii) Let $(x) = \frac{1}{x^2-5x+6}$, if $x = 2,3$ then $f(2)$ and $f(3)$ are not defined. Hence f is defined for all real numbers except at $x = 2$ and 3. So domain of $f = \mathbb{R} - \{2,3\}$

- ➢ An arrow diagram is a visual representation of a relation.
- ➢ If $n(A) = p, n(B) = q$ then the total number of relations that exist from A to B is 2^{pq}.
- ➢ A relation which contains no elements is called a "**Null relation**"

Exercise 1.4

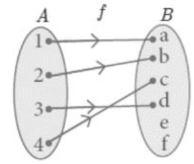
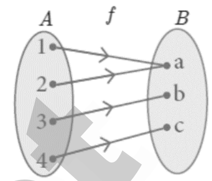
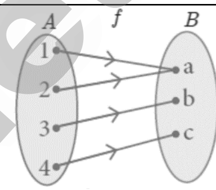
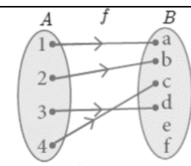
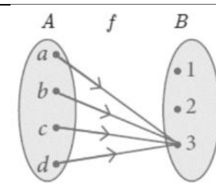
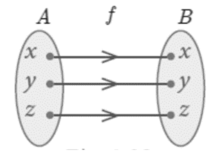
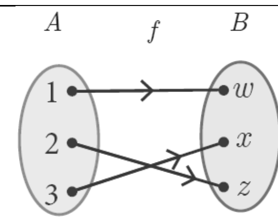
Concept corner

Note: Any equation represented in a graph is usually called a curve.

- ✓ **Representation of functions**
  a) a set of ordered pairs b) a table form
  c) An arrow diagram d) a graphical form.
- ✓ **Vertical line test:** A curve drawn in a graph represents a function, if every vertical line intersects the curve in at most one point.
- ✓ **Horizontal Line Test:** A function represented in a graph in one – one, if every horizontal line intersects the curve in at most one point.
- ✓ Every function can be represented by a curve in a graph. But not every curve drawn in a graph will represent a function.
- ✓ If $f: A - B$ is an onto function then, the range of $f = B$

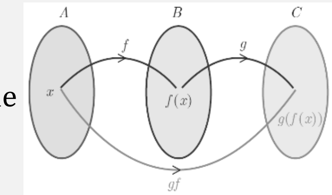
Note: A one-one and onto function is also called a one-one correspondence.

Types of functions

Sl.No	Name	Definition	Mapping Example
1	One-One function (Injection)	A function $f: A \to B$ is called one-one function if distinct elements of A have distinct images in B.	
2	Many-one function	A function $f: A \to B$ is called many-one function if two or more elements of A have same image in B	
3	Onto function (Surjection)	A function $f: A \to B$ is said to be onto function if the range of f is equal to the co-domain of f.	
4	Into function	A function $f: A \to B$ is called an into function if there exists at least one element in B which is not the image of any element of A	
5	Constant function	A function $f: A \to B$ is called a constant function if the range of f contains only one element. That is, $f(x) = c$ for all $x \in A$ and for some fixed $c \in B$.	
6	Identity function	Let A be a non-empty set. Then the function $f: A \to A$ defined by $f(x) = x$ for all $x \in A$ is called an identity function on A and is denoted by I_A.	
7	Bijection	If a function $f: A \to B$ is both one-one and onto, then f is called a bijection from A to B	
8	Real – Valued function	A function $f: A \to B$ is called a real valued function if the range of f is a subset of the set of all real numbers R. That is $f(A) \subseteq R$	

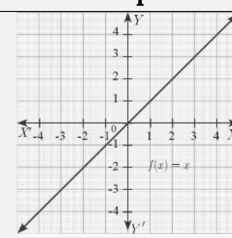
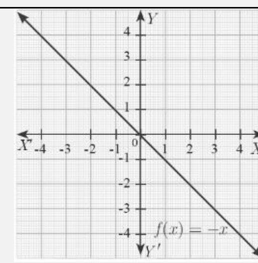
Exercise 1.5

Concept corner

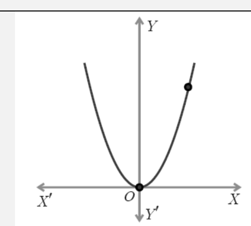
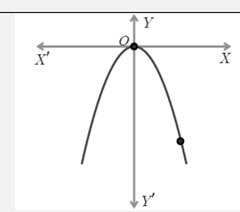


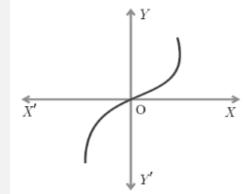
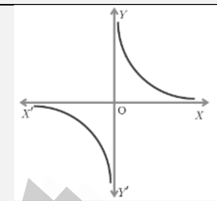
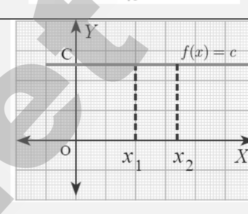
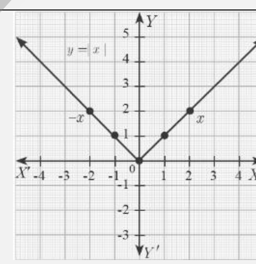
Definition: Let $f: A \to B$ and $g: B \to C$ be two functions. Then the composition of f and g denoted by $g \circ f$ is defined as the function $g \circ f(x) = g(f(x))$ for all $x \in A$.

- ✓ The composition $g \circ f(x)$ exists only when range of f is a subset of g
- ✓ $f \circ g \neq g \circ f$ Composition of function is not commutative.
- ✓ Composition of three functions is always associative. That is $f \circ (g \circ h) = (f \circ g) \circ h$.
- ✓ A function $f: R \to R$ defined by $f(x) = mx + c$, $m \neq 0$ is called a **linear function.** Some specific linear functions and their graphs are given below.

No.	Function	Domain and Definition	Graph
1	The identity function	$f: R \to R$ defined by $f(x) = x$	
2	Additive inverse function	$f: R \to R$ defined by $f(x) = -x$	

- ✓ A function $f: R \to R$ defined by $f(x) = ax^2 + bx + c$ $(a \neq 0)$ is called a **quadratic function.**

Function, Domain, Range and Definition	Graph
$f: R \to R$ defined by $f(x) = x^2, x \in R, f(x) \in [0, \infty)$	
$f: R \to R$ defined by $f(x) = -x^2, x \in R. f(x) \in (-\infty, 0]$	

A function $f: R \to R$ defined by $f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$ is called a **cubic function.**	
A function $f: R - \{0\} \to R$ defined by $f(x) = \frac{1}{x}$ is called a **reciprocal function.**	
A function $f: R \to R$ defined by $f(x) = c$ for all $x \in R$ is called a **constant function.**	
Modulus or Absolute Valued Function: $f: R \to [0, \infty)$ defined by $f(x) = \lvert x \rvert = \begin{cases} x \,;\, x \geq 0 \\ -x \,;\, x < 0 \end{cases}$	

✓ Modulus function is not a linear function but it is composed of two linear functions x and $-x$

2. Numbers and Sequences

Introduction for Exercise 2.1

Concept corner

Theorem 1 - Euclid's division Lemma : Let a and b $(a > b)$ be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r, 0 \leq r < b$.

Note:

- The remainder is always less than the divisor.
- If $r = 0$ then $a = bq$ so b divides a.
- Conversly, if b divides a then $a = bq$

Generalised form of Euclid's division lemma:

If a and b are any two integers then there exist unique integers q and r such that $a = bq + r$, where $0 \leq r < |b|$

Theorem 2: If a and b are positive integers such that $a = bq + r$, then every common divisor of a and b is a common divisor of b and r and vice – versa.

Euclid's Division Algorithm :

To find Highest Common Factor of two positive integers a and b where $a > b$.

Step-1: Using Euclid's division lemma $a = bq + r; 0 \leq r < b$ where q is the quotient , r is the remainder if $r = 0$ then b is the Highest Common Factor of a and b.

Step-2: Otherwise applying Euclid's division lemma divide b by r to get $b = rq_1 + r_1,$ $0 \leq r_1 < r$.

Step-3: If $r_1 = 0$ then r is the highest common factor of a and b.

Step-4: Otherwise using Euclid's division lemma repeat the process until we get the remainder zero. In that case, the corresponding divisor is the HCF of a and b.

Note:

- The above algorithm will always produce remainder zero at some stage. Hence the algorithm should terminate.
- Euclids division algorithm is a repeated application of division lemma until we get zero remainder.
- Highest Common Factor (HCF) of two positive numbers is denoted by (a, b)
- Highest Common Factor (HCF) is called as Greatest Common Divisor (GCD)

Theorem 3: If a, b are two positive integers with $a > b$ then GCD of (a, b) =GCD of $(a - b, b)$.

Highest common factor of three numbers : Let a, b, c be the given positive integers

(i) Find HCF of a, b call it as d, $d = (a, b)$

(ii) Find HCF of d and c.

This will be the HCF of the three given numbers a, b, c.

Note: Two positive integers are said to be relatively prime or co prime if their highest common factor is 1

Introduction for Exercise 2.2

Concept corner

Fundamental Theorem of Arithmetic:

Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.

In general, we conclude that given a composite number N, we decompose it uniquely in the form $N = p_1^{q_1} \times p_2^{q_2} \times p_3^{q_3} \times \ldots \times p_n^{q_n}$ Where $p_1, p_2, p_3, \ldots p_n$ are primes and $q_1, q_2, q_3 \ldots q_n$ are natural numbers.

Significance of the Fundamental Theorem of Arithmetic:

- If a prime number p divides ab then either p divides a or p divides b. That is p divides at least one of them.
- If a composite number n divides ab, then n neither divide a nor b.

  For example, 6 divides 4×3 but 6 neither divide 4 nor 3.

Introduction for Exercise 2.3

Concept corner

Modular Arithmetic: Modular Arithmetic is a system of arithmetic for integers where numbers wrap around a certain value.

Congruence Modulo: Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That $b - a = kn$ for some integer k. This can also be written as $a \equiv b \pmod{n}$

Here, the number n is called modulus. In other words,

$a \equiv b\ (mod\ n)$ means $a - b$ is divisible by n. [dividend = remainder (mod divisor)]

Example: $61 \equiv 5\ (mod\ 7)$ because $61 - 5 = 56$ is divisible by 7.

Note:

- When a positive integer is divided by n, then the possible remainder are $0,1,2,3, \ldots n-1$
- Thus, when we work with modulo n, we replace all the numbers by their remainders upon division by n, given by $0,1,2,3, \ldots, n-1$.

Connecting Euclid's Division lemma and Modular Arithmetic.

Let m and n be integers, where m is positive. Then by Euclid's division lemma, we can write $n = mq + r$ were $0 \le r < m$ and q is an integer,

$$n = mq + r$$
$$n - r = mq$$
$$n - r \equiv 0 (mod\ m)$$
$$n \equiv r\ (mod\ m)$$

Thus the equation $n = mq + r$ through Euclid's Division lemma can also be written as $n \equiv r\ (mod\ m)$.

Note: Two integers a and b are congruent modulo m, written as $a \equiv b\ (mod\ m)$, if they leave the same remainder when divided by m.

Modulo operations:

Theorem 5 : a, b, c and d are integers and m is a positive integer such that if $a \equiv b\ (mod\ m)$ and $c \equiv d\ (mod\ m)$ then (i) $(a + c) \equiv (b + d)(mod\ m)$

(ii) $(a - c) \equiv (b - d)(mod\ m)$ (iii) $(a \times c) \equiv (b \times d)(mod\ m)$

Theorem 6 : If $a \equiv b\ (mod\ m)$then (i) $ac \equiv bc\ (mod\ m)$

(ii) $a \pm c \equiv b \pm c\ (mod\ m)$for any integer c.

Note: While solving congruent equations, we get infinitely many solutions compared to finite number of solutions in solving a polynomial equation in Algebra.

Introduction for Exercise 2.4

Concept corner

Sequences : A real valued sequence is a function defined on the set of natural numbers and taking real values.

Term : Each element in the sequence is called a term of the sequence.

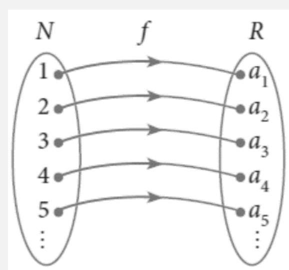
Finite sequence:

If the number of elements in a sequence is finite then it is called a Finite sequence.

Infinite sequence:

If the number of elements in a sequence is infinite then it is called an Infinite sequence.

Sequence as a function:



A sequence can be considered as a function defined on the set of natural numbers N. In particular a sequence is a function 00$f: N \to R$, where R is the set of all real numbers.

If the sequence is of the form a_1, a_2, a_3 , ... then we can associate the function to the sequence to the sequence a_1 , a_2, a_3 ... by $f(k) = a_k$, $k = 1,2,3, \ldots$

Note: Though all the sequences are functions not all the functions are sequences.

Introduction for Exercise 2.5

Concept corner

Arithmetic progression: Let a and d be real numbers. Then the numbers of the form $a, a+d$, $a+2d, a+3d, a+4d, \ldots$ is said to form Arithmetic progression denoted by A.P. The number $'a'$ is called the first term and $'d'$ is called the common difference.

(i) The General form	$a, a+d, a+2d, a+3d, \ldots \quad a+(n-1)d$.
(ii) n^{th} term	$t_n = a+(n-1)d$
(iii) Common difference	$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \cdots$ $d = t_n - t_{n-1}$ for $n = 2,3,4, \ldots$ The common difference of an $A.P$ can be positive, negative or zero
(iv) Total number of terms	$n = \left(\frac{l-a}{d}\right) + 1$

Note:

- The difference between any two consecutive terms of an $A.P$. is always constant. That constant value is called the common difference.
- If there are finite numbers of terms in an $A.P$ then it is called Finite Arithmetic progression. If there are infinitely many terms in an A.P. then it is called Infinite Arithmetic progression.
- An Arithmetic progression having a common difference of zero is called a **constant arithmetic progression**
- In a finite $A.P$ whose first term is a and last term l, then the number of terms in the $A.P$ is given by $l = a+(n-1)d$ gives $n = \left(\frac{l-a}{d}\right) + 1$

In an Arithmetic progression

- If every term is added or subtracted by a constant, then the resulting sequence is also an $A.P$.
- If every term is multiplied or divided by a non- zero number, then the resulting sequences is also an A.P.
- If the sum of three consecutive terms of an A.P is given, then they can be taken as $a-d, a$ and $a+d$. Here the common difference is d.
- If the sum of four consecutive terms of an $A.P$ is given then, they can be taken as $a-3d, a-d, a+d$ and $a+3d$. Here common difference is $2d$.

Condition for three numbers to be in $A.P$.

Three non zero numbers a, b, c are in $A.P$ if and only if $2b = a+c$

Introduction for Exercise 2.6

Concept corner

Series	The sum of the terms of a sequence is called series. Let $a_1, a_2, a_3, \ldots a_n \ldots$ be the sequence of real numbers. Then the real number $a_1 + a_2 + a_3 + \cdots$ is defined as the series of real numbers.
Finite series	If a series has finite number of terms then it is called a Finite series.
Infinite series	If a series has infinite number of terms then it is called an infinite series.
Arithmetic series	A series whose terms are in Arithmetic progression is called Arithmetic series.

Sum of n terms of an $A.P.$

- The sum of first n terms of a Arithmetic progression denoted by S_n is given by,

  $$S_n = a + (a+d) + (a+2d) + \cdots + a + (n-1)d = \frac{n}{2}[2a + (n-1)d]$$

- If the first term a, and the last term $l(n^{th}\ term)$ are given then, $S_n = \frac{n}{2}(a+l)$

Introduction for Exercise 2.7

Concept corner

Geometric progression

Definition	A Geometric progression is a sequence in which each term is obtained by multiplying a fixed non-zero number to the preceding term except the first term. The fixed number is called common ratio. The common ratio is usually denoted by r.
General form	Let a and $r \neq 0$ be real numbers. $a, ar, ar^2, \ldots ar^{n-1}$ is called General form of $G.P.$ a is called first term, r is called common ratio.
General term	$t_n = a\,r^{n-1}$

☑ If we consider the ratio of successive terms of the $G.P.$ then we have.

$$\frac{t_2}{t_1} = \frac{ar}{a} = r\ ;\ \frac{t_3}{t_2} = \frac{ar^2}{ar} = r\ ;\ \frac{t_4}{t_3} = \frac{ar^3}{ar^2} = r\ ; \frac{t_5}{t_4} = \frac{ar^4}{ar^3} = r$$

Thus the ratio between any two consecutive terms of the Geometric progression is always constant and that constant is the common ratio of the given progression.

☑ When the product of three consecutive terms of a G.P. are given, we can take the three terms as $\frac{a}{r}, a, ar$.

☑ When the products of four consecutive terms are given for a $G.P.$ then we can take the four terms as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

☑ When each term of a Geometric progression is multiplied or divided by a non – zero constant then the resulting sequence is also a Geometric progression.

Condition for three numbers to be in G. P.

Three non – zero numbers a, b, c are in $G.P.$ if and only if $b^2 = ac$

Total amount for compound interest is

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Where, A is the amount, P is the principal, r is the rate of interest and n is the number of years.

Introduction for Exercise 2.8

Concept corner

Geometric Series: A series whose terms are in Geometric progression is called Geometric series.

Sum to n terms of a G.P:

Sum to n terms of a G.P is $S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$

$r \neq 1, r > 1$	$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$
$r = 1$	$S_n = a + a + a + \cdots + a$ $S_n = na$
$r < 1$	$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$

The sum of infinite terms of a G.P is given by $a + ar + ar^2 + \cdots = \frac{a}{1-r}, -1 < r < 1$

Introduction for Exercise 2.9

Concept corner

Special Series: There are some series whose sum can be expressed by explicit formulae. Such series are called special series.

Sum of first n natural numbers	$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
Sum of first n odd natural numbers	$1 + 3 + 5 + \cdots + (2n - 1) = \frac{n}{2} \times 2n = n^2$
Sum of squares of first n natural numbers	$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
Sum of cubes of first n natural numbers	$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

- Sum of divisors of one number excluding itself is the other. Such pair of numbers is called **Amicable numbers** or **Friendly Numbers**
- The sum of first n natural numbers are also called '**Triangular Numbers**' because they form triangle shapes.
- The sum of squares of first n natural numbers are also called **Square Pyramidal Numbers** because they form pyramid shapes with square base.

3. Algebra

Introduction for Exercise 3.1

Concept corner

Definition:

- ✓ **Linear equation in two variables:** Any first degree equation containing two variables x and y is called a linear equation in two variables. The general form of linear equation in two variables x and y is $ax + by + c = 0$, where atleast one of a, b is non-zero and a, b, c are real numbers. A linear equation in two variables of the form $ax + by + c = 0$ represents a straight line.

- ✓ **Linear equation in three variables:** Any first degree equation containing three variables x, y and z is called a linear equation in three variables. The general form of linear equation in three variables x, y and z is $ax + by + cz + d = 0$, where atleast one of a, b, c is non-zero and a, b, c, d are real numbers. A linear equation in three variables of the form $ax + by + cz + d = 0$ represents a plane.

  - ✓ A system of linear equations in three variables x, y, z has the general form
    $a_1x + b_1y + c_1z + d_1 = 0$
    $a_2x + b_2y + c_2z + d_2 = 0$
    $a_3x + b_3y + c_3z + d_3 = 0$

  - ✓ **Procedure for solving (System of linear equations in three variables)**
    Step 1: By taking any two equations from the given three first multiply by some suitable non-zero constant to make the co-efficient of one variable (either x or y or z) numerically equal.
    Step 2: Eliminate one of the variable whose co-efficient are numerically equal from the equations.
    Step 3: Eliminate the same variable from another pair.
    Step 4: Now we have two equations in in two variables.
    Step 5: Solve them using any method studied in earlier classes.
    Step 6: The remaining variable is then found by substituting in any one of the given equation.

Note:

- If you obtain a false equation. Such as $0 = 1$, in any of the steps then the system has no solution.
- If you do not obtain a false solution, but obtain an identity, such as $0 = 0$ then the system has infinitely many solutions.

Introduction for Exercise 3.2

Concept corner

Way of finding GCD of two polynomials $f(x)$ & $g(x)$:

Step 1: First divide $f(x)$ by $g(x)$ to obtain $f(x) = g(x)q(x) + r(x)$

$q(x) \rightarrow$ Quotient , $r(x) \rightarrow$ Remainder then $deg[r(x)] < deg[g(x)]$

Step 2: If the reminder $r(x)$ is non zero divide, $g(x)$ by $r(x)$ to obtain $g(x) = r(x)q(x) + r_1(x)$ where $r_1(x)$the new remainder is. Then $\deg[r_1(x)] < \deg[r(x)]$ If the remainder $r_1(x)$ is zero, then $r(x)$ is the required GCD.

Step 3: If $r_1(x)$ is non zero, then continue the process until we get zero as remainder the divisor at this stage will be the required GCD.

Note :

If $f(x)$ and $g(x)$ are two polynomials of same degree then the polynomial carrying the highest co-efficient will be dividend. In case, if both have the same to coefficient then compare the next least degree's co-efficient and proceed with the division.

Introduction for Exercise 3.3

Concept corner

Relationship between LCM and GCD

$$f(x) \times g(x) = LCM[f(x), g(x)] \times GCD[f(x), g(x)]$$

Introduction for Exercise 3.4

Concept corner

Definition: An expression is called a **rational expression** if it can be written in the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. A rational expression is the ratio of two polynomials.

Reduction of Rational Expression:

(i) Factorize the numerator and the denominator

(ii) If there are common factors in the numerator and the denominator, cancel them

(iii) The resulting expression will be a rational expression in its lowest form

Excluded value: A value that makes a rational expression (in its lowest form) undefined is called an excluded value.

To find excluded value: (i) To reduce the given expression in its lowest form

(ii) Say it is $\frac{p(x)}{q(x)}$; consider $q(x) = 0$ and find the values of x.

Introduction for Exercise 3.5

Concept corner

OPERATIONS OF RATIONAL EXPRESSIONS

If $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ are two rational expressions where $q(x) \neq 0, s(x) \neq 0$ then

Multiplication of rational expressions: $\frac{p(x)}{q(x)} \times \frac{r(x)}{s(x)} = \frac{p(x) \times r(x)}{q(x) \times s(x)}$

Division of rational expressions: $\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} \times \frac{s(x)}{r(x)} = \frac{p(x) \times s(x)}{q(x) \times r(x)}$

Introduction for Exercise 3.7

Concept corner

- The square root of a given positive real number is another number which when multiplied with itself is the given number.
- The following two methods are used to find the square root of a given expression
  (i) Factorization method (ii) Division method

Introduction for Exercise 3.8

Concept corner

Finding the Square Root of a Polynomial by Division Method

The long division method in finding the square root of a polynomial is useful, when degree of the polynomial is higher.

Note: Before proceeding to find the square root of a polynomial one has to ensure that the degrees of variables are in descending or ascending order.

Introduction for Exercise 3.9

Concept corner

- An expression of degree 2 is called a **Quadratic Expression** which is expressed as $p(x) = ax^2 + bx + c$, $a \neq 0$ and a, b, c are real numbers.
- **Zeros of a quadratic expression**: let $p(x)$ be a polynomial. $x = a$ is called zeros of $p(x)$ if $p(a) = 0$.
- Roots of quadratic expression. Let $ax^2 + bx + c = 0 (a \neq 0)$ be a quadratic equation. The values of x such that the expression $ax^2 + bx + c$ becomes zero are called **roots of the quadratic equation** $ax^2 + bx + c = 0$. The roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **General form of a Quadratic Equation**: $x^2 -$ (Sums of the roots) $x +$(Product of the roots) $= 0$
  Here we take α and β be the roots of a quadratic equation then $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
- **Note:** $ax^2 + bx + c = 0 (a \neq 0)$ be the quadratic equation then
  Sum of the roots $= -\frac{b}{a}$, Product of the roots $= \frac{c}{a}$
- $ax^2 + bx + c = 0$ can equivalently be expressed as $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, since $a \neq 0$

Introduction for Exercise 3.10

Concept corner

Solving a quadratic equation by factorization method :

Step 1: Write the equation in general form $ax^2 + bx + c = 0$

Step 2: By splitting the middle term, factorize the given equation

Step 3: After factorizing the given quadratic equation can be written as product of two linear factors.

Step 4: Equate each linear factor to zero and solve for x.

Introduction for Exercise 3.11

Concept corner

Solving a Quadratic Equation by Completing the Square Method

Step 1: Write the quadratic equation in general form $ax^2 + bx + c = 0$

Step 2: Divide both sides of the equation by the coefficient of x^2 if it is not 1.

Step 3: Shift the constant term to the right hand side.

Step 4: Add the square of one-half of the coefficient of x to both sides

Step 5: Write the left hand side as a square and simplify the right hand side

Step 6: Take the square root on both sides and solve for x

Solving a Quadratic Equation by Formula Method

The formula for finding roots of a quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Introduction for Exercise 3.12

Concept corner

Solving problems Involving Quadratic Equations.

Step 1: Convert the word problem to a quadratic equation form for the given conditions.

Step 2: Solve the quadratic equation obtained in any one of the three methods (Factorization method, Completing the square method, Formula method).

Step 3: Relate the mathematical solution obtained to the statement asked to the question.

Introduction for Exercise 3.13

Concept corner

Nature of Roots of a Quadratic Equation

Values of discriminant $\Delta = b^2 - 4ac$	Nature of roots
$\Delta > 0$	Real and unequal roots
$\Delta = 0$	Real and equal roots
$\Delta < 0$	No real root

Introduction for Exercise 3.14

Concept corner

The Relation between Roots and Co- efficient of a Quadratic Equation

Let α and β are the roots of the equation $ax^2 + bx + c = 0$ then,

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \ \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\therefore \alpha + \beta = -\frac{b}{a} = \frac{-\text{Co-efficient of } x}{\text{Co-efficient of } x^2},$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Co-efficient of } x^2}$$

$\therefore$ Quadratic Equation $= x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Introduction for Exercise 3.15

Concept corner

- **Independent variable** represents a quantity that is manipulated in a given situation where as a **dependent variable** represents a quantity whose value depends on how the independent variable is manipulated.
- A constant is a quantity that assumes a fixed value throughout in a specific mathematical context.

Types of Variation	Equation	Graph
Direct Variation	$\frac{y}{x} = k$	straight line
Indirect Variation	$xy = k$	smooth curve known as a Rectangular Hyperbola

The linear equation of straight line is $y = mx + c$, where m is the slope of the straight line and c is the y-intercept. Also, the equation reduces to $y = mx$ when the straight line passes through the origin. As the graph of direct variation refer to straight line and its general form is $y = kx$, we can conclude that '**constant of proportionality**' is nothing but '**slope**' of its straight line.

Introduction for Exercise 3.16

Concept corner

Quadratic graphs

Note: For a quadratic equation, the axis is given by $x = -\frac{b}{2a}$, and the vertex is given by $\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$ when $\Delta = b^2 - 4ac$ is the discriminant of the quadratic equation $ax^2 + bx + c = 0$

Finding the Nature of Solution of Quadratic Equations Graphically

To obtain the roots of the quadratic equation $ax^2 + bx + c = 0$ graphically we first draw the graph of $y = ax^2 + bx + c$

The solutions of the quadratic equations are the x coordinates of the points intersection of the curve with x axis.

To determine the nature of solutions of a quadratic equation, we can use the following procedure.

If the graph of the given quadratic equation	intersect the X- axis at two distinct points,	Then the given equation has **two real and unequal roots.**
	touch the X- axis at only one point,	then the given equation has only one root which is same as saying **two real and equal roots.**
	does not intersect the X-axis at any point,	then the given equations has **no real root.**

Solving Quadratic Equations through Intersection of Lines:

If the straight line	**intersects the parabola** at **two distinct points,**	then the x coordinates of those points will be the **roots** of the given quadratic equation.
	just **touch the parabola** at **only one point,**	then the x coordinate of the common point will be the **single root** of the quadratic equation.
	doesn't intersect or touch the parabola ,	then the quadratic equation will have **no real roots.**

Introduction for Exercise 3.17

Concept corner

Definition: A **Matrix** is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns.

- **Order of a matrix**: If a matrix A has m rows and n coloumns, then the order of A is $m \times n$

Types of matrices :

Name	Definition	Example
Row matrix	A matrix that has only one row. Order $= 1 \times n$	$A = [5 \quad 3 \quad 4 \quad 1]$ Order $= 1 \times 4$
Column matrix	A matrix that has only one column Order $= m \times 1$	$A = \begin{bmatrix}1\\2\\5\end{bmatrix}$ Order $= 3 \times 1$
Square matrix	A matrix in which have equal number of rows and columns. Order $= m \times m$	$A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ Order $= 2 \times 2$
Diagonal Elements	In a square matrix, the elements of the form, $a_{11}, a_{22}, a_{33}, \ldots a_{ii}$ are called leading diagonal elements.	$\begin{bmatrix}1 & 3\\4 & 5\end{bmatrix}$ here $1, 5$ are leading diagonal elements.
Diagonal matrix	A square matrix which have 0 value for elements above and below the leading diagonal, Order $= m \times m$	$A = \begin{bmatrix}3 & 0 & 0\\0 & 1 & 0\\0 & 0 & 2\end{bmatrix}$ Order $= 3 \times 3$
Scalar matrix	A diagonal matrix which have equal, non-zero constant value for all elements along the leading diagonal Order $= m \times m$	$A = \begin{bmatrix}7 & 0 & 0\\0 & 7 & 0\\0 & 0 & 7\end{bmatrix}$ Order $= 3 \times 3$
Identity (or) Unit matrix (I)	A diagonal matrix which have value 1 for all elements along the leading diagonal Order $= m \times m$	$I_2 = A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
Zero matrix (or) Null matrix (O)	A matrix which have all element value as 0 Order $= m \times n$	$O_2 = A = \begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$
Lower Triangular Matrix	A square matrix in which all entries above the leading diagonal are zero is called a lower triangular matrix.	$A = \begin{bmatrix}8 & 0 & 0\\4 & 5 & 0\\-11 & 3 & 1\end{bmatrix}$
Upper Triangular Matrix	A square matrix in which all entries below the leading diagonal are zero is called a upper triangular matrix.	$A = \begin{bmatrix}1 & 7 & -3\\0 & 2 & 4\\0 & 0 & 7\end{bmatrix}$

- **Transpose of a matrix:** It is obtained by interchanging rows and coloumns of a matrix of the given A is called transpose of A and is denoted by A^T (read as A transpose)

If $A = \begin{bmatrix}1 & 5 & 6\\3 & 2 & 4\end{bmatrix}_{2\times3}$, then $A^T = \begin{bmatrix}1 & 3\\5 & 2\\6 & 4\end{bmatrix}_{3\times2}$ We note that $(A^T)^T = A$

- **Equal Matrices:** Two marices A and B are said to be equal if and only if they have the same order and each element of matrix A is equal to the corresponding element of matrix B. That is, $a_{ij} = b_{ij}$ for all i, j.
- **The negative of a matrix:** The negative of a matrix $A_{m\times n}$ denoted by $(-A)_{m\times n}$ is the matrix formed by replacing each element in the matrix $A_{m\times n}$ with its additive inverse.

Introduction for Exercise 3.18

Concept corner

Operations on Matrices:

Addition and Subtraction of matrices: Two matrices can be added or subtracted if they have same order. To add or subtract two matrices, simply add or subtract the corresponding elements.

Example: $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix} = \begin{bmatrix} a+g & b+h & c+i \\ d+j & e+k & f+l \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

Multiplication of matrix by a Scalar: We can multiply the elements of the given matrix A by a non-zero number k to obtain a new matrix kA whose elements are multiplied by k. The matrix kA is called scalar multiplication of A.

Thus, if $A = (a_{ij})_{m\times n}$ then, $kA = (ka_{ij})_{m\times n}$ for all $i = 1,2,\ldots,m$ and for all $j = 1,2,\ldots,n$

Properties of Matrix Addition and Scalar Multiplication:

Let A, B, C be $m \times n$ matrices and p,q be the two non-zero scalars (numbers). Then we have the following properties:

1	$A + B = B + A$	Commutative property of matrix addition
2	$A + (B + C) = (A + B) + C$	Associative property of matrix addition
3	$(pq)A = p(qA)$	Associative property of scalar multiplication
4	$IA = A$	Scalar Identity property where I is the unit matrix
5	$p(A + B) = pA + pB$	Distributive property of scalar and two matrices
6	$(p + q)A = pA + qA$	Distributive property of two scalars with a matrix

Additive identity: Null matrix (or) zero matrix is the identity of matrix addition.

Let A be any matrix

$A + O = O + A = A$

Where O is the null matrix or zero matrix of same order as that of A.

Additive inverse: If A be any given matrix then $-A$ is the additive inverse of A.

$A + (-A) = (-A) + A = O$

Introduction for Exercise 3.19

Concept corner

Properties of multiplication of matrix

a) Matrix multiplication is not commutative in general

Order of $A = m \times n$

Order of $B = n \times p$

Here $AB = \begin{matrix} m \times n & \\ & n \times p \end{matrix} = m \times p$

AB is defined

$BA = n \times p$

$m \times n \; [p \neq m]$

$\therefore$ BA is not defined, $\boldsymbol{AB \neq BA}$

b) Matrix multiplication is distributive over matrix addition

Right distributive property

Order of $A = m \times n$

Order of $B = n \times p$

Order of $C = n \times p$ then,

$A(B + C) = AB + AC$

Left distributive property

Order of $A = m \times n$

Order of $B = m \times n$

Order of $C = n \times p$ then,

$\boldsymbol{(A + B)C = AC + BC}$

c) Matrix multiplication is always associative

Order of $A = m \times n$

Order of $B = n \times p$

Order of $C = p \times q$

$\boldsymbol{(AB)C = A(BC)}$

d) Multiplication of a matrix by a unit matrix

$\boldsymbol{AI = IA = A}$, Here A & I must be in same order

Note:

$AB = O$ does not necessarily imply that $A = O$ or $B = O$ or both $A, B = O$

Note:

- If x and y are two real numbers such that $xy = 0$ then either $x = 0$ or $y = 0$. But this condition may not be true with respect to two matrices.
- If A and B are any two non zero matrices, then $(A + B)^2 \neq A^2 + 2AB + B^2$
- However if $AB = BA$ then $(A + B)^2 = A^2 + 2AB + B^2$

4. Geometry

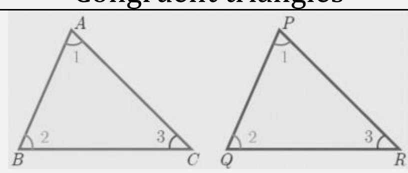
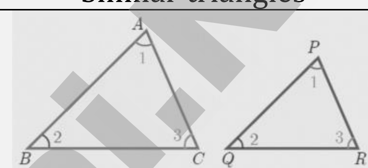
Introduction for Exercise 4.1

Concept corner

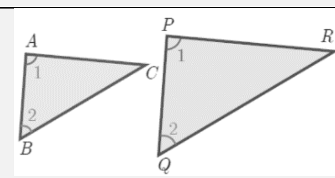
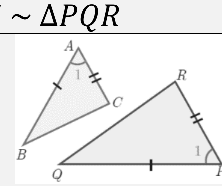
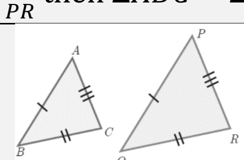
- Two figures are said to be similar if every aspect of one figure is proportional to other figure.

Congruency and similarity of triangles

- Congruency is a particular case of similarity. In both the cases, three angles of one triangle are equal to the three corresponding angles of the other triangle.
- But in congruent triangles, the corresponding sides are equal. While in similar triangles, the corresponding sides are proportional.
- The triangles ABC and PQR are similar can be written as $\Delta ABC \sim \Delta PQR$

Congruent triangles	Similar triangles
 $\Delta ABC \cong \Delta PQR$ $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ $AB = PQ, BC = QR, CA = RP$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$ Same shape and same size	 $\Delta ABC \sim \Delta PQR$ $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ $AB \neq PQ, BC \neq QR, CA \neq RP$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} > 1$ or < 1 Same shape but not same size

Criteria of Similarity

AA Criterion of similarity	If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar, because the third angle in both triangles must be equal. Therefore, AA similarity criterion is same as the AAA similarity criterion	 So, if $\angle A = \angle P = 1$ and $\angle B = \angle Q = 2$ then $\Delta ABC \sim \Delta PQR$
SAS Criterion of similarity	If one angle of a triangle is equal to one angle of another triangle and if the sides including them are proportional then the two triangles are similar.	 Thus, if $\angle A = \angle P = 1$ and $\frac{AB}{PQ} = \frac{AC}{PR}$ then $\Delta ABC \sim \Delta PQR$
SSS Criterion of similarity	If three sides of a triangle are proportional to the three corresponding sides of another triangle, then the two triangles are similar.	 So if, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ then $\Delta ABC \sim \Delta PQR$

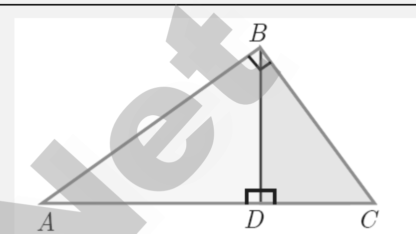
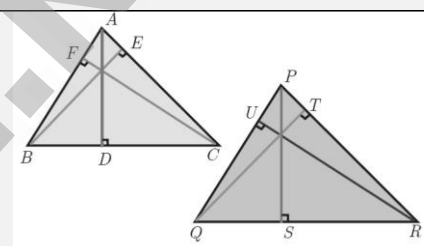
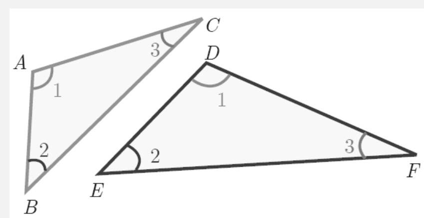
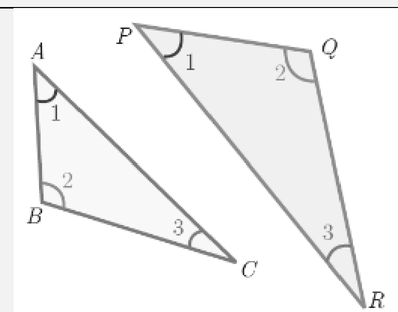
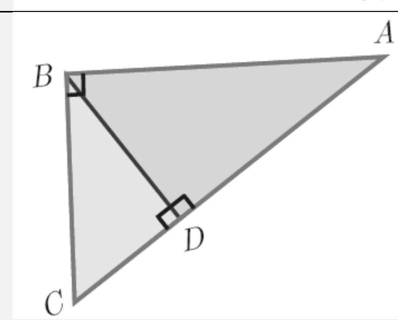
Definition:

1. Two triangles are said to be similar if their corresponding sides are proportional.
2. The triangles are equiangular if the corresponding angles are equal.

Note:

- If we change exactly one of the four given lengths, then we can make these triangles are similar
- A pair of equiangular triangles are similar.
- If two triangles are similar, then they are equiangular.

Some useful results on Similar Triangles:

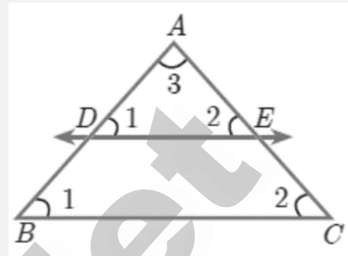
1	A perpendicular line drawn from the vertex of a right angled triangle divides the triangle into two triangles similar to each other and also to original triangle. $\Delta ADB \sim \Delta BDC$, $\Delta ABC \sim \Delta ADB$, $\Delta ABC \sim \Delta BDC$	
2	If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of their corresponding altitudes. If $\Delta ABC \sim \Delta PQR$ then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AD}{PS} = \frac{BE}{QT} = \frac{CF}{RU}$	
3	If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of the corresponding perimeters. $\Delta ABC \sim \Delta DEF$ then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AB+BC+CA}{DE+EF+FD}$	
4	The ratio of the area of two similar triangles are equal to the ratio of the squares of their corresponding sides $\frac{area\ (\Delta ABC)}{area\ (\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$	
5	If two triangles have common vertex and their bases are on the same straight line, the ratio between their areas is equal to the ratio between the length of their bases. $\frac{area\ (\Delta ABD)}{area\ (\Delta BDC)} = \frac{AD}{DC}$	

Introduction for Exercise 4.2

Concept corner

Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem

Statement: A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.



Proof:

Given: In ΔABC, D is a point on AB and E is a point on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

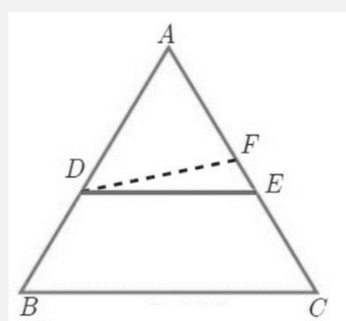
Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \backsim \Delta ADE$	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E
	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
	Hence proved	

Corollary: If in ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then (i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Theorem 2: Converse of Basic Proportionality Theorem

Statement: If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.



Proof:

Given: In ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$

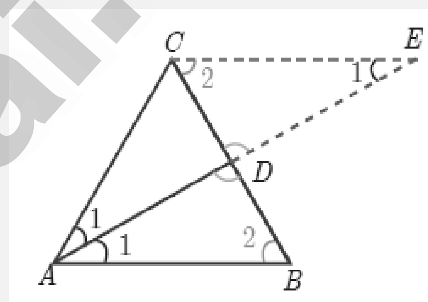
To prove: $DE \parallel BC$

Construction: Draw $BF \parallel DE$

No.	Statement	Reason
1.	In $\Delta ABC,\ BF \parallel DE$	Construction
2.	$\frac{AD}{DB} = \frac{AE}{EC}$ ……….. (1)	Thales theorem (In ΔABC taking D in $AB$and in AC)
3.	$\frac{AD}{EC} = \frac{AF}{FC}$ ………… (2)	Thales theorem (In ΔABC taking F in AC)
4.	$\frac{AE}{EC} = \frac{AF}{FC}$ $\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$ $\frac{AE+EC}{EC} = \frac{AF+FC}{FC}$ $\frac{AC}{EC} = \frac{AC}{FC}$ $EC = FC$ Therefore, $E = F$ Thus $DE \parallel BC$	From (1) and (2) Adding 1 to both sides Cancelling AC on both sides F lies between E and C. Hence Proved

Theorem 3: Angle Bisector Theorem

Statement: The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle. PTA-5



Proof:

Given : In $\Delta ABC,\ AD$ is the internal bisector

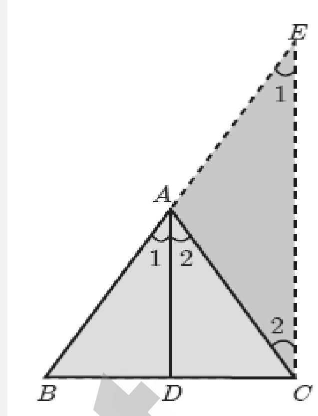
To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles $AC = CE$ …… (1)	In $\Delta ACE,\ \angle CAE = \Delta CEA$
3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved.

Theorem 4: Converse of Angle Bisector Theorem

Statement: If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex. PTA-3, 4



Proof:

Given : ABC is a triangle.

AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

That is $\frac{AB}{AC} = \frac{BD}{DC}$ .... (1)

To prove : AD bisects $\angle A$ i.e. $\angle 1 = \angle 2$

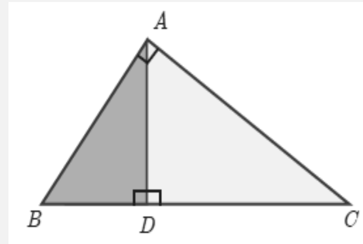
Construction : Draw $CE \parallel DA$. Extend BA to meet at E.

No.	Statement	Reason
1.	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC = \angle 1$	Since $DA \parallel CE$ and AC is transversal, corresponding angles are equal
3.	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$ and AC is transversal, Alternate angles are equal
4.	$\frac{BA}{AE} = \frac{BD}{DC}$ .... (2)	In ΔBCE by thales theorem
5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7.	$AC = AE$ ....(3)	Cancelling AB
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9.	AD bisects $\angle A$	Since, $\angle 1 = \angle BAD = \angle 2 = \angle DAC$. Hence proved

Note: If C_1 , C_2, ... are points on the circle, then all the triangles ΔBAC_1 , ΔBAC_2 , ... are with same base and the same vertical angle.

Introduction for Exercise 4.3

Concept corner



Theorem 5: Pythagoras Theorem

Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof: PTA-4

Given : In ΔABC, $\angle A = 90°$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ...(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity

Adding (1) and (2) we get

$AB^2 + AC^2 = BC \times BD + BC \times DC$

$= BC \times (BD + DC)$

$= BC \times BC$

$AB^2 + AC^2 = BC^2$

Hence the theorem is proved.

Converse of Pythagoras Theorem

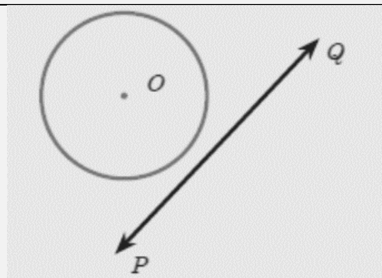
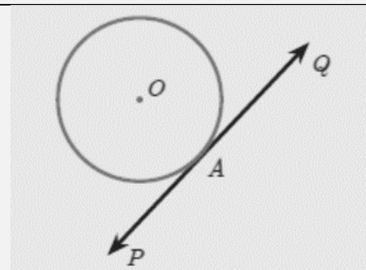
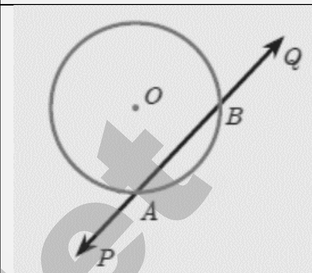
Statement: If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is a right angle triangle.

Note:

- In a right angles triangle, the side opposite to 90° (the right angle) is called the hypotenuse.
- The other two sides are called legs of the right angled triangle.
- The hypotenuse will be the longest side of the triangle.

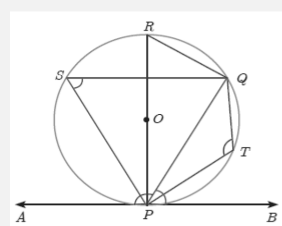
Introduction for Exercise 4.4

Concept corner

	Figure 1	Figure 2	Figure 3
			
(i)	Straight line PQ does not touch the circle.	Straight line PQ touches the circle at a common point A	Straight line PQ intersects the circle at two points A and B.
(ii)	There is no common point between the **straight line** and circle	PQ is called the **tangent** to the circle at A	The line PQ is called a **secant** of the circle
(iii)	Thus the number of **point of intersection** of a line and circle is **zero.**	Thus the number of **points of intersection** of a line and circle is **one.**	Thus the number of **points of intersection** of a line and circle is **two**

Definition: If a line touches the given circle at only one point then it is called **tangent to the circle.**

Theorem 6: Alternate Segment theorem



Statement: If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Proof:

Given : A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.

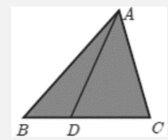
To prove : (i) $\angle QPB = \angle PSQ$ and (ii) $\angle QPA = \angle PTQ$

Construction : Draw the diameter POR. Draw QR, QS and PS.

No.	Statement	Reason
1.	$\angle RPB = 90°$ Now, $\angle RPQ + \angle QPB = 90°$ ... (1)	Diameter RP is perpendicular to tangent AB.
2.	In $\Delta RPQ, \angle PQR = 90°$ ... (2)	Angle in a semicircle is 90°.

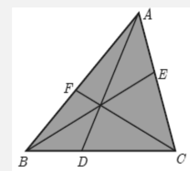
3.	$\angle QRP + \angle RPQ = 90°$ ... (3)	In a right angled triangle, sum of the two acute angles is 90°.
4.	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\angle QPB = \angle QRP$ ... (4)	From (1) and (3).
5.	$\angle QRP = \angle PSQ$ ... (5)	Angles in the same segment are equal.
6.	$\angle QPB = \angle PSQ$ ... (6)	From (4) and (5); Hence (i) is proved.
7.	$\angle QPB + \angle QPA = 180°$ ... (7)	Linear pair of angles.
8.	$\angle PSQ + \angle PTQ = 180°$ ...(8)	Sum of opposite angles of a cyclic quadrilateral is 180°.
9.	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From (7) and (8).
10.	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$ from (6)
11.	$\angle QPA = \angle PTQ$	Hence (ii) is proved. This completes the proof.

Definition: A **cevian** is a line segment that extends from one vertex of a triangle to the opposite side. In the diagram, AD is a cevian, from A.



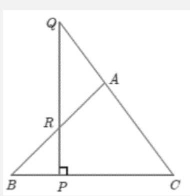
Ceva's Theorem (without proof)

Statement: Let ABC be a triangle and let D, E, F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed. This also works for the reciprocal of each of the ratios as the reciporcal of 1 is 1.



Note: The cevians do not necessarily lie within the triangle, although they do in the diagram

Menelaus Theorem (without proof)



Statement: A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1$ where all segments in the formula are directed segments.

Note:

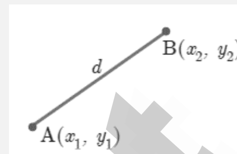
- Menelaus theorem can also be given as $BP \times CQ \times AR = -PC \times QA \times RB$
- If BP is replaced by PB (or) CQ by QC (or) AR by RA, or if any one of the six directed line segments BP, PC, CQ, QA, AR, RB is interchanged, then the product will be 1.
- Centroid is the point of concurrence of the median of a triangle.

5. Coordinate Geometry

Introduction for Exercise 5.1

Concept corner

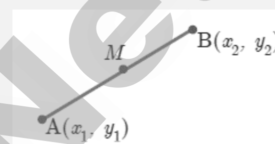
Distance between two points:



Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$|AB| = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

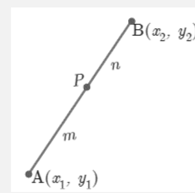
Mid – point of line segment:



The mid – point M, of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Section Formula

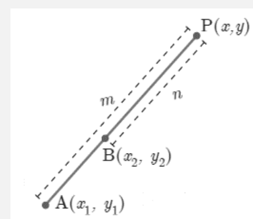
Internal Division:



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that point $P(x, y)$ divides AB **internally** in the ratio $m:n$.

Then the coordinates of P are given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

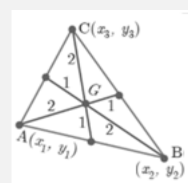
External Division:



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that point $P(x, y)$ divides AB **externally** in the ratio $m:n$.

Then the coordinates of P are given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$.

Centroid of a triangle:



The coordinates of the centroid (G) of a triangle with vertices $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Area of a Triangle:

Area of $\Delta ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ Sq. units.

Another form: Area of $\Delta ABC = \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$ sq.units

$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$ sq.units

Note: "As the area of a triangle can never be negative, we must take the absolute value, in case are happens to be negative".

Collinearity of three points:

Three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear if the area of $\Delta ABC = 0$.

Note: Another condition for collinearity:

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear points, then

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$ or $x_1y_2 + x_2y_3 + x_3y_1 = x_1y_3 + x_2y_1 + x_3y_2$.

Area of the quadrilateral:

Area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\} \text{ sq.units}$$

Note:

- To find the area of a quadrilateral, we divide it into triangular regions, which have no common area and then add the area of these regions.
- The area of the quadrilateral is never negative. That is, we always take the area of quadrilateral as positive.

Introduction for Exercise 5.2

Concept corner

Note: The inclination of a line or the **angle of inclination** of a line is the angle which a straight line makes with the positive direction of X axis measured in the counter – clockwise direction to the part of the line above the X axis. The inclination of the line is usually denoted by θ.

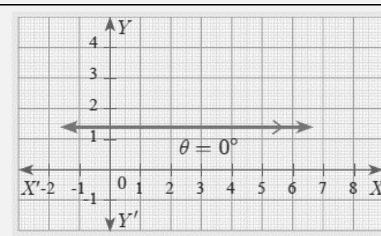
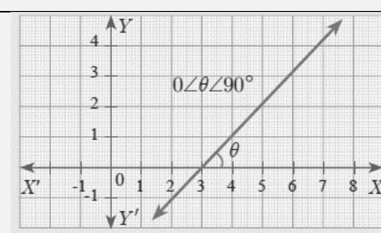
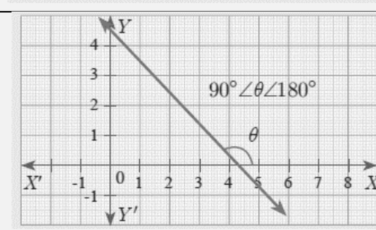
- The inclination of X axis and every line parallel to X axis is 0°
- The inclination of Y axis and every line parallel to Y axis is 90°

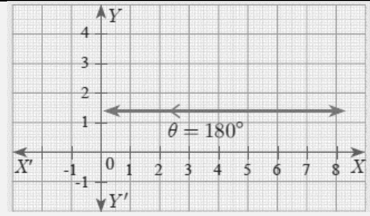
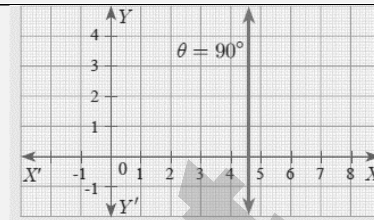
Definition: If θ is the angle of inclination of a non-vertical straight line, then $tan\theta$ is called the **slope** or gradient of the line and is denoted by m.

Therefore the slope of the straight line is $m = tan\theta,\ 0 \leq \theta \leq 180^\circ, \theta \neq 90^\circ$

Note: The slope of a vertical line is undefined.

Values of slopes

S.no	Condition	Slope	Diagram
(i)	$\theta = 0^\circ$	The line is parallel to the positive direction of X axis.	
(ii)	$0 < \theta < 90^\circ$	The line has positive slope (A line with positive slope rises from left to right)	
(iii)	$90^\circ < \theta < 180^\circ$	The line has negative slope (A line with negative slope falls from left to right).	

(iv)	$\theta = 180^\circ$	The line is parallel to the negative direction of X axis.	
(v)	$\theta = 90^\circ$	The slope is undefined.	

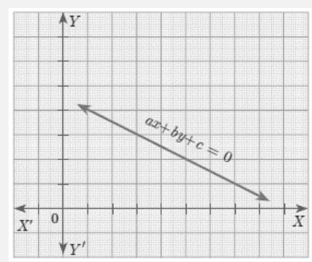
- Two non-vertical lines are parallel if and only if their slopes are equal.
- Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$.

Note:

- Let l_1 and l_2 be two lines with well-defined slopes m_1 and m_2 respectively, then
  (i) l_1 is parallel to l_2 if and only if $m_1 = m_2$. (ii) l_1 is perpendicular to l_2 if and only if $m_1 m_2 = -1$.
- In any triangle, exterior angle is equal to sum of the interior opposite angles.
- If the slopes of both the pairs of opposite sides are equal then the quadrilateral is a parallelogram.

Introduction for Exercise 5.3

Concept corner

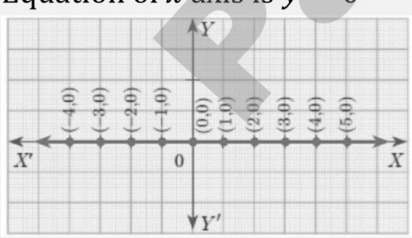


Straight line: Any first degree equation in two variables x and y of the form $ax + by + c = 0$ where a, b, c are real numbers and at least one of a, b is non-zero is called "**Straight line**" in xy plane.

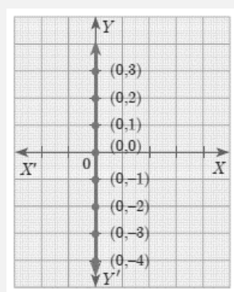
Equation of coordinate axes

The X axis and Y axis together are called coordinate axes.

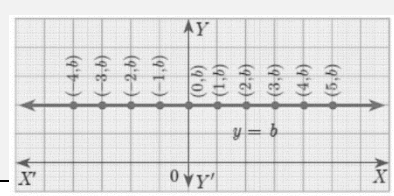
Equation of x axis is $y = 0$



Equation of y axis is $x = 0$



Equation of a straight line parallel to X axis



Let AB be a straight line parallel to X axis, which is at a distance 'b'

Then y coordinate of every point on 'AB' is 'b'.

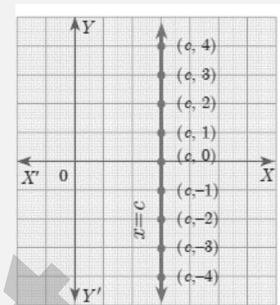
Therefore, the equation of AB is $y = b$

Note:

- If $b > 0$, then the line $y = b$ lies above the X axis
- If $b < 0$, then the line $y = b$ lies below the X axis
- If $b = 0$, then the line $y = b$ is the X axis itself.

Equations of a Straight line parallel to the Y axis

Let CD be a straight line parallel to Y axis, which is at a distance $'c'$. Then x coordinate of every point on CD is $'c'$. The equation of CD is $x = c$.



Note:

- If $c > 0$, then the line $x = c$ lies right to the side of the Y axis
- If $c < 0$, then the line $x = c$ lies left to the side of the Y axis
- If $c = 0$, then the line $x = c$ is the Y axis itself.

Slope – Intercept Form

Every straight line that is not vertical will cut the Y axis at a single point. The y coordinate of this point is called ***y* intercept** of the line.

A line with slope m and y intercept c can be expressed through the equation $y = mx + c$

- If a line with slope $m, m \neq 0$ makes x intercept d, then the equation of the straight line is $y = m(x - d)$.
- $y = mx$ represent equation of a line with slope m and passing through the origin.

Equation of Straight line in various forms:

	Name	Form
1	General form	$ax + by + c = 0$
2	Point – slope form	$y - y_1 = m(x - x_1)$
3	Slope - intercept	$y = mx + c$
4	Two point form	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

	Name	Form
5	Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
6	Parallel to Y axis	$x = c$
7	Parallel to X axis	$y = b$

Introduction for Exercise 5.4

Concept corner

General Form of a Straight Line

The linear equation (first degree polynomial in two variables x and y) $ax + by + c = 0$ (where a, b and c are real numbers such that at least one of a, b is non-zero) always represents a straight line. This is the general form of a straight line.

Now, let us find out the equations of a straight line in the following cases

(i) parallel to $ax + by + c = 0$

- The equation of all lines parallel to the line $ax + by + c = 0$ can be put in the form $\boldsymbol{ax + by + k = 0}$ for different values of k.

(ii) perpendicular to $ax + by + c = 0$

- The equation of all lines perpendicular to the line $ax + by + c = 0$ can be written as $\boldsymbol{bx - ay + k = 0}$ for different values of k.

(iii) The point of intersection of two intersecting straight lines

- Two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ where the coefficients are non-zero, are

(i) parallel if and only if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$; That is, $a_1b_2 - a_2b_1 = 0$

(ii) perpendicular if and only if $a_1a_2 + b_1b_2 = 0$

Slope of a straight line $ax + by + c = 0$:

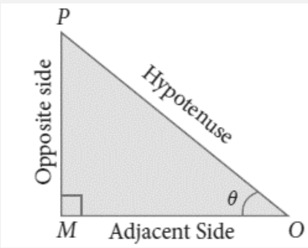
Slope $m = \frac{-\text{ coefficient of } x}{\text{coefficient of } y} = -\frac{a}{b}$, y intercept $= \frac{-\text{ constant term}}{\text{coefficient of } y} = -\frac{c}{b}$

6. Trigonometry

Introduction for Exercise 6.1

Concept corner

Trigonometric Ratios:



Let $0^\circ < \theta < 90^\circ$

In right anlge triangle OMP,

$$\sin\theta = \frac{Opposite\ side}{Hypotenuse} = \frac{MP}{OP}$$

$$\cos\theta = \frac{Adjacent\ side}{Hypotenuse} = \frac{OM}{OP}$$

From the above two ratios we can obtain other four trigonometric ratios as follows.

$$tan\theta = \frac{sin\theta}{cos\theta}; \quad cot\theta = \frac{cos\theta}{sin\theta}; \quad cosec\theta = \frac{1}{sin\theta}; \quad sec\theta = \frac{1}{cos\theta}$$

Note: All right triangles with θ as one of the angle are similar. Hence the trigonometric ratios defined through such right angle triangles do not depend on the triangle chosen.

Complementary angle

$sin(90^\circ - \theta) = cos\theta$	$cos(90^\circ - \theta) = sin\theta$	$tan(90^\circ - \theta) = cot\theta$
$cosec(90^\circ - \theta) = sec\theta$	$sec(90^\circ - \theta) = cosec\theta$	$cot(90^\circ - \theta) = tan\theta$

Table of Trigonometric Ratios for $0^0, 30^0, 45^0, 60^0, 90^0$

Trigonometric Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
$cosec\theta$	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
$\cot\theta$	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Note:

$(\sin\theta)^2 = \sin^2\theta$

$(\cos\theta)^2 = \cos^2\theta$

$(\tan\theta)^2 = \tan^2\theta$

$(\text{cosec}\,\theta)^2 = \text{cosec}^2\theta$

$(\sec\theta)^2 = \sec^2\theta$

$(\cot\theta)^2 = \cot^2\theta$

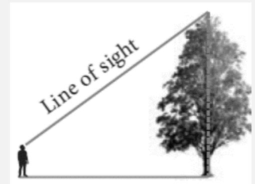
Trigonometric Identities:

Identity	Equal forms
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$ (or) $\cos^2\theta = 1 - \sin^2\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$ (or) $\sec^2\theta - \tan^2\theta = 1$
$1 + \cot^2\theta = cosec^2\theta$	$\cot^2\theta = cosec^2\theta - 1$ (or) $cosec^2\theta - \cot^2\theta = 1$

Note: Though the above identities are true for any angle θ, we will consider the six trigonometric ratios only for $0^\circ < \theta < 90^\circ$

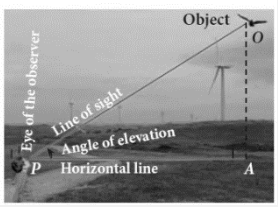
Introduction for Exercise 6.2

Concept corner



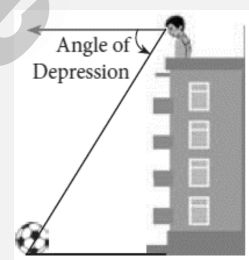
Line of sight:

The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.



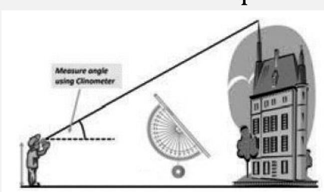
Angle of elevation:

The angle of elevation is an angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level. That is, the case when we raise our head to look at the object.



Angle of Depression:

The angle of depression is an angle formed by the line of sight with the horizontal when the point is below the horizontal level. That is, the case when we lower our head to look at the point being viewed.



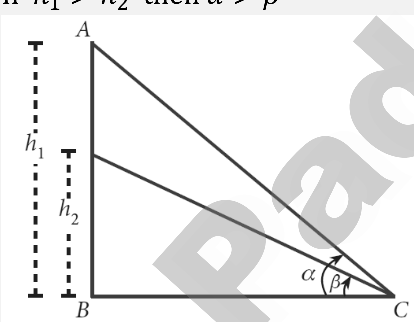
Clinometer:

The angle of elevation and depression are usually measured by a device called clinometer.

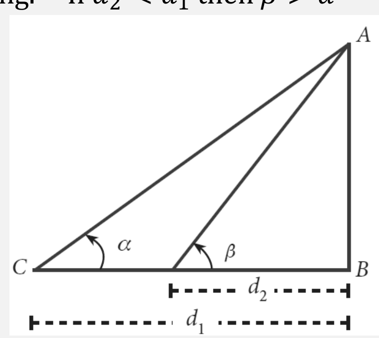
Note:

From a given point, when height of a object increases the angle of elevation increases.

If $h_1 > h_2$ then $\alpha > \beta$

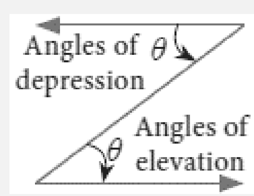


The angle of elevation increases as we move towards the foot of the vertical object like tower or building. If $d_2 < d_1$ then $\beta > \alpha$



Introduction for Exercise 6.3

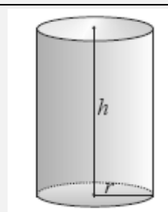
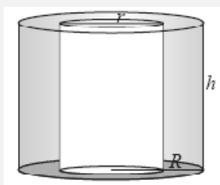
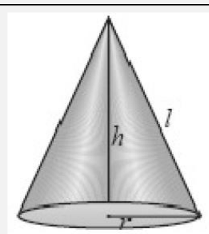
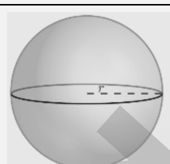
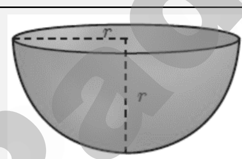
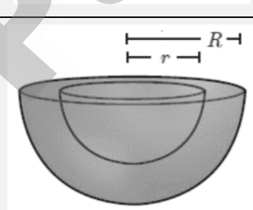
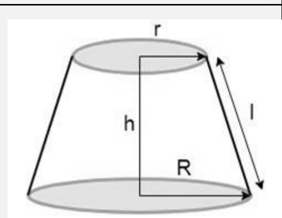
Concept corner

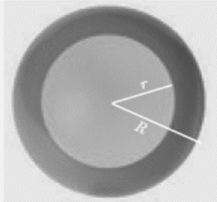
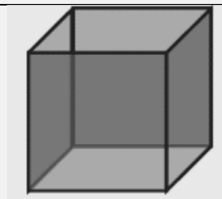


Note: Angle of Depression and angle of Elevation are equal because they are alternative angles.

7. Mensuration

Concept corner for Exercise 7.1 and 7.2

Sl.No	Name	Figure	CSA (sq.units)	TSA (sq.units)	Volume (cu.units)
1	Right circular cylinder		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2.	Hollow cylinder		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	$\pi(R^2-r^2)$ (Or) $\pi(R+r)(R-r)$
3	Right circular cone		πrl $\left(l=\sqrt{h^2+r^2}\right)$	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
5	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
6	Hollow hemisphere		$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
7	Frustum		$\pi(R+r)l$ $\left(l=\sqrt{h^2+(R-r)^2}\right)$	$\pi l(R+r)+\pi R^2+\pi r^2$	$\frac{1}{3}\pi h(R^2+r^2+Rr)$

Sl.No	Name	Figure	CSA (sq.units)	TSA (sq.units)	Volume (cu.units)
8	Hollow Sphere		$4\pi R^2$ = Outer Surface area	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
9	Cuboid		$2h(l+b)$	$2(lb+bh+lh)$	$l\times b\times h$
10	Cube		$4a^2$	$6a^2$	a^3

Introduction for Exercise 7.3

Concept corner

- A combined solid is said to be a solid formed by combining two or more solids.
- To calculate the surface area of the combined solid
  For example, if a cone is surmounted by a hemisphere, we need to just find out the C.S.A. of the hemisphere and C.S.A. of the cone separately and add them together.
- The volume of the solid formed by joining two basic solids will be the sum of the volumes of the individual solids.

Introduction for Exercise 7.4

Concept corner

- When one solid is Melted, Re-casted, and Reshaped into another solid, Volume will not be changed.
- Finding the missing parameter by equalizing the volume
- If the question is asked like "How many" & "Numbers required",

$$\text{Required Number} = \frac{\text{Volume of Bigger Shape}}{\text{Volume of Smaller Shape}}$$

- **Unit Conversion**
  10 cm = 1 dm, 100 cm = 1 m
  1 cm^3=1 ml
  1000 cm^3= 1 litre , 1000000 cm^3 =1m^3 = 1000 litres

8. Statistics and Probability

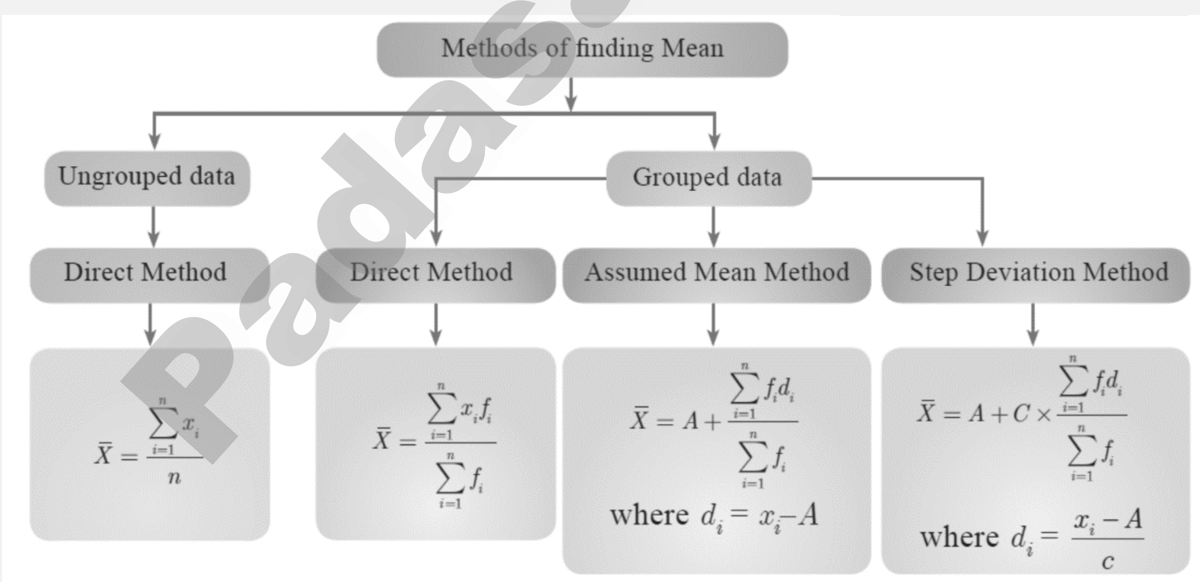
Introduction for Exercise 8.1

Concept corner

Note: Measures of Central Tendency: It is often convenient to have one number that represent the whole data. Such a number is called a Measures of Central Tendency.

The most common among them are Arithmetic Mean, Median, Mode.

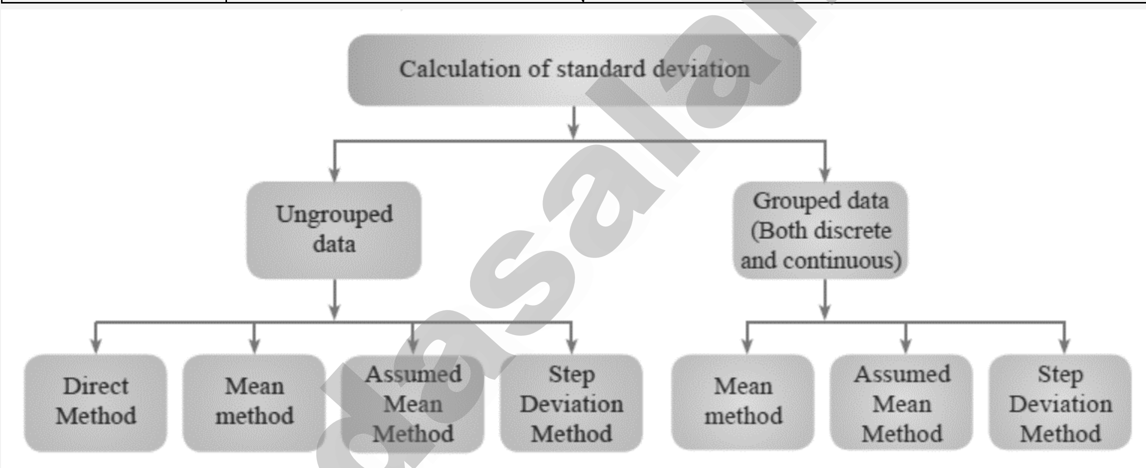
Data	The numerical representation of facts is called data.
Observation	Each entry in the data is called an observation.
Variable	The quantities which are being considered in a survey are called variables. Variables are generally denoted by x_i, where $i = 1,2,3,\ldots,n$.
Frequencies	The number of times, a variable occurs in a given data is called the frequency of that variable. Frequencies are generally denoted as f_i, where $i = 1,2,3,\ldots,n$.
Arithmetic Mean	The Arithmetic Mean or Mean of the given values is sum of all the observations divided by the total number of observations. It is denoted by $\bar{x}$ (pronounced as x bar) $\bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$



- Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) scattered throughout the data.
- Different Measures of Dispersion are

1. Range 2. Mean deviation 3. Quartile deviation
4. Standard deviation 5. Variance 6. Coefficient of Variation

Range	The difference between the largest value and the smallest value is called Range. Range $R = L - S$ Coefficient of range $= \frac{L-S}{L+S}$ (L – Largest value, S – Smallest value)
Deviation from the mean	For a given data with n observations $x_1, x_2, x_3 \ldots . x_n$ the deviations from the mean $\bar{x}$ are $x_1 - \bar{x}$, $x_2 - \bar{x}$,... $x_n - \bar{x}$
Squares of deviations from the mean	The squares of deviation from the mean $\bar{x}$ of the observations $x_1, x_2 . . x_n$ are $(x_1 - \bar{x})^2$, $(x_2 - \bar{x})^2$,... $(x_n - \bar{x})^2$ or $\sum_{i=1}^{n}(x_i - \bar{x})^2$ $(x_i - \bar{x})^2 \geq 0$ for all observations $x_i, i = 1,2,3, \ldots n$. If the deviations from the mean $(x_i - \bar{x})$ are small, then the squares of the deviations will be very small.
Variance	The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2 (read as sigma square) Variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$
Standard Deviation	The positive square root of Variance is called Standard deviation. That is, standard deviation is the positive square root of the mean of the squares of deviations of the given values from their mean. It is denoted by σ Standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$



Calculation of Standard Deviation for ungrouped data

(i)	Direct Method	$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$
(ii)	Mean Method	If $d_i = x_i - \bar{x}$ are the deviations, then $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$
(iii)	Assumed Mean Method	$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$
(iv)	Step deviation Method	$\sigma = c \times \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

Calculation of Standard Deviation for grouped data

(i)	Mean Method	$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$, where $N = \sum_{i=1}^{n} f_i$
(ii)	Assumed Mean Method	$d_i = x - A$, $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

Calculation of Standard deviation for continuous frequency distribution

(i)	Mean Method	$\sigma = \sqrt{\frac{\Sigma f_i(x_i-\bar{x})^2}{N}}$, where x_i = Middle value of the i^{th} class f_i =Frequency of the i^{th} class
(ii)	Shortcut Method (or) Step deviation method	$d_i = \frac{x_i-A}{c}$, $\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

- The SD of first 'n' natural numbers, $\sigma = \sqrt{\frac{n^2-1}{12}}$
- The SD **will not change** when we add or subtract some fixed constant to all the values
- SD of a collection of data gets multiplied or divided by the quantity k, if each item is multiplied or divided by k
- If the frequency of initial class is zero, then the next class will be considered for the calculation of range.
- The range of a set of data does not give the clear idea about the dispersion of the data from measures of Central Tendency. For this, we need a measure which depend upon the deviation from the measures of Central Tendency.
- $(x_i - \bar{x}) \geq 0$ for all observations $x_i, i = 1,2,3, \ldots n$. If the deviations from the mean $(x_i - \bar{x})$ are small, then the squares of the deviations will be very small.

Note: While computing standard deviation, arranging data in ascending order is not mandatory.

- If the data values are given directly then to find standard deviation we can use the formula $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$
- If the data values are not given directly but the squares of the deviations from the mean of each observation is given then to find standard deviation we can use the formula $\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$

Introduction for Exercise 8.2

Concept corner

Definition: For comparing two or more data for corresponding changes the relative measure of standard deviation called **Coefficient of variation.**

Coefficient of variation of first data (C.V1)= $\frac{\sigma_1}{\bar{x}_1} \times 100\%$

Coefficient of variation of second data (C.V2)= $\frac{\sigma_2}{\bar{x}_2} \times 100\%$

i) The data with **lesser** coefficient of variation is more consistent or stable than the other data.

ii) The data with **greater** coefficient of variation is inconsistent.

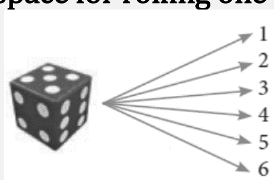
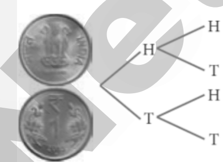
iii) The data have equal coefficient of variation values one data depends on the other.

To Find the Square root:	**Example: To find the square root of 75**
$\sqrt{X} = \sqrt{S} + \frac{(X-S)}{2\sqrt{S}}$ X –the number you want the square root S – the closet square number you know to X	$X = 75, S = 81$ (nearest square) $\sqrt{S} = 9$ $\sqrt{75} = \sqrt{81} + \frac{(75-81)}{2(\sqrt{81})} = 9 + \frac{-6}{2(9)} = 9 - \frac{6}{18}$ $= 9 - 0.333 = 8.667$

Introduction for Exercise 8.3

Concept corner

- A **random experiment** is an experiment in which
  (i) The set of all possible outcomes are known (ii) Exact outcome is not known
- **Sample space:** The set of all possible outcomes in a random experiment is called a sample space. It is generally denoted by S
- **Sample point :** Each element of a sample space is called a sample point.
- **Tree diagram:** Tree diagram allow us to see visually all possible outcomes of an random experiment. Each branch in a tree diagram represent a possible outcome.

Sample space for rolling one die	Sample space for toss two coins
	

Events	Explanation	Example
Equally likely events	Two or more events are said to be **equally likely** if each one of them has an equal chance of occuring.	Head and tail are equally likely events in **tossing a coin.**
Certain events	In an experiment, the event which surely occur is called **certain event.**	When we **roll a die,** the event of getting any natural number from 1 to 6 is a certain event.
Impossible events	In an experiment if an event has no scope to occur then it is called an **impossible event.**	When we **toss two coins**, the event of getting three heads is an impossible event.
Mutually exclusive events	Two or more events are said to be **mutually exclusive** if they don't have common sample points. i.e., events A, B are said to be mutually exclusive if, $A \cap B = \emptyset$.	When we **roll a die** the events of getting odd numbers and even numbers are mutually exclusive events.
Exhaustive events	The collection of events whose union is the whole sample space are called **exhaustive events.**	When we **toss a coin twice**, the collection of events of getting two heads, exactly one head, no head are exhaustive events.
Complementary events	The **complement of an event** A is the event representing collection of sample points not in A. It is denoted A' or A^c or $\bar{A}$ The event A and its complement A' are mutually exclusive and exhaustive.	When we **roll a die**, the event 'rolling 5 or 6' and the event of rolling 1, 2, 3 or 4 are complementary events.

Elementary Event: If an event E consists of only one outcome then it is called an elementary event.

Probability of an event:

In a random experiment, let S be the sample space and $E \subseteq S$. Then if E is an event, the probability of occurrence of E is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to occurence of } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

- $P(E) = \frac{n(E)}{n(S)}$
- $P(S) = \frac{n(S)}{n(S)} = 1.$ The probability of sure event is 1.
- $P(\emptyset) = \frac{n(\emptyset)}{n(S)} = \frac{0}{n(s)} = 0.$ The probability of impossible event is 0.
- Since E is a subset of S and $\emptyset$ is a subset of any set,

  $\emptyset \subseteq E \subseteq S$

  $P(\emptyset) \leq P(E) \leq P(S)$

  $0 \leq P(E) \leq 1$

  Therefore, the probability value always lies from 0 to 1.
- The complement event of E is $\bar{E}$.

  Let $P(E) = \frac{m}{n}$

  (Where m is the number of favorable outcomes of E and n is the total number of possible outcomes).

  $$P(\bar{E}) = \frac{\text{Number of outcomes unfavourable to occurance of } E}{\text{Number of all possible outcomes}}$$

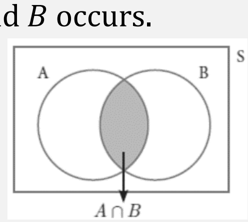
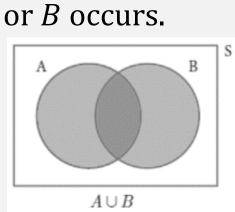
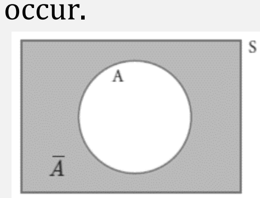
  $$P(\bar{E}) = \frac{n-m}{n} = 1 - \frac{m}{n}$$

  $$P(\bar{E}) = 1 - P(E)$$
- $P(E) + P(\bar{E}) = 1$

Introduction for Exercise 8.4

Concept corner

Algebra of events In a random experiment, let S be the sample space. Let $A \subseteq S$ and $B \subseteq S$ be the events in S. We say that

(i) $(A \cap B)$ is an event that occurs only when both A and B occurs.	(ii) $(A \cup B)$ is an event that occurs when either one of A or B occurs.	(iii) $\bar{A}$ is an event that occurs only when A doesn't occur.
		

- $A \cap \bar{A} = \emptyset$, $A \cup \bar{A} = S$
- If A, B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

- P (Union of mutually exclusive events)= $\sum(Probability\ of\ events)$

Verbal description of the event	Equivalent set theoretic notation
Not A	$\bar{A}$
A or B (at least one of Aor B)	$A \cup B$
A and B	$A \cap B$
A but not B	$A \cap \bar{B}$
Neither A nor B	$\bar{A} \cap \bar{B}$
At least one of A, B or C	$A \cup B \cup C$
Exactly one of A and B	$(A \cap \bar{B}) \cup (\bar{A} \cap B)$
All three of A, B and C	$A \cap B \cap C$
Exactly two of A, B *and* C	$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

Theorem 1: If A and B are two events associated with a random experiment, then prove that (i) $P(A \cap \bar{B}) = P$ (only A)= $P(A) - P(A \cap B)$(ii) $P(\bar{A} \cap B) = P$(only B)= $P(B) - P(A \cap B)$

Addition Theorem of Probability:

(i) If A and B are any two events then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) If A, B and C are any three events then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$