

## I CHOOSE THE CORRECT ANSWER

1. c 12
2. d quadratic
3. a 1
4. b  $\frac{1}{27}$
5. a straight line
6. c -1, 2
7. b point of contact
8. d  $5\sqrt{2}$ cm
9. c (3, 5)
10. b  $\frac{3}{2}$
11. a 12cm
12. c  $3\pi$
13. a 0
14. c 33.25

## II ANSWER ANY 10 ONLY

15. Given  $R = \{ (x,y) / y = x + 3, x \in \{0,1,2,3,4,5\} \}$

$\therefore$  Relation =

$\{ (0,3), (1,4), (2,5), (3,5), (4,7), (5,8) \}$

Domain =  $\{ 0, 1, 2, 3, 4, 5 \}$  and

Range =  $\{ 3, 4, 5, 6, 7, 8 \}$

16.  $f \circ f(k) = (2k-1) \circ (2k-1) = 2(2k-1) - 1$

Thus  $f \circ f(k) = 4k - 3$

But, it is given that  $f \circ f(k) = 5$

Therefore  $4k - 3 = 5$

$$4k = 8 \Rightarrow k = 2$$

17.  $1 + 2 + 3 + \dots + k = 325$

$$1^3 + 2^3 + 3^3 + \dots + k^3$$

$$\Rightarrow \left( \frac{k(k+1)}{2} \right)^2 = (\sum k)^2$$

$$= 325^2$$

$$= 105625$$

18.  $2A + B = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$
- $$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

19.  $\Rightarrow \Delta = b^2 - 4ac \quad \Rightarrow \Delta = (11)^2 - 4(15)(2)$

$$\Rightarrow \Delta = 121 - 120 \quad \Rightarrow \Delta = 1$$

Therefore  $\Delta > 0$ ,

$\therefore$  The roots are real and unequal.

20. By Pythagoras theorem we have

$$PN^2 = PE^2 + NE^2$$

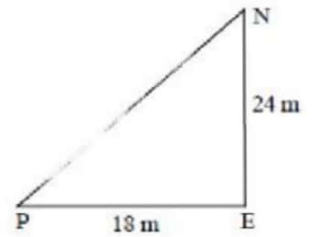
$$\Rightarrow x^2 = 18^2 + 24^2$$

$$\Rightarrow x^2 = 324 + 576$$

$$\Rightarrow x^2 = 900$$

$$\Rightarrow X = \sqrt{900}$$

$$\Rightarrow X = 30 \text{ m.}$$



21. Slope of the first line

$$\Rightarrow x - 2y + 3 = 0$$

$$\Rightarrow \text{Slope of straight line } (m_1) = \frac{-a}{b} = \frac{-1}{-2}$$

$$= \frac{1}{2}$$

Slope of the second line

$$\Rightarrow 6x + 3y + 8 = 0$$

$$\Rightarrow \text{Slope of straight line } (m_2) = \frac{-a}{b} = \frac{-6}{3}$$

$$\therefore m_1 \times m_2 = \frac{1}{2} \times \frac{-6}{3}$$

$$= -1$$

Therefore the lines are perpendicular.

22. Slope of line  $(m) = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{0 - \sqrt{5}}{0 - 5}$
- $$\Rightarrow m = \frac{-\sqrt{5}}{-5} \Rightarrow m = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

23.  $\Rightarrow \text{LHS} = \tan^2 \theta - \sin^2 \theta$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta = \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} = \frac{\sin^2 \theta \times \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \times \sin^2 \theta = \tan^2 \theta \sin^2 \theta = \text{RHS.}$$

24. Surface area of Sphere =  $154 \text{ m}^2$

$$\Rightarrow 4\pi r^2 = 154 \quad \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow r^2 = 154 \times \frac{7}{22} \times \frac{1}{4} = \frac{49}{4} \quad \Rightarrow r = \frac{7}{2}$$

$$\Rightarrow \text{Diameter} = 2r = 2 \times \frac{7}{2}$$

$\therefore$  The diameter of the sphere = 7cm

25. Ratio of their volumes

$$\Rightarrow \frac{4}{3} \pi R^3 : \frac{4}{3} \pi r^3$$

$$\Rightarrow \pi R^3 : \pi r^3$$

$$\Rightarrow R^3 : r^3$$

$$\Rightarrow 4^3 : 7^3 = 64 : 243$$

26. S.D. of first 21 natural numbers

$$= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}}$$

$$= \sqrt{\frac{440}{12}} = \sqrt{36.66} = 6.05$$

27. Sample Space (S)

$$= \{ HHH, HHT, HTH, THH, TTT, TTH, THT, HTT \}$$

$$n(S) = 8$$

Let A = probability of getting two consecutive tails

$$A = \{ HTT, TTH, TTT \}$$

$$\Rightarrow n(A) = 3 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

28.  $p^2 \times q^1 \times r^4 \times s^3 = 315000$   
 $= 3^2 \times 7^1 \times 5^4 \times 2^3$

$$\therefore p = 3, q = 7, r = 5, s = 2$$

**III ANSWER ANY 10 ONLY**

29. Given : A = { 1, 2, 3, 4, 5, 6, 7 }, B = { 2, 3, 5, 7 } and C = { 2, }

LHS : A x ( B - C )

$$(B - C) = \{ 2, 3, 5, 7 \} - \{ 2 \}$$

$$= \{ 3, 5, 7 \}$$

$$A \times (B - C) = \{ 1, 2, 3, 4, 5, 6, 7 \} \times \{ 3, 5, 7 \}$$

$$= (1,3), (2,3), (3,3), (4,3), (5,3), (6,3), (7,3), (1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (7,5), (1,7), (2,7), (3,7), (4,7), (5,7), (6,7), (7,7) \} \text{-----(1)}$$

RHS : ( A x B ) - ( A x C )

$$(A \times B) = \{ \{ 1, 2, 3, 4, 5, 6, 7 \} \times \{ 2, 3, 5, 7 \} \}$$

$$= \{ (1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5), (6,7), (7,2), (7,3), (7,5), (7,7) \}$$

$$(A \times C) = \{ 1, 2, 3, 4, 5, 6, 7 \} \times \{ 2 \}$$

$$= \{ (1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2) \}$$

$$(A \times B) - (A \times C) = \{ (1,3), (2,3), (3,3), (4,3), (5,3), (6,3), (7,3), (1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (7,5), (1,7), (2,7), (3,7), (4,7), (5,7), (6,7), (7,7) \} \text{-----(2)}$$

From (1) and (2) LHS = RHS

30. (i) If  $x = 1, f(1) = 3(1) + 2 = 5$   
 If  $x = 2, f(2) = 3(2) + 2 = 8$   
 If  $x = 3, f(3) = 3(3) + 2 = 11$

The images of 1, 2, 3, are 5, 8, 11 respectively.

If x is the pre image of 29, then  $f(x) = 29$

$$3x + 2 = 29 \Rightarrow x = 9$$

Similarly x is the pre image of 53, then  $f(x) = 53$

$$3x + 2 = 53 \Rightarrow x = 17$$

The pre - images of 29 and 53 are 9 and 17 respectively.

Since different elements of  $\mathbb{N}$  have different images in the co - domain, the function f is one - one function.

The co - domain f is  $\mathbb{N}$

But the range of  $f = \{ 5, 8, 11, 14, 17, \dots \}$  is a proper subset of  $\mathbb{N}$ .

$\therefore f$  is not an onto function. That is  $f$  is an into function. Thus  $f$  is one - one and into function.

31.  $S_1 = \frac{n}{2} ( 2a + ( n - 1 ) d )$   
 $S_2 = \frac{2n}{2} ( 2a + ( 2n - 1 ) d )$   
 $S_3 = \frac{3n}{2} ( 2a + ( 3n - 1 ) d )$   
 $3 ( S_2 - S_1 )$   
 $= 3 [ \frac{2n}{2} ( 2a + ( 2n - 1 ) d ) - \frac{n}{2} ( 2a + ( n - 1 ) d ) ]$   
 $= \frac{3n}{2} [ ( 4a + 2( 2n - 1 ) d ) - ( 2a + ( n - 1 ) d ) ]$   
 $= \frac{3n}{2} [ 2a + 3nd - d ]$   
 $= \frac{3n}{2} [ 2a + ( 3n - 1 ) d ]$

Therefore  $3 ( S_2 - S_1 ) = S_3$

32.  $3 + 33 + 3 + \dots, + n$  terms  
 $= 3 ( 1 + 11 + 111 + \dots + n \text{ terms} )$   
 $S_n = \frac{3}{9} ( 9 + 99 + 999 + \dots + n \text{ terms} )$   
 $= \frac{3}{9} ( ( 10 + 10^2 + 10^3 + \dots + n \text{ terms} ) - n )$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{1}{3} \left( \frac{10(10^n - 1)}{9} - n \right)$$

$$S_n = \frac{10(10^n - 1)}{27} - \frac{n}{3}$$

33. Let  $\alpha$  &  $\beta$  are the roots of the equation  $2y^2 - ay + 64 = 0$

$$\alpha = 2\beta \text{ (given) -----(1)}$$

$$\alpha + \beta = \frac{-b}{a} \text{ -----(2)}$$

$$= \frac{-(-a)}{2} = \frac{a}{2} \qquad \alpha\beta = \frac{c}{a} = \frac{64}{2}$$



$\alpha\beta = 32$

By (1)  $\Rightarrow 2\beta \times \beta = 32$

$2\beta^2 = 32$

$\beta = \sqrt{16} \quad \beta = \pm 4$

But  $\alpha = 2\beta$  (given)

$\Rightarrow \alpha = 2(4) \quad \Rightarrow \alpha = 8$

**If  $\alpha = 8$  &  $\beta = 4$**

$\Rightarrow \alpha + \beta = \frac{a}{2}$

$\Rightarrow 8 + 4 = \frac{a}{2} \quad \Rightarrow a = 24$

**If  $\alpha = 8$  &  $\beta = -4$**

$\Rightarrow \alpha + \beta = \frac{a}{2}$

$\Rightarrow 8 - 4 = \frac{a}{2} \quad \Rightarrow a = 8$

Therefore  $a = 8$  ( or )  $24$

34.  $x = \frac{a^2 + 3a - 4}{3a^2 - 3}$

$= \frac{(a+4)(a-1)}{3(a^2-1)} = \frac{(a+4)}{3(a+1)}$

$x^2 = \left(\frac{(a+4)}{3(a+1)}\right)^2 = \frac{(a+4)^2}{9(a+1)^2}$

$y = \frac{a^2 + 2a - 8}{2a^2 - 2a - 4}$

$= \frac{(a+4)(a-2)}{2(a^2-a-2)} = \frac{(a+4)}{2(a+1)}$

$y^2 = \left(\frac{(a+4)}{2(a+1)}\right)^2 = \frac{(a+4)^2}{4(a+1)^2}$

$y^{-2} = \frac{4(a+1)^2}{(a+4)^2}$

$x^2 y^{-2} = \frac{(a+4)^2}{9(a+1)^2} \times \frac{4(a+1)^2}{(a+4)^2}$

$x^2 y^{-2} = \frac{4}{9}$

35.  $A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \times \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$

$5A = \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} \quad 7I_2 = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

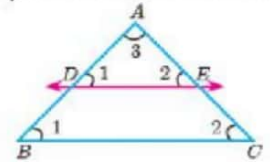
$A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

$A^2 - 5A + 7I_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

36. **Statement :** If a straight line is drawn parallel to one side of a triangle intersecting the other

two sides, then divides the sides in the same ratio.

**Given :** In a  $\Delta ABC$ , D is a point on AB and E is a point on AC



**To Prove :**  $\frac{AD}{DB} = \frac{AE}{EC}$

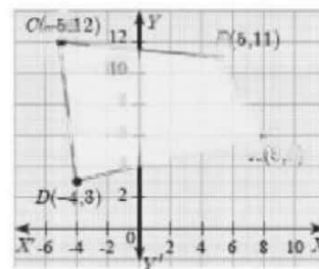
**Construction:**

Draw a line  $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$ $\Delta ABC \sim \Delta ADE$	Both triangles have a common angle By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	Split AB and AC using the points D and E.
4.	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
Hence proved		

37. Area of the quadrilateral

$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$  Sq. Units



Area of quadrilateral

$= \frac{1}{2} \begin{vmatrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 11 & 12 & 3 & 6 \end{vmatrix}$

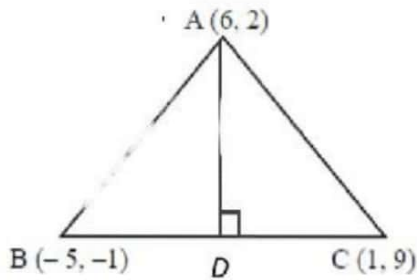
$$= \frac{1}{2} [(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)]$$

$$= \frac{1}{2} [(109) - (-49)] = \frac{1}{2} (109 + 49)$$

$$= \frac{1}{2} (158) = 79$$

Therefore area of quadrilateral = 79 sq. units.

38. /



Equation of Median AM

Here 'M' is the mid point of BC

Therefore  $x_1 = -5, x_2 = 1, y_1 = -1$  and  $y_2 = 9$

Therefore mid point of BC =  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right) = \left(\frac{-4}{2}, \frac{8}{2}\right) = (-2, 4)$$

Therefore equation of median AM is

A = (6, 2) and M = (-2, 4)

$$\Rightarrow \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\Rightarrow -8y + 16 = 2x - 12$$

$$\Rightarrow x + 4y - 14 = 0$$

39.  $\tan 30^\circ = \frac{1800}{BD}$

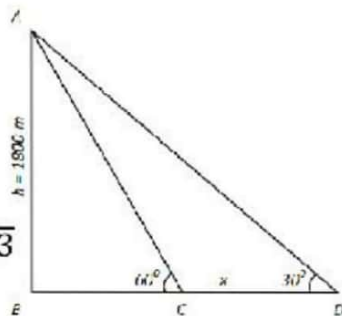
$$BD = 1800\sqrt{3}$$

$$\tan 60^\circ = \frac{1800}{BC}$$

$$BC = 600\sqrt{3}$$

$$DC = 1800\sqrt{3} - 600\sqrt{3}$$

$$DC = 1200\sqrt{3} = 2078.4m$$



40.  $\Rightarrow$  Volume of frustum

$$= \frac{1}{3} \pi h(R^2 + r^2 + Rr) \text{ Cubic units.}$$

$$= \frac{1}{3} \times \pi \times 45 (28^2 + 7^2 + 28 \times 7)$$

$$= \frac{1}{3} \times \pi \times 45 (784 + 49 + 196)$$

$$= \frac{1}{3} \times \pi \times 45 \times 1029$$

$$= 15435 \pi \text{ cm}^3 \text{ (or)}$$

$$= 15435 \times \frac{22}{7} = 48510 \text{ cm}^3$$

Therefore the volume of frustum = 48510 cm<sup>3</sup>

41. S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$$\Rightarrow n(S) = 36$$

Let A be the event of getting even number on the 1<sup>st</sup> die

- A = {(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$$\Rightarrow n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Let B be the probability of getting face sum 8

- B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$$\Rightarrow n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

- (A ∩ B) = {(2, 6), (4, 4), (6, 2)}

$$\Rightarrow n(A \cap B) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{5}{9}$$

42. Volume of cone =  $1005 \frac{5}{7}$  cu.cm

$$\frac{1}{3} \pi r^2 h = \frac{7040}{7} \text{ -----(1)}$$

Base area of cone =  $201 \frac{1}{7}$  sq.cm

$$\pi r^2 = \frac{1408}{7} \text{ -----(2)}$$

$$\frac{1}{3} \times \frac{1408}{7} \times h = \frac{7040}{7} \text{ (by (2))}$$

$$h = \frac{7040 \times 7 \times 3}{7 \times 1408} = 15 \text{ cm}$$

$$\pi r^2 = \frac{1408}{7} \Rightarrow \frac{22}{7} \times r^2 = \frac{1408}{7}$$

$$r = 8 \text{ cm}$$

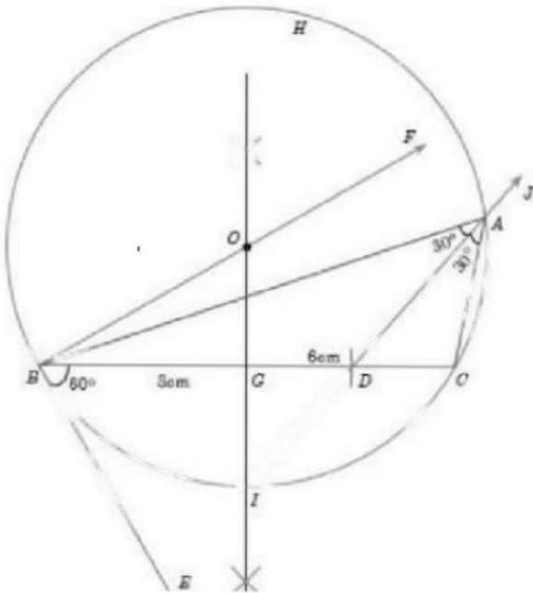
Slant height (l) =  $\sqrt{h^2 + r^2}$

$$\Rightarrow l = \sqrt{15^2 + 8^2} \Rightarrow l = \sqrt{225 + 64}$$

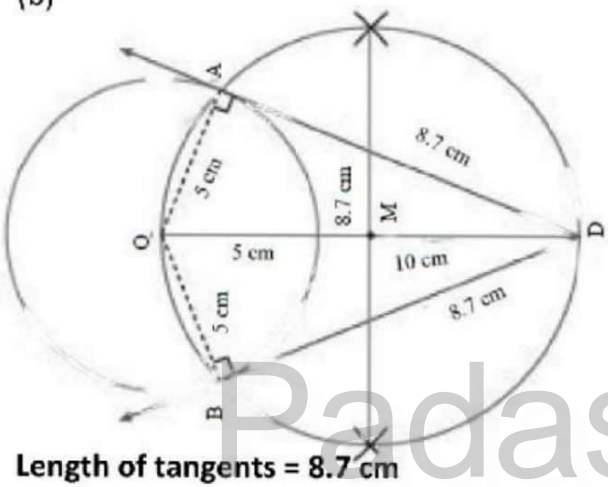
$$\Rightarrow l = \sqrt{289} \Rightarrow l = 17 \text{ cm}$$



43. (a)

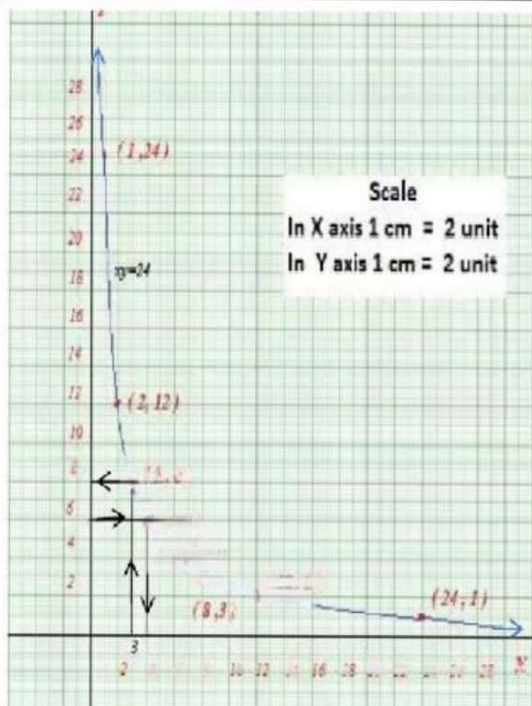


(b)



44. (a)

X	1	2	3	4	6	8	12	24
y	24	12	8	6	4	3	2	1



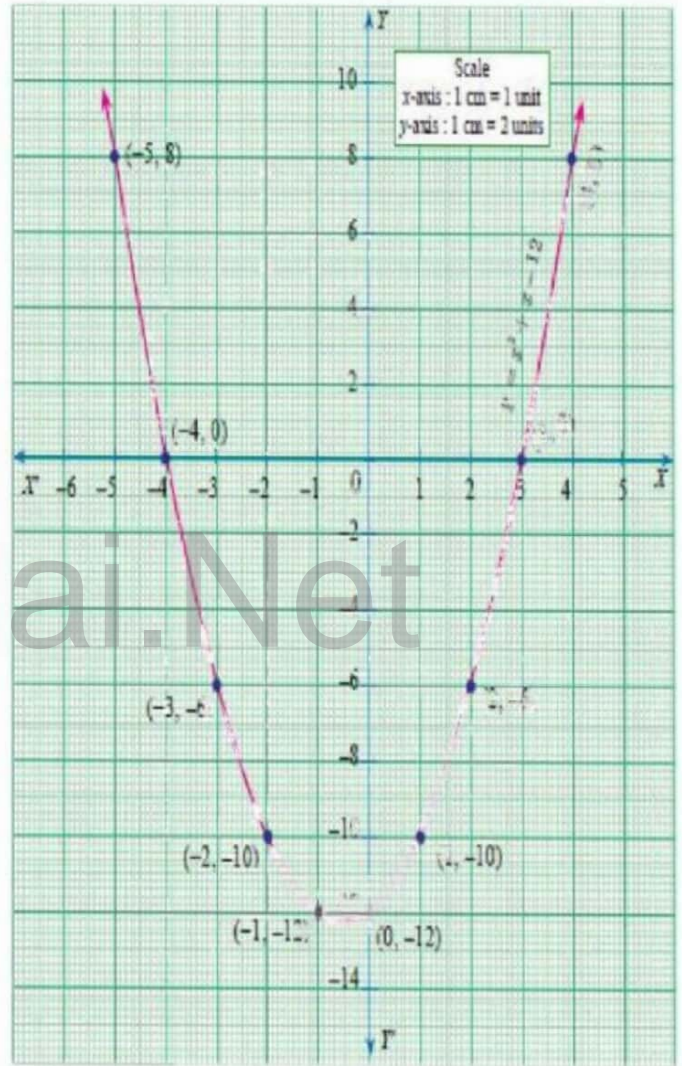
Solution: From the graph

(i) If  $x = 3$  then,  $y = 8$

(ii) If  $y = 6$  then,  $x = 4$

(b)

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
X <sup>2</sup>	25	16	9	4	1	0	1	4	9	16	25
X	-5	-4	-3	-2	-1	0	1	2	3	4	5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
Y	8	0	-6	-10	-12	-12	-10	-6	0	8	18



Solution set =  $\{-4, 3\}$

Therefore the roots are real and unequal.