1. Relations and Functions

1 mark Questions

1. If $n(A \times B) = 6$ and $A = \{1,3\}$ then $n(B)$ is SEP-21
   (A) 1 (B) 2 **(C) 3** (D) 6
2. $A = \{a, b, p\}, B = \{2,3\}, C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is PTA-3
   (A) 8 (B) 20 **(C) 12** (D) 16
3. If $A = \{1,2\}, B = \{1,2,3,4\}, C = \{5,6\}$ and $D = \{5,6,7,8\}$ then state which of the following statement is true SEP-20
   (A) $(\boldsymbol{A \times C}) \subset (\boldsymbol{B \times D})$ (B) $(B \times D) \subset (A \times C)$
   (C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$
4. If there are 1024 relations from a set $A = \{1,2,3,4,5\}$ to a set B, then the number of element in B is
   (A) 3 **(B) 2** (C) 4 (D) 8 PTA-2, JUL-22
5. The range of the relations $R = \{(x, x^2)|x$ is a prime number less than $13\}$ is PTA-4, JUL-22
   (A)$\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$ **(C) {4,9,25,49,121}** (D) $\{1,4,9,25,49,121\}$
6. If the ordered pairs $(a + 2,4)$ and $(5, 2a + b)$ are equal then (a, b) is PTA-6, MAY-22
   (A) $(2, -2)$ (B) $(5,1)$ (C) $(2,3)$ **(D)** $(\boldsymbol{3, -2})$
7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is
   (A) m^n (B) n^m **(C)** $\boldsymbol{2^{mn} - 1}$ (D) 2^{mn}
8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b respectively. PTA-1
   (A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)
9. Let $A = \{1,2,3,4\}$ and $B = \{4,8,9,10\}$. A function $f: A \to B$ given by $f = \{(1,4), (2,8), (3,9), (4,10)\}$ is a PTA-4
   (A) Many-one function (B) Identity function
   (C) One-to-one function (D) Into function
10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
    (A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ **(C)** $\boldsymbol{\frac{2}{9x^2}}$ (D) $\frac{1}{6x^2}$
11. If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to PTA-2
    (A) 7 (B) 49 (C) 1 (D) 14
12. Let f and g be two functions given by
    $f = \{(0,1), (2,0), (3, -4), (4,2), (5,7)\}$
    $g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ then the range of $f \circ g$ is
    (A) $\{0,2,3,4,5\}$ (B) $\{-4,1,0,2,7\}$ (C) $\{1,2,3,4,5\}$ **(D)** $\{\boldsymbol{0, 1, 2}\}$
13. Let $f(x) = \sqrt{1 + x^2}$ then
    (A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
    (C) $\boldsymbol{f(xy) \leq f(x).f(y)}$ (D) None of these
14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
    (A) $(-1,2)$ **(B)** $(\boldsymbol{2, -1})$ (C) $(-1, -2)$ (D) $(1,2)$ PTA-6
15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is
    (A)linear (B) cubic (C) reciprocal **(D) quadratic** PTA-5

2 mark Questions

1. **Find $A \times B, A \times A$ and $B \times A$ (iii) $A = \{m, n\}; B = \emptyset$** PTA-1

(iii) $A = \{m, n\}; B = \emptyset$

$A \times B = \{ \quad \}$

$A \times A = \{m, n\} \times \{m, n\}$

$= \{(m, m), (m, n), (n, m), (n, n)\}$

$B \times A = \{ \quad \}$

2. **Let $A = \{1, 2, 3\}$ and $B = \{x | x$ is a prime number less than 10$\}$. Find $A \times B$ and $B \times A$.** MAY-22

$A = \{1,2,3\}$

$B = \{x|x$ is a prime number less than $10\}$

$= \{2,3,5,7\}$

$A \times B = \{1,2,3\} \times \{2,3,5,7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3),$
$(2,5), (2,7), (3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{2,3,5,7\} \times \{1,2,3\}$

$= \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (5,1), (5,2), (5,3), (7,1), (7,2), (7,3)\}$

3. **Let $A = \{1, 2, 3, 4, \dots, 45\}$ and R be the relation defined as "is square of a number" on A.Write R as a subset of $A \times A$. Also, find the domain and range of R.** SEP-21

Given $A = \{1,2,3,4, \dots, 45\}$

$A \times A = \{(1,1), (1,2), (1,3), (1,4) \dots \dots (45,45)\}$

Then, R be the relation defined as is "square of a number" on A.

Hence, $R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$

So $R \subseteq A \times A$

The domain of $R = \{1, 2, 3, 4, 5, 6\}$

The range of $R = \{1, 4, 9, 16, 25, 36\}$

4. **A Relation R is given by the set $\{(x, y)/y = x + 3, \ x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range (PTA-5)**

$R = \{(x, y)/y = x + 3, x \in \{0,1,2,3,4,5\}\}$

Here domain $(x) = \{0,1,2,3,4,5\}$

Co-domain $(y) = x + 3$

$y_0 = 0 + 3 = 3$, $\quad y_3 = 3 + 3 = 6$

$y_1 = 1 + 3 = 4$, $\quad y_4 = 4 + 3 = 7$

$y_2 = 2 + 3 = 5$, $\quad y_5 = 5 + 3 = 8$

$R = \{(0,3), (1,4), (2,5), (3,6), (4,7), (5,8)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{3, 4, 5, 6, 7, 8\}$

5. **Show that the function $\boldsymbol{f:\mathbb{N}\to\mathbb{N}}$ defined by $\boldsymbol{f(m)=m^2+m+3}$ is one – one function** SEP-20

The function $f:\mathbb{N}\to\mathbb{N}$ defined by

$f(m)=m^2+m+3$

$m=1, f(1)=(1)^2+1+3=1+1+3=5$

$m=2, f(2)=(2)^2+2+3=4+2+3=9$

$m=3, f(3)=(3)^2+3+3=9+3+3=15$

$m=4, f(4)=(4)^2+4+3=16+4+3=23$

Since different elements of N have different images in the codomain the function of $\boldsymbol{f}$ **is one-one function.**

6. **Write the domain of the following real functions** PTA-6

i) $\boldsymbol{f(x)=\frac{2x+1}{x-9}}$ **iii)** $\boldsymbol{g(x)=\sqrt{x-2}}$

i) $f(x)=\frac{2x+1}{x-9}$

If $x=9$ then $f(-9)$ is not defined

Hence f is defined for all real numbers except at $x=9$.

So domain of $f=R-\{9\}$

iii) $g(x)=\sqrt{x-2}$

If $x\in(-\infty,2)$ $g(x)$ is not real

If $x\in[2,\ \infty)$ $g(x)$ is real

∴ the Domain is $[2,\infty)$

5 mark Questions

1. **Let $\boldsymbol{A=\{x\in\mathbb{W}|x<2\}, B=\{x\in\mathbb{N}|1<x\leq 4\}}$ and $\boldsymbol{C=\{3,5\}}$. Verify that**

(ii) $\boldsymbol{A\times(B\cap C)=(A\times B)\cap(A\times C)}$ SEP-21, PTA-5

LHS: $B\cap C=\{2,3,4\}\cap\{3,5\}=\{3\}$

$A\times(B\cap C)=\{0,1\}\times\{3\}$

$=\{(\mathbf{0},\mathbf{3}),(\mathbf{1},\mathbf{3})\}$ ....(1)

RHS: $A\times B=\{0,1\}\times\{2,3,4\}\ =\{(\mathbf{0},\mathbf{2}),(\mathbf{0},\mathbf{3}),(\mathbf{0},\mathbf{4}),(\mathbf{1},\mathbf{2}),(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{4})\}$

$A\times C=\{0,1\}\times\{3,5\}\ =\{(0,3),(0,5),(1,3),(1,5)\}$

$(A\times B)\cap(A\times C)\ =\{(0,2),(\mathbf{0},\mathbf{3}),(0,4),(1,2),(\mathbf{1},\mathbf{3}),(1,4)\}\cap\{(\mathbf{0},\mathbf{3}),(0,5),(\mathbf{1},\mathbf{3}),(1,5)\}$

$=\{(\mathbf{0},\mathbf{3}),(\mathbf{1},\mathbf{3})\}$ ....(2)

From (1) and (2),

$$\boldsymbol{A\times(B\cap C)=(A\times B)\cap(A\times C)}$$

2. **If $\boldsymbol{A=\{5,6\}, B=\{4,5,6\}, C=\{5,6,7\}}$, show that $\boldsymbol{A\times A=(B\times B)\cap(C\times C)}$.** **(JUL-22)**

$A\times A=(B\times B)\cap(C\times C)$

LHS: $A\times A=\{5,6\}\times\{5,6\}$

$=\{(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{6}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6})\}$ ……….(1)

RHS:

$B\times B=\{4,5,6\}\times\{4,5,6\}$

$=\{(4,4),(4,5),(4,6),(5,4),(\mathbf{5},\mathbf{5}),$

$(\mathbf{5},\mathbf{6}),(6,4),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6})\}$

$C\times C=\{5,6,7\}\times\{5,6,7\}=\{(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{6}),(5,7),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6}),(6,7),(7,5),(7,6),(7,7)\}$

$(B\times B)\cap(C\times C)=\{(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{6}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{6})\}$……….(2)

From (1) and (2), $\boldsymbol{A\times A=(B\times B)\cap(C\times C)}$

3. **Let $\boldsymbol{A=\{x\in\mathbb{W}|x<2\}, B=\{x\in\mathbb{N}|1<x\leq 4\}}$ and $\boldsymbol{C=\{3,5\}}$. Verify that**

(i) $\boldsymbol{A\times(B\cup C)=(A\times B)\cup(A\times C)}$ **(PTA-2)**

$A=\{x\in\mathbb{W}|x<2\}=\{0,1\},\quad B=\{x\in\mathbb{N}|1<x\leq 4\}=\{2,3,4\},\quad C=\{3,5\}$

LHS:

$B\cup C=\{2,3,4\}\cup\{3,5\}=\{2,3,4,5\}$

$A\times(B\cup C)=\{0,1\}\times\{2,3,4,5\}$

$=\{(\mathbf{0},\mathbf{2}),(\mathbf{0},\mathbf{3}),(\mathbf{0},\mathbf{4}),(\mathbf{0},\mathbf{5}),(\mathbf{1},\mathbf{2}),(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{4}),(\mathbf{1},\mathbf{5})\}$ ……..(1)

RHS:

$A\times B=\{0,1\}\times\{2,3,4\}=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A\times C=\{0,1\}\times\{3,5\}=\{(0,3),(0,5),(1,3),(1,5)\}$

$(A\times B)\cup(A\times C)=\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}\cup\{(0,3),(0,5),(1,3),(1,5)\}$

$=\{(\mathbf{0},\mathbf{2}),(\mathbf{0},\mathbf{3}),(\mathbf{0},\mathbf{4}),(\mathbf{0},\mathbf{5}),(\mathbf{1},\mathbf{2}),(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{4}),(\mathbf{1},\mathbf{5})\}$ ……….(2)

From (1) and (2), $\boldsymbol{A\times(B\cup C)=(A\times B)\cup(A\times C)}$

4. **Let $\boldsymbol{A}$ = The set of all natural numbers less than 8, $\boldsymbol{B}$ = The set of all prime numbers less than 8, $\boldsymbol{C}$ = The set of even prime number, Verify that (i) $(\boldsymbol{A} \cap \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \times \boldsymbol{C}) \cap (\boldsymbol{B} \times \boldsymbol{C})$ (SEP-20)**

A = The set of all natural numbers less than 8 = $\{1,2,3,4,5,6,7\}$

B = The set of all prime numbers less than 8 = $\{2,3,5,7\}$

C = The set of even prime number = $\{2\}$

(i) $(\boldsymbol{A} \cap \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \times \boldsymbol{C}) \cap (\boldsymbol{B} \times \boldsymbol{C})$

LHS: $A \cap B = \{1,2,3,4,5,6,7\} \cap \{2,3,5,7\}$

$= \{2,3,5,7\}$

$(A \cap B) \times C = \{2,3,5,7\} \times \{2\} = \{(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{2}),(\mathbf{5},\mathbf{2}),(\mathbf{7},\mathbf{2})\}$ ....(1)

RHS:

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{2,3,5,7\} \times \{2\} = \{(2,2),(3,2),(5,2),(7,2)\}$

$(A \times C) \cap (B \times C) = \{(\mathbf{2},\mathbf{2}),(\mathbf{3},\mathbf{2}),(\mathbf{5},\mathbf{2}),(\mathbf{7},\mathbf{2})\}$ ....(2)

From (1) and (2), $(\boldsymbol{A} \cap \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \times \boldsymbol{C}) \cap (\boldsymbol{B} \times \boldsymbol{C})$

5. **Let $\boldsymbol{A}$ = The set of all natural numbers less than 8, $\boldsymbol{B}$ = The set of all prime numbers less than 8, $\boldsymbol{C}$ = The set of even prime number, Verify that**

(ii) $\boldsymbol{A} \times (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) - (\boldsymbol{A} \times \boldsymbol{C})$ MAY-22

LHS: $B - C = \{\cancel{2},3,5,7\} - \{\cancel{2}\} = \{3,5,7\}$

$A \times (B - C) = \{1,2,3,4,5,6,7\} \times \{3,5,7\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),$
$(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$............(1)

RHS: $A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),$
$(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),(7,2),(7,3),(7,5),(7,7)\}$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\} = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C)$

$= \{\cancel{(1,2)},(1,3),(1,5),(1,7),\cancel{(2,2)},(2,3),(2,5),(2,7),\cancel{(3,2)},(3,3),(3,5),(3,7),\cancel{(4,2)},(4,3),$
$(4,5),(4,7),\cancel{(5,2)},(5,3),(5,5),(5,7),\cancel{(6,2)},(6,3),(6,5),(6,7),\cancel{(7,2)},(7,3),(7,5),(7,7)\}$
$- \{\cancel{(1,2)},\cancel{(2,2)},\cancel{(3,2)},\cancel{(4,2)},\cancel{(5,2)},\cancel{(6,2)},\cancel{(7,2)}\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),$
$(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$...........(2)

From (1) and (2) , $\boldsymbol{A} \times (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) - (\boldsymbol{A} \times \boldsymbol{C})$

6. **Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible. (ii) $\{(x, y) | y = x + 3, x, y$ are natural numbers $<10\}$ (JUL-22)**

(ii) $\{(x, y) | y = x + 3, x, y$ are natural numbers $<10\}$

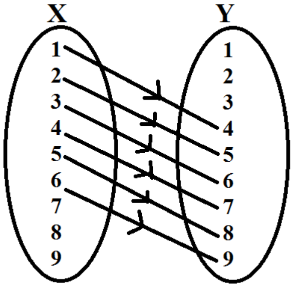
Given, x, y are natural numbers < 10

$X = \{1,2,3,4,5,6,7,8,9\}$, $y = x + 3$

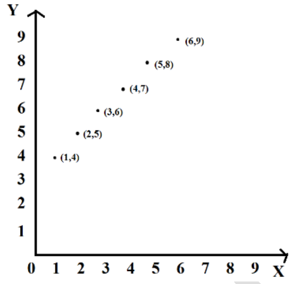
Here $y_1 = 4,\quad y_2 = 5,\quad y_3 = 6,$

$y_4 = 7,\quad y_5 = 8,\quad y_6 = 9$

(a) Arrow diagram



(b) graph



(c) Roster Form

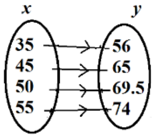
$R = \{(1,4), (2,5), (3,6),$
$(4,7), (5,8), (6,9)\}$

7. **The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants. (i) Check if this relation is a function. (ii) Find a and b (iii) Find the height of a person whose forehand length is 40cm (iv) Find the length of forehand of a person if her height is 53.3 inches.** PTA-4

Length x of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

Given $y = ax + b$

(i) Arrow diagram



Each element in x is associated with a unique element in y

Yes, this relation is a function

(ii) find a and b

From the table

$35a + \cancel{b} = 56$ ....(1)

$45a + \cancel{b} = 65$ ....(2)

$(-)\quad(-)\quad(-)$

$-10a = -9$

$a = \frac{9}{10} = 0.9$

$a = 0.9$ substitute in (1)

$35(0.9) + b = 56$

$31.5 + b = 56$

$b = 56 - 31.5 = 24.5$

$\boldsymbol{a = 0.9}$ **and** $\boldsymbol{b = 24.5}$

(iii) Length = 40cm, $a = 0.9, b = 24.5$

$y = ax + b$

$= (0.9)(40) + 24.5 \quad = 60.5$

The height of a person whose forehand length is 40 cm **= 60.5 inches.**

(iv) Height = 53.3 inches

$y = ax + b$

$53.3 = (0.9)x + 24.5 \quad = 0.9x + 24.5$

$53.3 - 24.5 = 0.9x$

$28.8 = 0.9x$

$x = \frac{28.8}{0.9} = 32 \Rightarrow x = 32$ cm

The length of forehand of a person = **32 cm**

8. **A function $f : [-5, 9] \to \mathbb{R}$ is defined as follows: $f(x) = \begin{cases} 6x+1; & -5 \le x < 2 \\ 5x^2-1; & 2 \le x < 6 \\ 3x-4; & 6 \le x \le 9 \end{cases}$** PTA-4

Find (ii) $f(7) - f(1)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$$f(x) = \begin{cases} 6x+1; & -5 \le x < 2 \text{ ; Where } x = -5,-4,-3,-2,-1,0,1 \\ 5x^2-1; & 2 \le x < 6 \text{ ; Where } x = 2,3,4,5 \\ 3x-4; & 6 \le x \le 9 \text{ ; Where } x = 6,7,8,9 \end{cases}$$

(ii) $f(7) - f(1)$

When $x = 7$

$f(x) = 3x - 4$

$f(7) = 3(7) - 4 = 21 - 4 = 17$

When $x = 1$

$f(x) = 6x + 1$

$f(1) = 6(1) + 1 = 6 + 1 = 7$

$\therefore f(7) - f(1) = 17 - 7 = \mathbf{10}$

(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

When $x = -2$, $f(x) = 6x + 1$

$f(-2) = 6(-2) + 1 = -12 + 1 = -11$

When $x = 6$, $f(x) = 3x - 4$

$f(6) = 3(6) - 4 = 18 - 4 = 14$

When $x = 4$, $f(x) = 5x^2 - 1$

$f(4) = 5(4)^2 - 1 = 80 - 1 = 79$

$\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{79-11} = \frac{-36}{68} = -\frac{\mathbf{9}}{\mathbf{17}}$

9. **The distance S an object travels under the influence of gravity in the time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Verify whether the function $S(t)$ is one-one or not.** PTA-3

Given $S(t) = \frac{1}{2}gt^2 + at + b$ (a, b constants)

Now take $t = 1,2,3, \ldots$ seconds

$t = 1$, $S(1) = \frac{1}{2}g(1)^2 + a(1) + b$

$= \frac{1}{2}g + a + b = \mathbf{0.5g + a + b}$

$t = 2$, $S(2) = \frac{1}{2}g(2)^2 + a(2) + b$

$= \mathbf{2g + 2a + b}$

$t = 3$, $S(3) = \frac{1}{2}g(3)^2 + a(3) + b$

$= \mathbf{4.5g + 3a + b}$

Since distinct elements of A have distinct image in B.

Yes, it is an one-one function.

10. **The function $'t'$ which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find**

PTA-1

(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C) = 212$

(v) the temperature when the Celsius value is equal to the Fahrenheit value

The function t is defined by, $t(C) = F$, where $F = \frac{9}{5}C + 32$

(i) $t(0) = \frac{9}{5}(0) + 32 = \mathbf{32°F}$

(ii) $t(28) = \frac{9}{5}(28) + 32$

$= 9(5.6) + 32$

$= 50.4 + 32$

$= \mathbf{82.4°F}$

(iii) $t(-10) = \frac{9}{5}(-10) + 32$

$= -18 + 32$

$= \mathbf{14°F}$

(iv) When $t(C) = 212$

$\frac{9}{5}C + 32 = 212$

$\frac{9}{5}C = 212 - 32 = 180$

$C = \frac{180\times5}{9} = \mathbf{100°C}$

(v) we know that

$t(C) = F$ where $F = \frac{9}{5}C + 32$

$t(F) = C$ where $C = \frac{9}{5}F + 32$

If the temperatures are same then two $'t'$s in the formula should represent the same temperature. So then we multiply each side by $\left(-\frac{5}{4}\right)$

$t = \frac{9}{5}t + 32°$

$t - \frac{9}{5}t = 32°$

Multiply each side by $\left(-\frac{5}{4}\right)$

$-\frac{5}{4}\left(t - \frac{9}{5}t\right) = 32° \times \left(-\frac{5}{4}\right)$

$-\frac{5}{4}t + \frac{9}{4}t = -40°$

$\frac{-5t+9t}{4} = -40°$

$\frac{4t}{4} = -40°$

$\boldsymbol{t = -40°}$

11. **If $f(x) = x^2 - 1, g(x) = x - 2$ find a, if $g \circ f(a) = 1$**

PTA-2

Given $f(x) = x^2 - 1, g(x) = x - 2$

$g \circ f(x) = g(f(x)) = g(x^2 - 1)$

$= x^2 - 1 - 2$

$= x^2 - 3$

Given $g \circ f(a) = 1$

Hence $a^2 - 3 = 1$

$a^2 = 1 + 3$

$a^2 = 4$

$\boldsymbol{a = \pm 2}$

12. **If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?** PTA-6

$f: R \to R$ defined by $f(x) = x^5$

$f \circ f(x) = f(f(x))$

$= f(x^5)$

$= (x^5)^5 = x^{25}$

$f \circ f(1) = (1)^{25} = 1$

$f \circ f(2) = (2)^{25}$

$f \circ f(3) = (3)^{25}$

Since each elements in f have distinct images, f is one-one

$g: R \to R$ defined by $g(x) = x^4$

$g \circ g(x) = g(g(x)) = g(x^4)$

$= (x^4)^4$

$= x^{16}$

$g \circ g(-1) = (-1)^{16} = 1$

$g \circ g(1) = (1)^{16} = 1$

$g \circ g(2) = (2)^{16}$

Thus two distinct elements -1

and 1 have same images.

Hence g is not one-one

$f \circ g(x) = f(g(x))$

$= f(x^4)$

$= (x^4)^5 = x^{20}$

$f \circ g(1) = (1)^{20} = 1$

$f \circ g(-1) = (-1)^{20} = 1$

Thus two distinct elements -1 and 1 have same

images. Hence **$f \circ g$ is not one-one.**

13. Consider the functions $\boldsymbol{f(x), g(x), h(x)}$ as given below, show that $(\boldsymbol{f \circ g) \circ h = f \circ (g \circ h)}$ in each case.

(iii) $\boldsymbol{f(x) = x - 4, g(x) = x^2}$ **and** $\boldsymbol{h(x) = 3x - 5}$

PTA-2

$f \circ g(x) = f(g(x))$

$= f(x^2) = x^2 - 4$

Then $(f \circ g) \circ h(x) = f \circ g(h(x))$

$= f \circ g(3x - 5)$

$= (3x - 5)^2 - 4$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$……..(1)

$(g \circ h)x = g(h(x))$

$= g(3x - 5) = (3x - 5)^2$

$= 9x^2 - 30x + 25$

$f \circ (g \circ h)(x) = f(9x^2 - 30x + 25)$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$ ………..(2)

From (1) and (2), $\boldsymbol{(f \circ g) \circ h = f \circ (g \circ h)}$

2. Numbers and Sequences

1 mark Questions

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
   (A) $1 < r < b$ (B) $0 < r < b$ **(C) $0 \leq r < b$** (D) $0 < r \leq b$
2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are PTA-5, SEP-20
   (A) 0, 1, 8 (B) 1, 4, 8 (C) 0, 1, 3 (D) 1, 3, 5
3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
   (A) 4 **(B) 2** (C) 1 (D) 3 MAY-22
4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
   (A)1 (B) 2 **(C) 3** (D) 4 SEP-21,PTA-4,JUL-22
5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
   (A) 2025 (B) 5220 (C) 5025 **(D) 2520**
6. $7^{4k} \equiv$ _____$(mod\ 100)$ PTA-1
   (A) 1 (B) 2 (C) 3 (D) 4
7. Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is SEP-21, MDL
   (A) 3 (B) 5 (C) 8 **(D) 11**
8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.
   (A) 4551 (B) 10091 **(C) 7881** (D) 13531
9. If 6 times of 6th term of an A.P is equal to 7 times the 7th term, then the 13th terms of the A.P is
   (A) 0 (B) 6 (C) 7 (D) 13 PTA-4
10. An A.P consists of 31 terms. It is 16th term is m, then the sum of all the terms of this A.P is
    (A) 16m (B) 62m **(C) 31m** (D) $\frac{31}{2}$ m PTA-5
11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120? MDL
    (A) 6 (B) 7 **(C) 8** (D) 9
12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \cdots + 2^0$ which of the following is true? PTA-6, SEP-20
    (A) B is 2^{64} more than A (B) A and B are equal
    (C) B is larger than A by 1 **(D) A is larger than B by 1**
13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots$ is PTA-2
    (A) $\frac{1}{24}$ **(B) $\frac{1}{27}$** (C) $\frac{2}{3}$ (D) $\frac{1}{81}$
14. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P then the sequence $t_6, t_{12}, t_{18}, \ldots$ is
    (A) a Geometric Progression
    (B) an Arithmetic Progression
    (C) neither an Arithmetic Progression nor a Geometric Progression
    (D) a constant sequence
15. The value of $(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \cdots + 15)$ is PTA-3
    (A) 14400 (B) 14200 **(C) 14280** (D) 14520

2 mark Questions

1. **A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.**

PTA-1

Given: A man has 532 flower pots.

Each row contains 21 flower pots.

Thus, Dividend $= 532$

Divisor $= 21$

By Euclid division lemma,

$532 = 21(25) + 7$

The quotient $= 25$, remainder$= 7$

$$\begin{array}{r|l} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

$a = bq + r\ , 0 \leq r < b$

∴ **25 rows** are completed and

7 flower pots are left over

2. **If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?**

SEP-20

Given: $m, n \in N$ and $2^n \times 5^m$

$n = 1, m = 1 \Rightarrow 2^1 \times 5^1 = 2 \times 5 = 10$

$n = 1, m = 2 \Rightarrow 2^1 \times 5^2 = 2 \times 25 = 50$

$n = 2, m = 3 \Rightarrow 2^2 \times 5^3 = 4 \times 125 \quad = 500$

∴ 2^n is always even.

So that, the product of 5 is in always end digit is 0.Hence, **No value** of $2^n \times 5^m$ end with the digit 5.

3. **If $13824 = 2^a \times 3^b$ then a and b.**

MAY-22

Given $13824 = 2^a \times 3^b$

The number 13824 can be factorized

As, $13824 = 2^9 \times 3^3$

Hence, $2^a \times 3^b = 2^9 \times 3^3$

∴ $a = 9$ and $b = 3$

$$\begin{array}{r|l} 3 & 13824 \\ \hline 3 & 4608 \\ \hline 3 & 1536 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

4. **Find the least number that is divisible by the first ten natural numbers.**

JUL-22

The first ten natural numbers are, 1,2,3,4,5,6,7,8,9,10.

Given: the number is divisible by first ten natural numbers.

Thus, LCM of 1,2,3,4,5,6,7,8,9, and 10

$= 1 \times 2^3 \times 3^2 \times 5 \times 7$

$= 8 \times 9 \times 35$

$= 2520$

∴ The least number is **2520**

5. If x is congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17 ?

Given: $x \equiv 13\ (mod 17)$ $[\text{If } a \equiv b (mod\ m) \text{ then } a \times c \equiv b \times c\ (mod\ m)]$ PTA-2

Multiply by 7

$$7x = 91\ (mod\ 17)$$

$$\begin{array}{r|l} & 5 \\ 17 & 88 \\ & 85 \\ & 3 \end{array}$$

$$7x - 3 \equiv 91 - 3\ (mod\ 17)$$

$$7x - 3 \equiv 88 (mod\ 17)$$

$$7x - 3 \equiv 3\ (mod\ 17) \qquad [\because 88 \equiv 3\ (mod\ 17)]$$

$\therefore 7x - 3$ is congruent to **3 modulo 17.**

6. Find the 19^{th} term of an $A.P.$ $-11, -15, -19, \ldots$ MDL, JUL-22

Given, $A.P$ is $-11, -15, -19, \ldots$

$$a = -11, d = t_2 - t_1 = -15 + 11$$

$$d = -4$$

n^{th} term of $A.P$ $t_n = a + (n-1)d$

$$n = 19 \quad \Rightarrow t_{19} = -11 + (19-1)(-4)$$

$$= -11 + 18(-4)$$

$$= -11 - 72$$

$$t_{19} = -83$$

7. Which term of an A. P. $16, 11, 6, 1, \ldots$ is -54 ? MAY-22

Given: $A.P$ is $16, 11, 6, 1, \ldots$

$$t_n = -54 \quad , \quad a = 16,$$

$$d = t_2 - t_1 = 11 - 16 = -5$$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \left(\frac{-54-16}{-5}\right) + 1 \quad = \left(\frac{-70}{-5}\right) + 1$$

$$n = 14 + 1 = 15$$

$$\therefore \boldsymbol{t_{15} = -54}$$

8. **Find the middle term(s) of an $A.P.$ $9, 15, 21, 27, \dots, 183$.**

PTA-1

Given: $A.P$ is $9,15,21,27,\dots 183$.

$a = 9, \quad d = t_2 - t_1 = 15 - 9$

$l = 183\,, \; d = 6$

The number of term in $A.P$.

$$n = \frac{l-a}{d} + 1$$

$$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1$$

$$= 29 + 1$$

$$n = 30$$

$n = 30$ even,

The middle term $= \frac{n^{th}}{2}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term

$= \frac{30}{2}$ term and $\frac{30}{2} + 1$ term

$= 15^{th}$ term and 16^{th} term

$t_n = a + (n-1)d$

$n = 15 \Rightarrow$

$t_{15} = 9 + (15-1)(6) = 9 + (14)(6)$

$t_{15} = 93$

$n = 16 \Rightarrow$

$t_{16} = 9 + (16-1)(6) = 9 + (15)(6)$

$t_{16} = 99$

$\therefore$ The middle terms are $\boldsymbol{t_{15} = 93}$, $\boldsymbol{t_{16} = 99}$

9. **If $3 + k, 18 - k, 5k + 1$ are in $A.P.$ then find k.**

SEP-21, PTA-3, 5

Given: $3 + k, 18 - k, 5k + 1$ are in $A.P$.

ie, $d = t_2 - t_1 = t_3 - t_2$

$18 - k - (3 + k) = 5k + 1 - (18 - k)$

$18 - k - 3 - k = 5k + 1 - 18 + k$

$15 - 2k = 6k - 17$

$15 + 17 = 6k + 2k$

$32 = 8k$

$k = 4$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each, successive row contains two additional seats than its front row. How many seats are there in the last row?

PTA-4

Given: 30 rows were allotted in the theatre

$n = 30$

20 seats in the front row then $a = 20$

2 seats increased in each row.

Thus, $1^{st}, 2^{nd}, 3^{rd}, \ldots$ 30 rows are 20,22,24, … respectively.

It is an $A.P.$ $d = t_2 - t_1 = 22 - 20 = 2$

To find: t_{30}

$t_n = a + (n-1)d$

$t_{30} = 20 + (30-1)2$

$= 20 + (29)(2)$

$= 20 + 58$

$t_{30} = 78$

78 seats in the last row.

5 mark Questions

1. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Given, 1230 and 1926 leaving remainder 12 in each case, when divided by largest number.

JUL-22

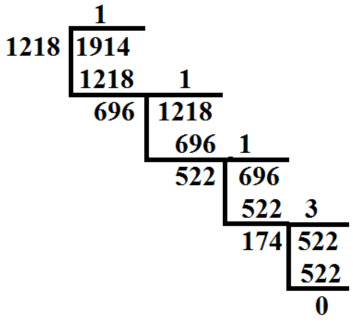
$1230 - 12 = 1218$ and $1926 - 12 = 1914$.

Let $a = 1914$ and $b = 1218$ $a > b$

By using Euclid's lemma, $a = bq + r, 0 \leq r < b$

$1914 = 1218(1) + 696$

The remainder $696 \neq 0$



$1218 = 696(1) + 522$

The remainder $522 \neq 0$

$696 = 522\ (1) + 174$

The remainder $174 \neq 0$

$522 = 174\ (3) + 0$

The remainder is 0

∴ The largest number is **174** which divides 1230 and 1926 and leaves remainder 12.

2. **If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.**

JUL-22

Given:

$$9t_9 = 15t_{15} \quad (\because t_n = a + (n-1)d)$$

$$9[a + (9-1)]d = 15[a + (15-1)d]$$

$$9(a + 8d) = 15\ (a + 14d)$$

$$9a + 72d = 15a + 210d$$

$$9a - 15a + 72d - 210d = 0$$

$$-6a - 138d = 0$$

$$-6(a + 23d) = 0$$

$$6[a + (24-1)d] = 0$$

$$6t_{24} = 0$$

Hence proved.

3. **The sum of three consecutive terms that are in $A.P.$ is 27 and their product is 288. Find the three terms.**

SEP-21

Let the three consecutive terms be

$a - d,\ a,\ a + d$

Given: $a - d + a + a + d = 27$

$$3a = 27$$

$$a = 9$$

Also, $(a-d)(a)(a+d) = 288$

$$(a^2 - d^2)a = 288$$

$$(9^2 - d^2) = \frac{288}{9}$$

$$81 - d^2 = 32$$

$$-d^2 = 32 - 81$$

$$-d^2 = -49$$

$$d = \pm 7$$

When $a = 9, d = 7$ the $A.P$ is

$9 - 7,\ 9,\ 9 + 7$

2, 9, 16

When $a = 9, d = -7$

$9 + 7,\ 9,\ 9 - 7$

16, 9, 2.

4. **The ratio of 6^{th} and 8^{th} term of an $A.P.$ is $7:9$. Find the ratio of 9^{th} term to 13^{th} term.**

MAY-22

Given: $t_6 : t_8 = 7:9 \Rightarrow \quad \frac{t_6}{t_8} = \frac{7}{9}$

$\frac{a+(6-1)d}{a+(8-1)d} = \frac{7}{9} \qquad [\because t_n = a + (n-1)d]$

$9(a + 5d) = 7(a + 7d)$

$9a + 45d = 7a + 49d$

$9a - 7a = 49d - 45d$

$2a = 4d$

$a = 2d$ ....(1)

To find, $\quad t_9 : t_{13} = \frac{t_9}{t_{13}}$

$= \frac{a+(9-1)d}{a+(13-1)d}$

$= \frac{a+8d}{a+12d}$

$= \frac{2d+8d}{2d+12d}$

$= \frac{10d}{14d} = \frac{5}{7}$

$\therefore \boldsymbol{t_9 : t_{13} = 5:7}$

5. **A man joined a company as Assistant Manager. The company gave him a starting salary of ₹60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?**

PTA-6

The starting salary of man is ₹ 60,000

His salary increased 5% annually.

P = 60000 , $r = 5\%, n = 5$ years

$A = P\left(1 + \frac{r}{100}\right)^n$

$= 60000\left(1 + \frac{5}{100}\right)^5$

$= 60000\left(\frac{21}{20}\right)^5$

$= 60000\left(\frac{21\times21\times21\times21\times21}{20\times20\times20\times20\times20}\right)$

$= \frac{12252303}{160}$

$= 76576.89$

$A =$ ₹ 76577

His salary will be after 5 years is ₹ 76577

6. **Find the sum of the Geometric series $3+6+12+\cdots+1536$** PTA-3

Given geometric series

$$3+6+12+\cdots+1536$$

$$a=3, r=\frac{t_2}{t_1}=\frac{6}{3}=2\,, l=1536$$

$$t_n=ar^{n-1} \qquad (n \text{ term is } 1536)$$

$$1536=3(2)^{n-1}$$

$$\frac{1536}{3}=2^{n-1}$$

$$512=2^{n-1}$$

$$2^9=2^{n-1}$$

$$9=n-1$$

$$n=10$$

$$S_n=a\left[\frac{r^n-1}{r-1}\right], r>1$$

$$S_{10}=3\left[\frac{2^{10}-1}{2-1}\right]=3(1024-1)=3(1023)$$

$$=\mathbf{3069}$$

7. **If $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots n$ terms then prove that $(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}\right]$** PTA-1

Given: $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots+n$ terms 5M

Multiply by $(x-y)$

$$(x-y)S_n=[(x-y)(x+y)+(x-y)(x^2+xy+y^2)+(x-y)(x^3+x^2y+xy^2+y^3)+\cdots+n$$

$$=[(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+\cdots n \text{ terms}]$$

$$(x-y)S_n=[(x^2+x^3+x^4+\cdots+n \text{ terms})-(y^2+y^3+y^4+\cdots n \text{ terms})]$$

$x^2+x^3+x^4+\cdots+n$ terms	$y^2+y^3+y^4+\cdots n$ terms
Here $a=x^2, r=\frac{x^3}{x^2}=x$	Here $a=y^2, r=\frac{y^3}{y}=y$

$$S_n=\frac{a(r^n-1)}{r-1}$$

$$(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}\right]-\left[\frac{y^2(y^n-1)}{y-1}\right]$$

$$(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}\right]$$

Hence proved.

8. Find the sum of the following series

PTA-5

(vi) $\mathbf{10^3 + 11^3 + 12^3 + \cdots + 20^3}$

$10^3 + 11^3 + 12^3 + \cdots + 20^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 20^3) - (1^3 + 2^3 + \cdots + 9^3)$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$= \left[\frac{20(20+1)}{2}\right]^2 - \left[\frac{9(9+1)}{2}\right]^2$

$= \left[\frac{20(21)}{2}\right]^2 - \left[\frac{9(10)}{2}\right]^2$

$= [10(21)]^2 - [9(5)]^2$

$= (210)^2 - (45)^2$

$= 44100 - 2025$

$= \mathbf{42075}$

9. **Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, … ,24cm. How much area can be decorated with these colour papers?**

PTA-1

Given: The size of 15 square colour papers are 10cm, 11cm, 12cm, … 24cm

The area of square $= a^2$

The colour paper decorated area

$= 10^2 + 11^2 + 12^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2) \quad -(1^2 + 2^2 + 3^2 + \cdots + 9^2)$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$= \frac{24(24+1)(24\times2+1)}{6} - \frac{9(9+1)(2\times9+1)}{6}$

$= 4(25)(49) - 3(5)(19)$

$= 4900 - 285$

$= \mathbf{4615}\ cm^2$

3. Algebra

1 Mark Question

1. A system of three linear equations in three variables is inconsistent if their planes PTA-1, JUL-22
(A) Intersect only at a point (B) intersect in a line
(C) Coincides with each other **(D) do not intersect**

2. The solution of the system $x + y - 3z = -6,\ -7y + 7z = 7,\quad 3z = 9$ is JUL-22
(A) $\boldsymbol{x = 1, y = 2, z = 3}$ (B) $x = -1, y = 2, z = 3$
(C) $x = -1, y = -2, z = 3$ (D) $x = 1, y = -2, z = 3$

3. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is PTA-4, MAY-22
(A) 3 **(B) 5** (C) 6 (D) 8

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is PTA-5
(A) $\boldsymbol{\frac{9y}{7}}$ (B) $\frac{9y^3}{21y-21}$ (C) $\frac{21y^2-42y+21}{3y^3}$ (D) $\frac{7(y^2-2y+1)}{y^2}$

5. $y^2 + \frac{1}{y^2}$ is not equal to PTA-6, JUL-22
(A) $\frac{y^4+1}{y^2}$ **(B) $\boldsymbol{\left(y + \frac{1}{y}\right)^2}$** (C) $\left(y - \frac{1}{y}\right)^2 + 2$ (D) $\left(y + \frac{1}{y}\right)^2 - 2$

6. $\frac{x}{x^2-25} - \frac{8}{x^2+6x+5}$ gives
(A) $\frac{x^2-7x+40}{(x-5)(x+5)}$ (B) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$ **(C) $\boldsymbol{\frac{x^2-7x+40}{(x^2-25)(x+1)}}$** (D) $\frac{x^2+10}{(x^2-25)(x+1)}$

7. The square root of $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to SEP-21
(A) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (B) $16\left|\frac{y^2}{x^2z^4}\right|$ (C) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ **(D) $\boldsymbol{\frac{16}{5}\left|\frac{xz^2}{y}\right|}$**

8. Which of the following should be added to make $x^4 + 64$ a perfect square MAY-22
(A) $4x^2$ **(B) $\boldsymbol{16x^2}$** (C) $8x^2$ (D) $-8x^2$

9. The solution of $(2x - 1)^2 = 9$ is equal to
(A) -1 (B) 2 **(C) $\boldsymbol{-1, 2}$** (D) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
(A) 100,120 (B) 10,12 **(C) – 120, 100** (D) 12,10

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in _______
(A) $A.P$ **(B) $\boldsymbol{G.P}$** (C) Both $A.P$ and $G.P$ (D) None of these

12. Graph of a linear equation is a SEP-21, PTA-2
(A) Straight line (B) circle (C) parabola (D) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is MAY-22
(A) 0 **(B) 1** (C) 0 or 1 (D) -2

14. For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is
(A) 2×3 (B) 3×2 (C) 3×4 **(D) $\boldsymbol{4 \times 3}$**

15. If A is 2×3 matrix and B is a 3×4 matrix, how many columns does AB have
(A) 3 **(B) 4** (C) 2 (D) 5

16. If number of columns and rows are not equal in a matrix then it is said to be a
(A) Diagonal matrix **(B) rectangular matrix**
(C) square matrix (D) Identity matrix

17. Transpose of a column matrix is SEP-20
(A) Unit matrix (B) diagonal matrix
(C) column matrix **(D) Row matrix**

18. Find the matrix X if $2X + \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 5 \end{bmatrix}$ PTA-6

(A) $\begin{bmatrix} -2 & -2 \\ 2 & -1 \end{bmatrix}$ **(B)** $\begin{bmatrix} \mathbf{2} & \mathbf{2} \\ \mathbf{2} & \mathbf{-1} \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

19. Which of the following can be calculated from the given matrices

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, (i) A^2 (ii) B^2 (iii) AB (iv) BA

(A) (i) and (ii) only (B) (ii) and (iii) only **(C) (ii) and (iv) only** (D) all of these

20. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$. Which of the following statements are correct?

(i) $AB + C = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$ (ii) $BC = \begin{bmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{bmatrix}$ (iii) $BA + C = \begin{bmatrix} 2 & 5 \\ 3 & 0 \end{bmatrix}$ (iv) $(AB)C = \begin{bmatrix} -8 & 20 \\ -8 & 13 \end{bmatrix}$

(A) (i) and (ii) only (B) (ii) and (iii) only (C) (ii) and (iv) only (D) all of these

2 Mark Questions

1. .Find the LCM of each pair of the following polynomials $a^2 + 4a - 12, a^2 - 5a + 6$ Whose GCD is $a - 2$ PTA-6

$$f(x) = a^2 + 4a - 12 = (a + 6)(a - 2)$$

$$g(x) = a^2 - 5a + 6 = (a - 3)(a - 2)$$

$$GCD: a - 2$$

$$LCM = \frac{f(x) \times g(x)}{GCD} = \frac{(a+6)\cancel{(a-2)} \times (a-3)(a-2)}{\cancel{(a-2)}}$$

$$LCM = \boldsymbol{(a + 6)(a - 3)(a - 2)}$$

2. If a polynomial $p(x) = x^2 - 5x - 14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$, find $q(x)$. PTA-2

$$\frac{p(x)}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{x^2-5x-14}{q(x)} = \frac{x-7}{x+2}$$

$$q(x) = \frac{x^2-5x-14}{x-7} \times x + 2$$

$$= \frac{\cancel{(x-7)}(x+2)}{\cancel{x-7}} \times (x + 2)$$

$$q(x) = (x + 2)(x + 2)$$

$$q(x) = x^2 + 4x + 4$$

3. **Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to get $\frac{3}{x^2-2x+4}$**

PTA-4

$$\frac{x^2+6x+8}{x^3+8} - p(x) = \frac{3}{x^2-2x+4}$$

$$\frac{x^2+6x+8}{x^3+2^3} - \frac{3}{x^2-2x+4} = p(x)$$

$$\frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)} - \frac{3}{x^2-2x+4} = p(x)$$

$$\frac{(x+4)}{(x^2-2x+4)} - \frac{3}{x^2-2x+4} = p(x)$$

$$\frac{x+4-3}{x^2-2x+4} = p(x)$$

$$p(x) = \boldsymbol{\frac{x+1}{x^2-2x+4}}$$

4. **Find the square root of the following rational expressions. $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$**

JUL-22

$$\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \left[\frac{4x^4y^{12}z^{16}}{x^8y^4z^4}\right]^{\frac{1}{2}}$$

$$= \left[\frac{4y^8z^{12}}{x^4}\right]^{\frac{1}{2}}$$

$$= \boldsymbol{2\left|\frac{y^4z^6}{x^2}\right|}$$

5. **Determine the quadratic equations, whose sum and product of roots are $-9, 20$**

SEP-21

$\alpha+\beta = -9\,, \alpha\beta = 20$

The general form

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$\boldsymbol{x^2+9x+20 = 0}$

6. **If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.**

PTA-6

First number $= x$, It's reciprocal $= \frac{1}{x}$

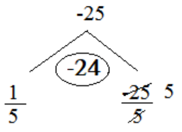
Difference $= \frac{24}{5}$

$x - \frac{1}{x} = \frac{24}{5}$

$\frac{x^2-1}{x} = \frac{24}{5}$

$5x^2 - 5 = 24x$

$5x^2 - 24x - 5 = 0$



$(5x+1)(x-5) = 0$

$\therefore x-5 = 0$	$5x+1 = 0$
$x = 5$	$5x = -1$
	$x = -\frac{1}{5}$

If the number is **5** and its reciprocal $\boldsymbol{\frac{1}{5}}$

If the number is $\boldsymbol{-\frac{1}{5}}$ and its reciprocal $\boldsymbol{-5}$

7. Determine the nature of the roots for the following quadratic equations

$15x^2 + 11x + 2 = 0$

SEP-21

Compare with $ax^2 + bx + c = 0$

$a = 15\,, b = 11\,, c = 2$

$\Delta = b^2 - 4ac$

$= 11^2 - 4(15)(2)$

$= 121 - 120$

$\Delta = 1,$ Here $\Delta > 0$

∴ The roots are **real and unequal**

8. If $A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ then find the transpose of A

PTA-2

Transpose of $A = A^T = \begin{bmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{bmatrix}$

9. If $A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$ then find the transpose of $-A$

SEP-20

$-A = \begin{bmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{bmatrix}$

Transpose of $-A = \begin{bmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{bmatrix}$

10. If $A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}, B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$ find the value of (i) $B - 5A$ (ii) $3A - 9B$

PTA-5

(i) $B - 5A$

$B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix},$

$5A = 5\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{bmatrix}$

$B - 5A = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{7} & \mathbf{-17} & \mathbf{-37} \\ \mathbf{-39} & \mathbf{-11} & \mathbf{-26} \end{bmatrix}$

(ii) $3A - 9B$

$3A = 3\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{bmatrix}$

$9B = 9\begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{bmatrix}$

$3A - 9B = \begin{bmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{bmatrix} - \begin{bmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{-63} & \mathbf{-15} & \mathbf{-45} \\ \mathbf{15} & \mathbf{-27} & \mathbf{-60} \end{bmatrix}$

11. If A is of order $p \times q$ and B is of order $q \times r$ what is the order of AB and BA?

PTA-1

Order of $A =$ $p \times q$

Order of $B =$ $q \times r$

Order of $AB =$ $\boldsymbol{p \times r}$

Order of BA is not defined because

Order of $B = q \times r$

Order of $A = p \times q$

Column of $B \neq$ row of A.

5 Mark Questions

1. Solve the following system of linear equations in three variables

(i) $x + y + z = 5; 2x - y + z = 9; x - 2y + 3z = 16$ SEP-21, PTA-5

$x + y + z = 5$ .... (1)

$2x - y + z = 9$ .... (2)

$x - 2y + 3z = 16$ .... (3)

Add (1) + (2)

$(1) \Rightarrow \ x + \cancel{y} + z = 5$

$(2) \Rightarrow 2x - \cancel{y} + z = 9$

$3x \quad + 2z = 14$ .... (4)

$(2) \times 2 \Rightarrow 4x - \cancel{2y} + 2z = 18$

$(3) \ \Rightarrow \ x - \cancel{2y} + 3z = 16$

$(-) \quad (+) \quad (-) \quad (-)$

$3x \quad - z = 2$ .... (5)

$(4) \Rightarrow \cancel{3x} + 2z = 14$

$(5) \Rightarrow \cancel{3x} - \ z = 2$

$(-) \quad (+) \quad (-)$

$3z = 12$

$3z = 12$

$z = \frac{12}{3} = 4$

Sub. $z = 4$ in (5)

$3x - z = 2$

$3x - 4 = 2$

$3x = 2 + 4$

$3x = 6 \Rightarrow x = \frac{6}{3} \Rightarrow x = 2$

Sub. $x = 2, z = 4$ in (1)

$x + y + z = 5$

$2 + y + 4 = 5$

$y = 5 - 2 - 4$

$y = 5 - 6$

$y = -1$

$\therefore \boldsymbol{x = 2, y = -1, z = 4}$

2. Vani, her father and her grand father have an average age of 53. One – half of her grandfather's age plus one-third of her father's age plus one fourth of vani's age is 65. Four years ago if vani's grandfather was four times as old as vani then how old are you they all now? PTA-2

Vani's age $= x$

Her father's age $= y$

Her grandfather's age $= z$

Average age $= 53 \Rightarrow \frac{x+y+z}{3} = 53$

$x + y + z = 53 \times 3$

$x + y + z = 159$...... (1)

Here $\frac{1}{2}z + \frac{1}{3}y + \frac{1}{4}x = 65$

$3x + 4y + 6z = 780$ …… (2)

Four years ago

Vani's age $= x - 4$

Her father's age $= y - 4$

Grandfather's age $= z - 4$

$$z - 4 = 4(x - 4)$$
$$z - 4 = 4x - 16$$
$$4x - z - 12 = 0$$
$$4x - z = 12 \text{ .... } (3)$$

$$\begin{array}{lrcrcrcr}
(1)\times 4 \Rightarrow & 4x & + & \cancel{4y} & + & 4z & = & 636 \\
(2) \Rightarrow & 3x & + & \cancel{4y} & + & 6z & = & 780 \\
 & (-) & & (-) & & (-) & & (-) \\
\hline
 & x & & & - & 2z & = & -144 \text{ .... } (4)
\end{array}$$

$$\begin{array}{lrcrcr}
(3)\times 2 \Rightarrow & 8x & - & \cancel{2z} & = & 24 \\
(4)\times 1 \Rightarrow & x & - & \cancel{2z} & = & -144 \\
 & (-) & & (+) & & (+) \\
\hline
 & 7x & & & = & 168 \\
\hline
\end{array}$$

$$x = \frac{168}{7} = 24$$

Sub. $x = 24$ in (5)

$$4x - z = 12$$
$$4(24) - z = 12$$
$$-z = 12 - 96$$
$$-z = -84$$
$$z = 84$$

Sub. $x = 24;\ z = 84$ in (1)

$$x + y + z = 159$$
$$24 + y + 84 = 159$$
$$y + 108 = 159$$
$$y = 159 - 108$$
$$y = 51$$

∴ Vani's age $= 24$

Her father's age $= 51$

Her grandfather's age $= 84$

3. Find the GCD of the given polynomials

(i) $f(x) = x^4 + 3x^3 - x - 3 \quad g(x) = x^3 + x^2 - 5x + 3$

SEP-20

$$\begin{array}{r|l}
 & \quad x + 2 \\
\hline
x^3 + x^2 - 5x + 3 & \cancel{x^4} + 3x^3 - 0x^2 - x - 3 \\
 & \cancel{x^4} + x^3 - 5x^2 + 3x \\
 & (-)\ (-)\ (+)\ (-) \\
\hline
 & \cancel{2x^3} + 5x^2 - 4x - 3 \\
 & \cancel{2x^3} + 2x^2 - 10x + 6 \\
 & (-)\ (-)\ (+)\ (-) \\
\hline
 & 3x^2 + 6x - 9 \\
\hline
 & 3[x^2 + 2x - 3]
\end{array}$$

$3[x^2 + 2x - 3] \neq 0$ here 3 is not a divisor of $g(x)$

$$\begin{array}{r|l}
 & \quad x - 1 \\
\hline
x^2 + 2x - 3 & \cancel{x^3} + x^2 - 5x + 3 \\
 & \cancel{x^3} + 2x^2 - 3x \\
 & (-)\ (-)\ (+) \\
\hline
 & \cancel{-x^2} - \cancel{2x} + \cancel{3} \\
 & \cancel{-x^2} - \cancel{2x} + \cancel{3} \\
 & (+)\ (+)\ (-) \\
\hline
 & 0
\end{array}$$

∴ GCD of $f(x)$ and $g(x)$ is $\mathbf{x^2 + 2x - 3}$

4. If $x = \frac{a^2+3a-4}{3a^2-3}$ and $y = \frac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2} PTA-3

$$x = \frac{a^2+3a-4}{3a^2-3}$$

$$= \frac{(a+4)(a-1)}{3(a^2-1^2)}$$

$$= \frac{(a+4)\cancel{(a-1)}}{3(a+1)\cancel{(a-1)}}$$

$$x = \frac{a+4}{3(a+1)}$$

$$y = \frac{a^2+2a-8}{2a^2-2a-4}$$

$$= \frac{a^2+2a-8}{2(a^2-a-2)} = \frac{(a+4)\cancel{(a-2)}}{2[\cancel{(a-2)}(a+1)]}$$

$$y = \frac{a+4}{2(a+1)}$$

$$x^2y^{-2} = \frac{x^2}{y^2} = \left(\frac{x}{y}\right)^2$$

$$\frac{x}{y} = \frac{\cancel{a+4}}{3\cancel{(a+1)}} \times \frac{2\cancel{(a+1)}}{\cancel{a+4}}$$

$$\frac{x}{y} = \frac{2}{3}$$

$$\left(\frac{x}{y}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\therefore x^2y^{-2} = \frac{\mathbf{4}}{\mathbf{9}}$$

5. Find the square root of the following polynomials by division method

(i) $x^4 - 12x^3 + 42x^2 - 36x + 9$ JUL-22

$$\begin{array}{r|lllll}
 & 1 & -6 & +3 & & \\
\hline
1 & \cancel{1} & -12 & +42 & -36 & +9 \\
 & \cancel{1} & & & & \\
 & (-) & & & & \\
\hline
2-6 & & -\cancel{12} & +42 & & \\
 & & -\cancel{12} & +36 & & \\
 & & (+) & (-) & & \\
\hline
2-12+3 & & & 6 & -\cancel{36} & +\cancel{9} \\
 & & & 6 & -\cancel{36} & +\cancel{9} \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & 0 &
\end{array}$$

$$\sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$$

6. Find the values of a and b if the following polynomials are perfect squares

(i) $4x^4 - 12x^3 + 37x^2 + bx + a$ PTA-4

$$\begin{array}{r|lllll}
 & 2 & -3 & +7 & & \\
\hline
2 & \cancel{4} & -12 & +37 & +b & +a \\
 & \cancel{4} & & & & \\
 & (-) & & & & \\
\hline
4-3 & & -\cancel{12} & +37 & & \\
 & & -\cancel{12} & +9 & & \\
 & & (+) & (-) & & \\
\hline
4-6+7 & & & \cancel{28} & +b & +a \\
 & & & \cancel{28} & -42 & +49 \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & & 0
\end{array}$$

5M

The given polynomial is perfect square then $a = 49, b = -42$

7. **Find the values of m and n if the following polynomials are perfect squares** MAY-22

(i) $36x^4 - 60x^3 + 61x^2 - mx + n$

$$\begin{array}{r|lllll}
 & 6 & -5 & +3 & & \\
\hline
6 & \cancel{36} & -60 & +61 & -m & +n \\
 & \cancel{36} & & & & \\
 & (-) & & & & \\
\hline
12\ -5 & & -\cancel{60} & +61 & & \\
 & & -\cancel{60} & +25 & & \\
 & & (+) & (-) & & \\
\hline
12-10+3 & & & \cancel{36} & -m & +n \\
 & & & \cancel{36} & -30 & 9 \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & 0 &
\end{array}$$

The given polynomial is perfect square $-m + 30 = 0 \Rightarrow -m = -30 \Rightarrow \boldsymbol{m = 30}$, $n - 9 = 0 \Rightarrow \boldsymbol{n = 9}$

8. **Solve the following quadratic equations by completing the square method** PTA-3

(ii) $\frac{5x+7}{x-1} = 3x + 2$

$$5x + 7 = (3x + 2)(x - 1)$$
$$5x + 7 = 3x^2 - 3x + 2x - 2$$
$$5x + 7 = 3x^2 - x - 2$$
$$3x^2 - 6x - 9 = 0$$
$$\div 3, \quad x^2 - 2x - 3 = 0$$
$$x^2 - 2x = 3$$
$$x^2 - 2(1)(x) = 3$$

Adding 1 on both side

$\left(-\frac{b}{2}\right)^2 = \left(\frac{-(-2)}{2}\right)^2$
Adding $(1)^2 = 1$

$$x^2 - 2(x) + 1 = 3 + 1$$
$$x^2 - 2(x) + 1 = 4$$
$$(x - 1)^2 = 2^2$$
$$(x - 1) = \pm 2$$

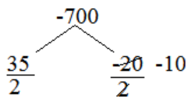
$x - 1 = +2$	$x - 1 = -2$
$x = 2 + 1$	$x = -2 + 1$
$x = 3$	$x = -1$

$\therefore \boldsymbol{x = \{3, -1\}}$

9. **A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.** PTA-4

Sister's age $= x$
Girl's age $= 2x$
After 5 years
Sister's age $= x + 5$
Girl's age $= 2x + 5$
Product $= 375$
$(x + 5)(2x + 5) = 375$
$2x^2 + 5x + 10x + 25 = 375$

$$2x^2 + 15x - 350 = 0$$
$$(2x + 35)(x - 10) = 0$$



$2x + 35 = 0$	
$2x = -35$	$x - 10 = 0$
$x = -\frac{35}{2}$	$x = 10$

$\therefore x = 10$ [Because x must be positive]
Sister's age $= x = \mathbf{10\ years}$
Girl's age $= 2x = 2(10) = \mathbf{20\ years}$

10.If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ are real and equal prove that either $a=0$ (or) $a^3+b^3+c^3=3abc$.

PTA-6

$\Delta = 0$

$a=c^2-ab, \quad b=-2(a^2-bc), \quad c=b^2-ac$

$$b^2-4ac=0$$

$$\begin{aligned} b^2-4ac &= \left(-2(a^2-bc)\right)^2-4(c^2-ab)(b^2-ac) \\ &= 4(a^4+b^2c^2-2a^2bc)-4(c^2b^2-ac^3-ab^3+a^2bc) \\ &= 4[a^4+b^2c^2-2a^2bc-c^2b^2+ac^3+ab^3-a^2bc] \\ &= 4[a^4+ac^3+ab^3-3a^2bc] \\ &= 4a[a^3+b^3+c^3-3abc] \end{aligned}$$

$4a(a^3+b^3+c^3-3abc)=0$

$\boxed{\because b^2-4ac=0}$

$4a=0 \quad | \quad a^3+b^3+c^3-3abc=0$

$\boldsymbol{a=0} \quad | \quad \boldsymbol{a^3+b^3+c^3=3abc}$

$\therefore$ Hence proved

11. If α, β are the roots of $7x^2+ax+2=0$ and if $\beta-\alpha=-\frac{13}{7}$. Find the values of a.

PTA-6, MAY-22

$$7x^2+ax+2=0$$

$$a=7, b=a, c=2$$

$$\alpha+\beta=\frac{-b}{a}=\frac{-a}{7}$$

$$\alpha\beta=\frac{c}{a}=\frac{2}{7}$$

$$(\beta-\alpha)=-\frac{13}{7}$$

Here $(\beta-\alpha)^2=\left(-\frac{13}{7}\right)^2$

$$\beta^2+\alpha^2-2\beta\alpha=\frac{169}{49}$$

$$(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta=\frac{169}{49}$$

$$(\alpha+\beta)^2-4\alpha\beta=\frac{169}{49}$$

$$\left(-\frac{a}{7}\right)^2-4\left(\frac{2}{7}\right)=\frac{169}{49}$$

$$\frac{a^2}{49}-\frac{8}{7}=\frac{169}{49}$$

$$\frac{a^2}{49}=\frac{169}{49}+\frac{8}{7}$$

$$\frac{a^2}{49}=\frac{169+56}{49}$$

$$a^2=\frac{225}{49}\times 49$$

$$a^2=225$$

$$a=\pm 15$$

$$\boldsymbol{a=15,-15}$$

12. Find the values of x, y, z if (ii) $\begin{pmatrix} x & y-z & z+3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$ PTA-5

$$\begin{pmatrix} x+y & y-z+4 & z+6 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$$

$$z+6=16$$
$$z=16-6$$
$$z=10$$

$$y-z+4=8$$
$$y-10=8-4$$
$$y-10=4$$
$$y=4+10$$
$$y=14$$

$$x+y=4$$
$$x+14=4$$
$$x=4-14$$
$$x=-10$$

$\therefore \boldsymbol{x=-10, y=14, z=10}$

13. Find the non-zero values of x satisfying the matrix equation PTA-4

$$\boldsymbol{x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}}$$

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

Comparing the elements

$$12x = 48$$

$$\Rightarrow x = \frac{48}{12}$$

$$\boldsymbol{x = 4}$$

12. If $A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$ verify that $(AB)^T = B^T A^T$ PTA-3

$$A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$$

LHS: $AB = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 30 \\ 43 & 3 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \text{............ (1)}$$

RHS: $A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}, \ B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}$ 5M

$$B^T A^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \text{... (2)}$$

$(1) = (2) \Rightarrow \boldsymbol{(AB)^T = B^T A^T}$, Hence proved.

4 . Geometry

1 mark Questions

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ **(C) $\angle B = \angle D$** (D) $\angle A = \angle F$

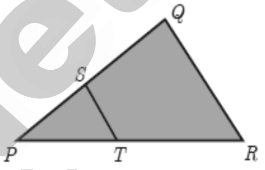
2. In ΔLMN, $\angle L = 60^\circ$, $\angle M = 50^\circ$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is SEP-20

(A) 40° **(B) 70°** (C) 30° (D) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90^\circ$ and $AC = 5\ cm$, then AB is PTA-4, MAY-22

(A) $2.5\ cm$ (B) $5\ cm$ (C) $10\ cm$ **(D) $5\sqrt{2}\ cm$**

4. In a given figure $ST \parallel QR$, $PS = 2\ cm$ and $SQ = 3\ cm$. Then the ratio of the area of ΔPQR to the area of ΔPST is



(A) 25 : 4 (B) 25 : 7

(C) 25 : 11 (D) 25 : 13

5. The perimeters of two similar triangles ΔABC and ΔPQR are $36\ cm$ and $24\ cm$ respectively. If $PQ = 10\ cm$, then the length of AB is PTA-5

(A) $6\frac{2}{3}cm$ (B) $\frac{10\sqrt{6}}{3}cm$ (C) $66\frac{2}{3}cm$ **(D) $15\ cm$**

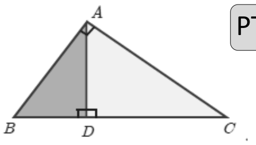
6. If in ΔABC, $DE \parallel BC$. $AB = 3.6\ cm$, $AC = 2.4\ cm$ and $AD = 2.1\ cm$ then the length of AE is

(A) $1.4\ cm$ (B) $1.8\ cm$ (C) $1.2\ cm$ (D) $1.05\ cm$ SEP-21, PTA-3, JUL-22

7. In a $\Delta\ ABC$, AD is the bisector of $\angle BAC$. If $AB = 8\ cm$, $BD = 6\ cm$ and $DC = 3\ cm$. The length of the side AC is PTA-6, MAY-22

(A) $6\ cm$ **(B) $4\ cm$** (C) $3\ cm$ (D) $8\ cm$

8. In the adjacent figure $\angle BAC = 90^\circ$ and $AD \perp BC$ then PTA-1



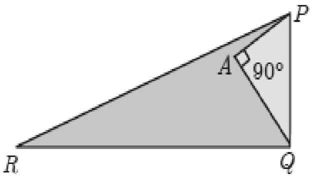
(A) $BD \cdot CD = BC^2$ (B) $AB \cdot AC = BC^2$

(C) $BD \cdot CD = AD^2$ (D) $AB \cdot AC = AD^2$

9. Two poles of heights $6\ m$ and $11\ m$ stand vertically on a plane ground. If the distance between their feet is $12\ m$, what is the distance between their tops?

(A) $13\ m$ (B) $14\ m$ (C) $15\ m$ (D) $12.8\ m$

10. In the given figure, $PR = 26\ cm$, $QR = 24\ cm$, $\angle PAQ = 90^\circ$, $PA = 6\ cm$ and $QA = 8\ cm$. Find $\angle PQR$ PTA-6



(A) 80° (B) 85°

(C) 75° **(D) 90°**

11. A tangent is perpendicular to the radius at the PTA-2

(A) centre **(B) point of contact** (C) infinity (D) chord

12. How many tangents can be drawn to the circle from an exterior point? SEP-21, JUL-22

(A) one **(B) two** (C) infinite (D) zero

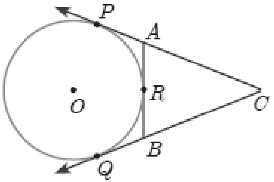
13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^{\circ}$ then the value of $\angle AOB$ is

(A) 100° **(B) 110°** (C) 120° (D) 130°

14. If figure CP amd CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11\ cm$ and $BC = 7\ cm$, then the length of BR is

MDL

(A) $6\ cm$ (B) $5\ cm$ (C) $8\ cm$ **(D) $4\ cm$**

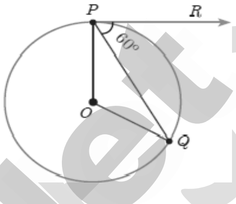


15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is

SEP-20

(A) 120° (B) 100°

(C) 110° (D) 90°



2 mark Questions

1. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE||BC$

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15cm$ find AE.

SEP-21

Given in $\Delta ABC, D$ and E are points an the sides AB & AC respectively such that $DE||BC$

$\therefore$ By Thales theorem, $\frac{AD}{DB} = \frac{AE}{EC}$

Let $EC = x$, $AE = 15 - x$

$$\frac{3}{4} = \frac{15-x}{x}$$

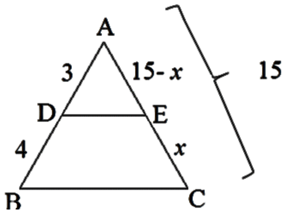
$$3x = 60 - 4x$$

$$3x + 4x = 60$$

$$7x = 60$$

$$x = \frac{60}{7} = 8.57$$

$AE = 15 - 8.57 = \mathbf{6.43cm}$



(ii) If $AD = 8x - 7, DB = 5x - 3, AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

By Thales theorem, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$$(8x - 7)(3x - 1) = (4x - 3)(5x - 3)$$

$$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$$

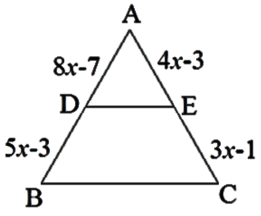
$$4x^2 - 2x - 2 = 0$$

$$\div 2 \Rightarrow \quad 2x^2 - x - 1 = 0$$

$$(x - 1)(2x + 1) = 0$$

$x = 1$ (or) $x = -\frac{1}{2} \Rightarrow \boldsymbol{x = 1}$

Since $x \neq -\frac{1}{2}$



2. **ABCD is a trapezium which $AB||DC$ and P, Q are points on AD and BC respectively, such that $PQ||DC$ if $PD = 18cm, BQ = 35cm$ and $QC = 15cm$, and find AD.** JUL-22

In trapezium $ABCD, AB \parallel CD \parallel PQ$

Join AC, meets PQ at R

In $\Delta ACD, PR \parallel CD$

By BPT, $\frac{AP}{PD} = \frac{AR}{RC}$

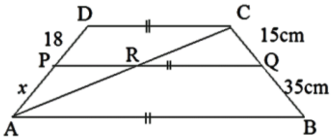
$\frac{x}{18} = \frac{AR}{RC}$ ....(1)

In $\Delta ABC, RQ \parallel AB$

By BPT, $\frac{BQ}{QC} = \frac{AR}{RC}$

$\frac{35}{15} = \frac{AR}{RC}$

$\frac{7}{3} = \frac{AR}{RC}$ ....(2)



From (1) and (2), $\frac{x}{18} = \frac{7}{3}$

$3x = 126$

$x = \frac{126}{3} = 42$

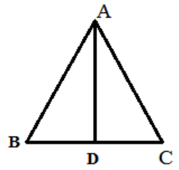
If $AP = x$

$AP = 42$

$AD = AP + PD = 42 + 18 = \mathbf{60\ cm}$

3. **Check whether $AD$is bisector of $\angle A$ of ΔABC in each of the following**

(i) $AB = 5cm, AC = 10cm, BD = 1.5cm$ and $CD = 3.5cm$ SEP-20



Given: In the ΔABC,

$\frac{AB}{AC} = \frac{5}{10}$

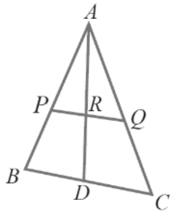
$\frac{AB}{AC} = \frac{1}{2}$..................... (1)

$\frac{BD}{DC} = \frac{1.5}{3.5}$

$\frac{BD}{DC} = \frac{15}{35}$

$\frac{BD}{DC} = \frac{3}{7}$ .... (2)

(1) & (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{DC}$

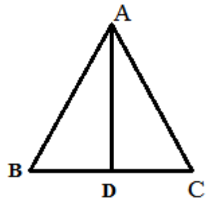


$\therefore AD$ is **not an angle bisector** of $\angle A$

(ii) $AB = 4cm,\ AC = 6cm, BD = 1.6cm$ and $CD = 2.4cm$

$\frac{AB}{AC} = \frac{4}{6}$

$\frac{AB}{AC} = \frac{2}{3}$............... (1)



$\frac{BD}{CD} = \frac{1.6}{2.4}$

$\frac{BD}{CD} = \frac{16}{24}$

$\frac{BD}{CD} = \frac{2}{3}$............... (2)

(1)& (2) $\frac{AB}{AC} = \frac{BD}{CD}$

$\Rightarrow AD$ is the angle **bisector** of $\angle A$

4. **A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?** JUL-22

Given, $BC = 18\text{m}, BA = 24\text{m}$

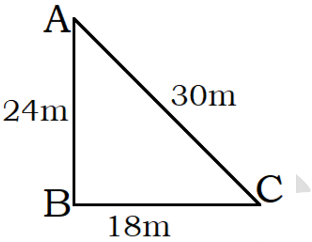
By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$$= 24^2 + 18^2$$

$$= 576 + 324$$

$$AC^2 = 900 = 30^2$$

$$\boldsymbol{AC = 30m}$$



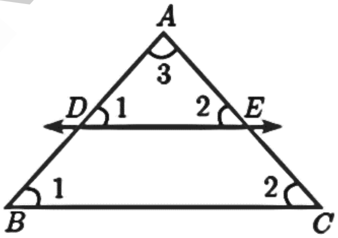
5 mark Questions

Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem

MAY-22

Statement: A straight line drawn parallel to a side of triangle intersectin the other two sides, divides the sides in the same ratio.

Proof:



Given: In ΔABC, D is a point on AB and E is a point on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \backsim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	By AAA similarity Corresponding sides are proportional Split AB and AC using the points D and E On simplification Cancelling 1 on both sides Taking reciprocals
Hence proved		

Corollary: If in ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

2. Theorem 3: Angle Bisector Theorem

PTA-5,SEP-20, JUL-22

Statement: The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Proof:

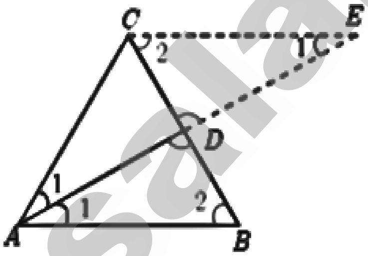
Given : In ΔABC, AD is the internal bisector

To prove:

$$\frac{AB}{AC} = \frac{BD}{CD}$$

Construction :

Draw a line through C parallel to AB. Extend AD to meet line through C at E

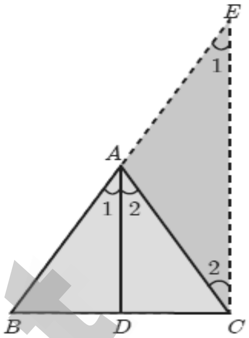


No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles $AC = CE$ …… (1)	In ΔACE, $\angle CAE = \Delta CEA$
3.	$\Delta ABD \backsim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved.

3. Theorem 4: Converse of Angle Bisector Theorem

PTA-3, 4

Statement: If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.



Proof:

Given : ABC is a triangle.

AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

That is $\frac{AB}{AC} = \frac{BD}{DC}$ .... (1)

To prove : AD bisects $\angle A$ i.e. $\angle 1 = \angle 2$

Construction : Draw $CE \parallel DA$. Extend BA to meet at E.

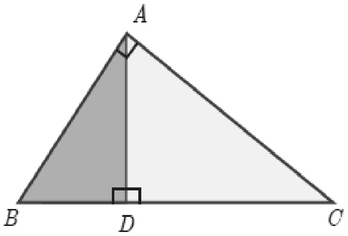
No.	Statement	Reason
1.	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC = \angle 1$	Since $DA \parallel CE$ and AC is transversal, corresponding angles are equal
3.	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$ and AC is transversal, Alternate angles are equal
4.	$\frac{BA}{AE} = \frac{BD}{DC}$ .... (2)	In ΔBCE by thales theorem
5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7.	$AC = AE$ ....(3)	Cancelling AB
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9.	AD bisects $\angle A$	Since, $\angle 1 = \angle BAD = \angle 2 = \angle DAC$. Hence proved

4. Theorem 5: Pythagoras Theorem

SEP-21, PTA-4

Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof:



Given : In $\Delta ABC,\ \angle A = 90°$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \backsim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ...(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \backsim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle ADC = 90°$ By AA similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = (BC \times BD) + (BC \times DC)$$
$$= BC \times (BD + DC)$$
$$= BC \times BC$$
$$AB^2 + AC^2 = BC^2$$

Hence the theorem is proved.

Converse of Pythagoras Theorem

Statement: If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is a right angle triangle.

5. Two triangle QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.

PTA-6

In ΔPQR and ΔSQR

$\angle P = \angle S = 90^\circ$ and ΔSQR

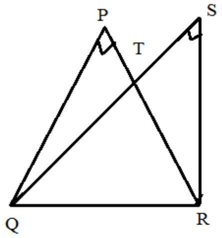
$\angle P = \angle S = 90^\circ$

And $\angle PTQ = \angle STR$ (vertically opposite angles)

Thus by AA criterion of similarity we have $\Delta PTQ \sim \Delta STR$

$$\frac{PT}{ST} = \frac{TQ}{TR}$$

$\Rightarrow PT \times TR = TQ \times ST$



6. Two vertical poles of heights $6m$ and $3m$ are erected above a horizontal ground AC. Find the value of y.

PTA-5

In $\Delta PAC, \Delta QBC$ are similar triangles

$$\frac{PA}{QB} = \frac{AC}{BC} = \frac{PQ}{QC}$$

$$\frac{6}{y} = \frac{AC}{BC}$$

$y(AC) = 6BC$ .... (1)

ΔACR and ΔABQ are similar triangles

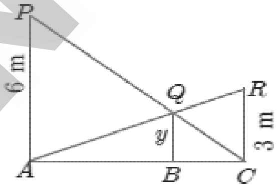
$$\frac{CR}{QB} = \frac{AC}{AB}$$

$$\frac{3}{y} = \frac{AC}{AB}$$

$3(AB) = (AC)y$ .... (2)

(1) & (2) $\Rightarrow 3AB = 6BC$

$$\frac{AB}{BC} = \frac{6}{3} = 2$$



$AB = 2BC$

$AC = AB + BC$

$AC = 2BC + BC$ $\quad (AB = 2BC)$

$AC = 3BC$

Substitute $AC = 3BC$ in (1) we get

$(3BC)y = 6BC$

$$y = \frac{6BC}{3BC}$$

$$\boldsymbol{y = 2\ m}$$

7. In figure $\angle QPR = 90^\circ$, PS is its bisector. If $ST \perp PR$, prove that $ST \times (PQ + PR) = PQ \times PR$.

PTA-2

Given: In the figure $\angle QPR = 90^\circ$,

PS is its bisector and $ST \perp PR$

$$\frac{PQ}{PR} = \frac{QS}{SR}$$ By Angle bisector theorem

$\frac{PQ}{PR} + 1 = \frac{QS}{SR} + 1$ Add 1 both side

$$\frac{PQ+PR}{PR} = \frac{QS+SR}{SR}$$

$\frac{PQ+PR}{PR} = \frac{QR}{SR}$............. (1)

In ΔPQR and ΔSTR

$\angle QPR = 90^0$, $\angle STR = 90^\circ$

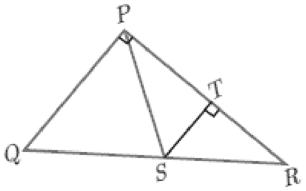
$\angle PRS = \angle TRS = \angle R$ is common ,

By AA similarity

$\therefore \frac{PQ}{ST} = \frac{QR}{SR} = \frac{PR}{TR}$ .... (2)

(1) & (2) $\Rightarrow \frac{PQ+PR}{PR} = \frac{PQ}{ST}$

$ST\ (PQ + PR) = PQ \times PR$. Hence proved



8. **The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.** PTA-3

In $\Delta ABC; \angle B = 90^0$

Let $AB = x \Rightarrow AC = 2x + 6$ *and*

$BC = 2x + 4$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16x + 16$

$x^2 + 16x - 24x + 16 - 36 = 0$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

$x = 10$ (or) $x = -2$

But $x \neq -2$

If $x = 10$

$\Rightarrow AC = 2x + 6$

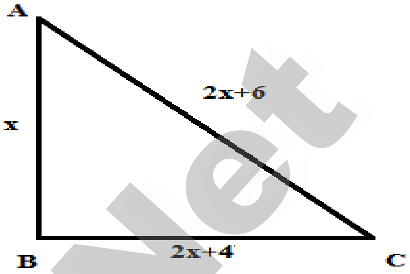
$= 20 + 6 = 26$

$\Rightarrow BC = 2x + 4$

$= 20 + 4 = 24$

$\therefore$ The sides are $\boldsymbol{AB = 10m}$;

$\boldsymbol{BC = 24m}$; $\boldsymbol{AC = 26m}$.



9. **The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that $QS = 3SR$. Prove that $2PQ^2 = 2PR^2 + QR^2$**

Given the ΔPQR, the perpendicular on the base QR at S, such that $QS = 3SR$

In $\Delta PQS \Rightarrow PQ^2 = PS^2 + QS^2$

$\Delta PSR \Rightarrow PR^2 = PS^2 + SR^2$

$\Rightarrow PS^2 = PR^2 - SR^2$

$QR = QS + SR$

$= 3SR + SR$

$QR = 4SR$

$\frac{QR}{4} = SR$

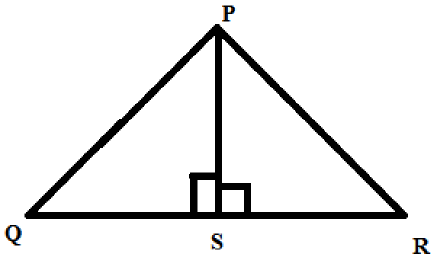
$PQ^2 = PR^2 - SR^2 + (3SR)^2$

$PQ^2 = PR^2 - SR^2 + 9SR^2$

$PQ^2 = PR^2 + 8SR^2$

$PQ^2 = PR^2 + \frac{8QR^2}{16}$

$\Rightarrow \boldsymbol{2PQ^2 = 2PR^2 + QR^2}$

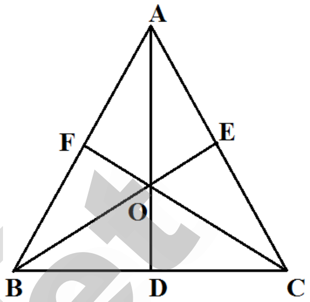


10. Show that the angle bisectors of a triangle are concurrent.

PTA-4

In the ΔABC, "O" is any point inside the Δ

The angle bisector $\angle AOB, \angle BOC,$ and $\angle AOC$ meet the sides AB, BC & CA at D, E & F respectively.



$\therefore$ In $\Delta BOC, OD$ is the bisector of $\angle BOC$

$\therefore \frac{OB}{OC} = \frac{BD}{DC}$ .... (1)

Similarly in the triangle AOC & AOB we get

$\frac{OC}{OA} = \frac{CE}{AE}$ .... (2)

$\frac{OA}{OB} = \frac{AF}{FB}$ .... (3)

$(1) \times (2) \times (3) \Rightarrow \frac{OB}{OC} \times \frac{OC}{OA} \times \frac{OA}{OB} = \frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{EB}$

$\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1$ .... (4)

If AD, BE & CF are the bisectors of $\angle A, \angle B$ & $\angle C$ then by ABT

$\frac{AB}{AC} = \frac{BD}{DC}; \frac{BC}{CA} = \frac{AF}{FB}; \frac{AB}{BC} = \frac{AE}{EC}$

$\frac{AB}{AC} \times \frac{BC}{CA} \times \frac{AB}{BC} = \frac{BD}{DC} \times \frac{AF}{FB} \times \frac{AE}{EC}$

$1 = 1$ (By (4))

$\therefore$ O is the point of concurrent.

The angle bisectors of a triangle concurrent

5. Coordinate Geometry

1 mark Questions

1. The area of triangle formed by the points $(-5,0), (0,-5)$ and $(5,0)$ is SEP-21,PTA-2
(A) 0 sq. units **(B) 25 sq. units** (C) 5 sq. units (D) none of these

2. A man walks near a wall, such that the distance between him and the wall is 10 units consider the wall to be the Y axis. The path travelled by the man
(A) $x = 10$ (B) $y = 10$ (C) $x = 0$ (D) $y = 10$

3. The straight line given by the equation $x = 11$ is PTA-1, SEP-20
(A) Parallel to X axis **(B) parallel to Y axis**
(C) passing through the origin (D) passing through the point $(0,11)$

4. If $(5,7), (3,p)$ and $(6,6)$ are collinear then the value of p is PTA-5, MAY-22
(A) 3 (B) 6 **(C) 9** (D) 12

5. The point of intersection $3x - y = 4$ and $x + y = 8$ is PTA-2, JUL-22
(A) $(5,3)$ (B) $(2,4)$ **(C) $(3,5)$** (D) $(4,4)$

6. The slope of the line joining $(12,3)$ and $(4,a)$ is $\frac{1}{8}$ the value of $'a'$ is PTA-3
(A) 1 (B) 4 (C) -5 **(D) 2**

7. The slope of the line which is perpendicular to line joining the points $(0,0)$ and $(-8,8)$ is MAY-22
(A) -1 **(B) 1** (C) $\frac{1}{3}$ (D) -8

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of the perpendicular bisector of PQ is PTA-6, JUL-22
(A) $\sqrt{3}$ **(B) $-\sqrt{3}$** (C) $\frac{1}{\sqrt{3}}$ (D) 0

9. If A is a point on the $y -$ axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is
(A) $8x + 5y = 40$ (B) $8x - 5y = 40$ (C) $x = 8$ (D) $y = 5$

10. The equation of the line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ PTA-4
(A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$ **(C) $3x + 7y = 0$** (D) $7x - 3y = 0$

11. Consider four straight lines
(i) $l_1: 3y = 4x + 5$ (ii) $l_2: 4y = 3x - 1$ (iii) $l_3: 4y + 3x = 7$ (iv) $l_4: 4x + 3y = 2$
Which of the following statement is true
(A) l_1 and l_2 are perpendicular (B) l_1 and l_4 are parallel
(C) l_2 and l_4 are perpendicular (D) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21$ which of the following is true. PTA-3
(A) The slope is 0.5 and the y intercept is 2.6 (B) The slope is 5 and the y intercept is 1.6
(C) The slope is 0.5 and the y intercept is 1.6 (D) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium it is necessary to show PTA-4
(A) Two sides are parallel **(B) Two parallel and two non- parallel sides**
(C) Opposite sides are parallel (D) All sides are of equal length

14. When proving that a quadrilateral is a parallelogram by using slopes you must find
(A) The slopes of two sides **(B) The slopes of two pair of opposite sides**
(C) The length of all sides (D) Both the length and slopes of two sides

15. $(2,1)$ is the point of intersection of two lines
(A) $x - y - 3 = 0, 3x - y - 7 = 0$ **(B) $x + y = 3,\ 3x + y = 7$**
(C) $3x + y = 3,\ x + y = 7$ (D) $x + 3y - 3 = 0, x - y - 7 = 0$

2 mark Questions

1. **What is the inclination of a line whose slope is**

PTA-3

(i) 0

$m = 0$

$\tan\theta = 0$

Angle of inclination is $\mathbf{0}^{\circ}$

(ii) 1

Slope $m = 1$

$\tan\theta = 1 \Rightarrow \quad \theta = 45^{\circ}$ $(\because \tan 45^{\circ} = 1)$

Angle of inclination is 45°.

2. **Find the slope of a line joining the points**

(ii) $(\sin\boldsymbol{\theta}, -\cos\boldsymbol{\theta})$ and $(-\sin\boldsymbol{\theta}, \cos\boldsymbol{\theta})$

PTA-2

$(x_1, y_1) = (\sin\theta, -\cos\theta)$
$(x_2, y_2) = (-\sin\theta, \cos\theta)$

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\cos\theta - (-\cos\theta)}{-\sin\theta - \sin\theta}$$

$$= \frac{2\cos\theta}{-2\,sin\,\theta}$$

$$= \frac{-\cos\theta}{\sin\theta} \quad = -\cot\theta$$

$$\boldsymbol{m = -\cot\theta}$$

(i) $(5, \sqrt{5})$With the origin (JUL-22)

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{0-\sqrt{5}}{0-5} = \frac{\sqrt{5}}{5}$$

$$\text{Slope} = \frac{\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$$

$$= \frac{\mathbf{1}}{\sqrt{\mathbf{5}}}$$

3. **Show that the given points are collinear: $(-3, -4), (7, 2)$ and $(12, 5)$**

SEP-21

Let the given points be $A(-3, -4), B(7,2)$ and $C(12,5)$

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{[2-(-4)]}{[7-(-3)]} = \frac{2+4}{7+3} = \frac{6}{10}$$

$$\boldsymbol{m} = \frac{\mathbf{3}}{\mathbf{5}}$$

$$\text{Slope of } BC = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-2}{12-7}$$

$$\boldsymbol{m} = \frac{\mathbf{3}}{\mathbf{5}}$$

Slope of AB = Slope of BC

$\therefore$ The given points are collinear.

4. **Find the equation of a straight line which has slope $-\frac{5}{4}$ and passing through to the point $(-1, 2)$**

Slope $m = -\frac{5}{4}$

MAY-22

Equation of the line passing through the point $(-1,2) \Rightarrow \quad y - y_1 = m(x - x_1)$

$$y - 2 = \left(-\frac{5}{4}\right)(x - (-1))$$

$$4(y-2) = -5(x+1)$$

$$4y - 8 = -5x - 5$$

$$5x + 4y + 5 - 8 = 0$$

The required equation is $\mathbf{5x + 4y - 3 = 0}$

5. **Find the intercept made by following lines on the coordinate areas.**

SEP-21

(i) $\mathbf{3x - 2y - 6 = 0}$

$3x - 2y = 6$ Dividing by 6

$$\frac{x}{2} + \frac{y}{-3} = 1$$

x Intercept $\Rightarrow$ **2**

y Intercept $\Rightarrow$ **−3**

5 mark Questions

1. **Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are $(-4, -2), (-3, k), (3, -2)$ and $(2, 3)$**

PTA-5, SEP-20

Area of quadrilateral $= 28$ square units

$$\frac{1}{2}\begin{bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix} = 28$$

$$[(-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12)] = 56$$

$$(-4k + 11) - (3k - 10) = 56$$

$$-4k + 11 - 3k + 10 = 56$$

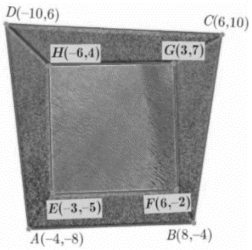
$$-7k = 56 - 21$$

$$-7k = 35$$

$$k = \frac{35}{-7}$$

$$\mathbf{k = -5}$$

2. **In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.** PTA-2



To find the area of patio we have to subtract area EFGH from area of ABCD

Area of ABCD $A(-4,-8), B(8,-4), C(6,10), D(-10,6)$

$$= \frac{1}{2}\begin{bmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{bmatrix}$$

$$= \frac{1}{2}[(16+80+36+80)-(-64-24-100-24)]$$

$$= \frac{1}{2}[212-(-212)]$$

$$= \frac{1}{2}[212+212] = \frac{1}{2}[424]$$

$= \mathbf{212}$ **Square units.**

Area of EFGH $E(-3,-5), F(6,-2), G(3,7), H(-6,4)$

$$= \frac{1}{2}\begin{bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{bmatrix}$$

$$= \frac{1}{2}[(6+42+12+30)-(-30-6-42-12)]$$

$$= \frac{1}{2}[90-(-90)]$$

$$= \frac{1}{2}[180]$$

$= \mathbf{90}$ **Square units.**

Area of the concrete patio $=$ Area of $ABCD$ $-$ Area of $EFGH = 212 - 90 = \mathbf{122}$ **sq.units.**

3. **The line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(8,12)$ and $(x,24)$. Find the value of x.** PTA-6

Slope of the line passing through the points $(-2,6)$ and $(4,8)$

Slope $m_1 = \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{8-6}{4-(-2)} = \frac{2}{4+2} = \frac{2}{6} = \frac{1}{3} \text{ ....(1)}$$

Slope of the line passing through the points $(8,12)$ and $(x,24)$

Slope $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{24-12}{x-8} = \frac{12}{x-8}$........ (2)

Since these lines are perpendicular to each other

$m_1 \times m_2 = -1 \Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$

$$\frac{4}{x-8} = -1$$

$$4 = -(x-8)$$

$$4 = -x+8$$

$$x = 8-4$$

$$x = 4$$

4. A quadrilateral has vertices at $A(-4,-2)$, $B(5,-1)$, $C(6,5)$ and $D(-7,6)$. Show that the mid-points of its sides form a parallelogram.

MAY-22

Midpoint of the side $AB = \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right)$

$= \left(\frac{1}{2}, \frac{-3}{2}\right) = P$

Midpoint of the side $BC = \left(\frac{5+6}{2}, \frac{-1+5}{2}\right)$

$= \left(\frac{11}{2}, \frac{4}{2}\right)$

$= \left(\frac{11}{2}, 2\right) = Q$

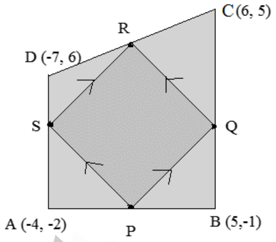
Midpoint of the side $CD = \left(\frac{6-7}{2}, \frac{5+6}{2}\right)$

$= \left(-\frac{1}{2}, \frac{11}{2}\right) = R$

Midpoint of the side $DA = \left(\frac{-7-4}{2}, \frac{6-2}{2}\right)$

$= \left(-\frac{11}{2}, \frac{4}{2}\right) = \left(-\frac{11}{2}, 2\right) = S$

Slope of opposite sides:



Slope of the PQ

$$= \frac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$$

Slope of RS

$$= \frac{2-\frac{11}{2}}{-\frac{1}{2}+\frac{11}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$$

Slope of $QR = \dfrac{\frac{11}{2}-2}{-\frac{1}{2}-\frac{11}{2}} = \dfrac{\frac{7}{2}}{-\frac{12}{2}} = -\dfrac{7}{12}$

Slope of $PS = \dfrac{2+\frac{3}{2}}{-\frac{11}{2}-\frac{1}{2}} = \dfrac{\frac{7}{2}}{\frac{-11-1}{2}} = \dfrac{7/2}{-12/2} = -\dfrac{7}{12}$

$\therefore\ PQ = RS\ ,\quad QR = PS$

Hence, mid-points of its sides form a parallelogram

5. A cat is located at the point $(-6,-4)$ is xy- plane. A bottle of milk is kept at $(5,11)$ The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

JUL-22

Equation of the path $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$(-6,-4)$ and $(5,11)$

$$\boxed{\begin{matrix}(x_1,y_1) = (-6,-4)\\ (x_2,y_2) = (5,11)\end{matrix}}$$

$\frac{y+4}{15} = \frac{x+6}{11}$

$11(y+4) = 15(x+6)$

$11y + 44 = 15x + 90$

$0 = 15x - 11y + 90 - 44$

The required equation is $\mathbf{15x - 11y + 46 = 0}$

6. Find the equation of a straight line which has slope $-\frac{5}{4}$ and passing through to the point $(-1,2)$

MAY-22

Slope $m = -\frac{5}{4}$

Equation of the line passing through the point $(-1,2) \Rightarrow \quad y - y_1 = m(x - x_1)$

$y - 2 = \left(-\frac{5}{4}\right)(x - (-1))$

$4(y-2) = -5(x+1)$

$4y - 8 = -5x - 5$

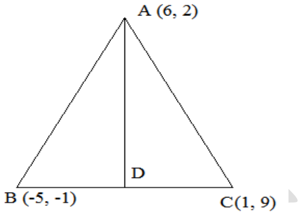
$5x + 4y + 5 - 8 = 0$

The required equation is $\mathbf{5x + 4y - 3 = 0}$

7. Find the equation of the median and altitude of triangle ABC through A where the vertices are $A(6,2), B(-5,-1)$ and $C(1,9)$

SEP-21, PTA-6

The median drawn passing through the vertex A intersect the side BC at the mid point.



$$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$D = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{8}{2}\right) = (-2,4)$$

$(x_1, y_1) = B(-5,-1)$
$(x_2, y_2) = C(1,9)$

Equation of the median AD:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$(x_1, y_1) = A(6,2)$
$(x_2, y_2) = D(-2,4)$

$$-8(y-2) = 2(x-6)$$

$$-8y+16 = 2x-12$$

$$0 = 2x+8y-12-16$$

$$2x+8y-28 = 0$$

$\div 2, \quad x+4y-14 = 0$

If a line passing through the vertex A is altitude, then it will be perpendicular to BC

Slope of BC $\quad m = \frac{y_2-y_1}{x_2-x_1} = \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

$$m_1 \times m_2 = -1$$

$$\frac{5}{3} \times m_2 = -1$$

$$m_2 = -1 \times \frac{3}{5}$$

$$= -\frac{3}{5}$$

Equation of altitude passing through A

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{5}(x-6)$$

$$5(y-2) = -3(x-6)$$

$$5y-10 = -3x+18$$

$$3x+5y-10-18 = 0$$

$$\mathbf{3x+5y-28 = 0}$$

8. Find the equation of a straight line joining the point of intersection of $3x+y+2=0$ and $x-2y-4=0$ to the point of intersection of $7x-3y=-12$ and $2y=x+3$ PTA-3

$3x+y+2=0$………….. (1)

$x-2y-4=0$…………….. (2)

$$2\times(1)\Rightarrow 6x+\cancel{2y}+\cancel{4}=0$$
$$(2)\Rightarrow \underline{\;x-\cancel{2y}-\cancel{4}=0\;}$$
$$\underline{7x \qquad\qquad =0}$$

$$x=\frac{0}{7}=0$$

$$x=0$$

sub $x=0$ in (1) we get

$3(0)+y+2=0$

$$y=-2$$

Point of intersection of the first two lines is $(0,-2)$

$7x-3y=-12$…………. (3)

$2y=x+3$

$x-2y=-3$……………. (4)

$$2\times(3)\Rightarrow 14x-\cancel{6y}=-24$$
$$-3\times(4)\Rightarrow\;\; 3x-\cancel{6y}=-9$$
$$\underline{\;(-)\quad(+)\qquad(+)\;}$$
$$\underline{11x \qquad = -15}$$

$$x=-\frac{15}{11}$$

Sub $x=-\frac{15}{11}$ in (4) we get

$$-\frac{15}{11}-2y=-3$$

$$-2y=-3+\frac{15}{11}$$

$$-2y=\frac{-33+15}{11}$$

$$-2y=-\frac{18}{11}$$

$$y=\frac{9}{11}$$

Point of intersection of other set of lines is $\left(\frac{-15}{11},\frac{9}{11}\right)$

To find the equation of the line passing through the points $(0,-2)$ and $\left(\frac{-15}{11},\frac{9}{11}\right)$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y+2}{\frac{9}{11}+2}=\frac{x-0}{-\frac{15}{11}-0}$$

$$\frac{y+2}{\frac{31}{11}}=\frac{x-0}{-\frac{15}{11}}$$

$$-15(y+2)=31(x-0)$$

$$-15y-30=31x$$

∴ The required equation is $\mathbf{31x+15y+30=0}$

9. The area of a triangle is 5 sq. Units. Two of its vertices are $(2,1)$ and $(3,-2)$. The third vertex is (x,y) where $y=x+3$. Find the coordinates of the third vertex. PTA-1

Given, area of triangle ABC is 5 sq. Units and $A(2,1), B(3,-2), C(x,y)$ where $y=x+3$

$$\text{Area of }\Delta=\frac{1}{2}\begin{bmatrix}2 & 3 & x & 2\\ 1 & -2 & y & 1\end{bmatrix}=5$$

$$(-4+3y+x)-(3-2x+2y)=10$$

$$x+3y-4-3+2x-2y=10$$

$$3x+y=17\text{ …… (1)}$$

Given $y=x+3$ sub in (1)

$3x+x+3=17$

$$4x=14\Rightarrow x=\frac{14}{4}\Rightarrow x=\frac{7}{2}$$

Substitute, $x=\frac{7}{2}$ in $y=x+3\Rightarrow\quad y=\frac{13}{2}$

∴ Third vertex is $\left(\frac{7}{2},\frac{13}{2}\right)$

6. Trigonometry

1 mark Questions

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to

(A) $\tan^2\theta$ **(B) 1** (C) $\cot^2\theta$ (D) 0

2. $\tan\theta\,\mathrm{cosec}^2\theta - \tan\theta$ is equal to PTA-3

(A) $\sec\theta$ (B) $\cot^2\theta$ (C) $\sin\theta$ **(D) $\cot\boldsymbol{\theta}$**

3. If $(\sin\alpha + \mathrm{cosec}\,\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to PTA-1

(A) 9 **(B) 7** (C) 5 (D) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \mathrm{cosec}\,\theta = b$, then the value of $b(a^2-1)$ is equal to

(A) $2\boldsymbol{a}$ (B) $3a$ (C) 0 (D) $2ab$

5. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to PTA-2

(A) 25 **(B) $\frac{1}{25}$** (C) 5 (D) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to PTA-1, 4

(A) $\frac{-3}{2}$ **(B) $\frac{3}{2}$** (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then

(A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \mathrm{cosec}\,\theta)$ is equal to

(A) 0 (B) 1 **(C) 2** (D) -1

9. $a\cot\theta + b\,\mathrm{cosec}\,\theta = p$ and $b\cot\theta + a\,\mathrm{cosec}\,\theta = q$ then $p^2 - q^2$ is equal to PTA-5

(A) $a^2 - b^2$ **(B) $\boldsymbol{b^2 - a^2}$** (C) $a^2 + b^2$ (D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure PTA-6, SEP-21

(A) 45° (B) 30° (C) 90° **(D) 60°**

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to

(A) $\sqrt{3}\,b$ **(B) $\frac{b}{3}$** (C) $\frac{b}{2}$ (D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m height. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to MAY-22

(A) 41.92 m **(B) 43.92 m** (C) 43 m (D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in meters) is

(A) $20, 10\sqrt{3}$ (B) $30, 5\sqrt{3}$ (C) 20, 10 **(D) $\boldsymbol{30, 10\sqrt{3}}$**

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(A) $\sqrt{2}x$ **(B)** $\boldsymbol{\frac{x}{2\sqrt{2}}}$ (C) $\frac{x}{\sqrt{2}}$ (D) $2\,x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is JUL-22

(A) $\boldsymbol{\frac{h(1+\tan\beta)}{1-\tan\beta}}$ (B) $\frac{h(1-\tan\beta)}{1+\tan\beta}$ (C) $h\tan(45^0-\beta)$ (D) none of these

2 mark Questions

1. **Prove the following identities (i)** $\boldsymbol{\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sec\theta+\tan\theta}$ SEP-20

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$$

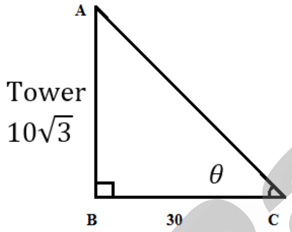
$$=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=\boldsymbol{\sec\theta+\tan\theta}$$

2. **Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $\boldsymbol{10\sqrt{3}m}$.** PTA-2, SEP-21, JUL-22



In right angle ΔABC

$AB=$ Tower

$=10\sqrt{3}m$

$BC=30m$

$$tan\theta=\frac{AB}{BC}$$

$$=\frac{10\sqrt{3}}{30}=\frac{\sqrt{3}}{3}=\frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

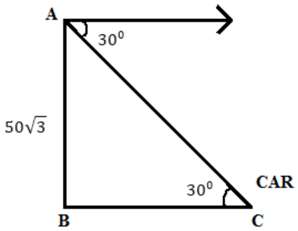
$$\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow$$

$$\tan30^{\circ}=\frac{1}{\sqrt{3}}$$

$$\therefore\theta=30^{\circ}$$

$\therefore$ The angle of elevation $\boldsymbol{\theta=30^{\circ}}$

3 . From the top of a rock $50\sqrt{3}m$ high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. PTA-6, MAY-22



AB = Height of the rock $= 50\sqrt{3}$

Angle of depression $= 30^{\circ}$

In right angle $\Delta\ ABC$,

$$\tan 30^{\circ} = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$$\frac{50\sqrt{3}}{BC} = \frac{1}{\sqrt{3}}$$

$$BC = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$

$$= 150m$$

The distance of the car from rock

$$= \mathbf{150\ m}$$

5 mark Questions

1. If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ PTA-3

Given: $\sqrt{3}\sin\theta - \cos\theta = 0$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^0 \qquad \left(\because \tan 30^{\circ} = \frac{1}{\sqrt{3}}\right)$$

LHS: $\tan 3\theta = \tan 3(30^{\circ}) = \tan 90^{\circ} = \infty$ ……………… (1)

RHS:

$$\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \frac{3\times\tan 30^{\circ} - \tan^3 30^{\circ}}{1-3\tan^2 30^{\circ}}$$

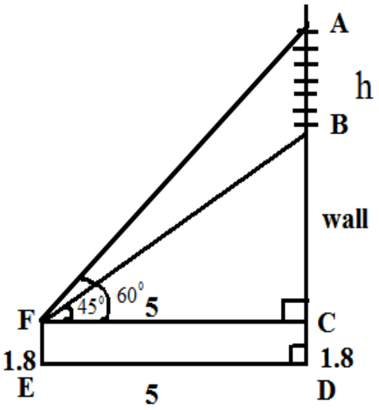
$$= \frac{3\times\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1-3\times\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{3}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\times\frac{1}{3}}$$

$$= \frac{\sqrt{3}-\left(\frac{1}{\sqrt{3}}\right)^3}{0} = \infty \text{ ……………. (2)}$$

(1) = (2)

$$\therefore \tan 3\theta = \frac{\mathbf{3\tan\theta - \tan^3\theta}}{\mathbf{1 - 3\tan^2\theta}}$$

2. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5m away from the wall, what is the height of the window? $(\sqrt{3} = \mathbf{1.732})$. JUL-22



Let AB = Window = h

EF = Man = $180cm$

$= 1.8m = CD$

$CF = 5m$

To find the height of the window

In right angle $\Delta\, BCF$

$$\tan 45^\circ = \frac{BC}{5}$$

$$1 = \frac{BC}{5}$$

$$\therefore BC = 5\text{m}$$

In right angle ΔACF

$$\tan 60^\circ = \frac{AC}{5}$$

$$\sqrt{3} = \frac{AC}{5}$$

$$AC = 5\sqrt{3}$$

$$BC + AB = 5\sqrt{3}$$

$$5 + h = 5\sqrt{3}$$

$$h = 5\sqrt{3} - 5$$

$$= (5 \times 1.732) - 5$$

$$= 8.660 - 5$$

$$h = 3.66\text{m}$$

Height of the window $\boldsymbol{h} = \mathbf{3.66m}$

3. From the top of the tower 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. $(\tan 38^\circ = 0.7813, \sqrt{3} = 1.732)$

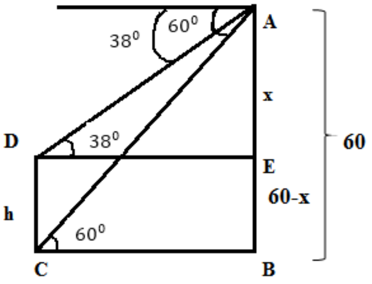
SEP-20

AB =Tower $= 60m$

CD =lamp post $= h$

$AE = x$

$CD = BE = 60 - x = h$



In right angle $\Delta\ AEC$

$\tan 38^\circ = \frac{AE}{DE} = 0.7813$

$DE = \frac{x}{0.7813}$……… (1)

In right angle $\Delta\, ABC$

$\theta = 60^\circ$

$\tan 60^\circ = \frac{AB}{BC} = \sqrt{3}$

$\frac{60}{BC} = \sqrt{3}$

$BC = \frac{60}{\sqrt{3}}$

$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{60\sqrt{3}}{3}$

$BC = 20\sqrt{3}$

$BC = DE$

$\therefore DE = 20\sqrt{3}$ ……………. (2)

From (1) & (2)

$DE \Rightarrow \frac{x}{0.7813} = 20\sqrt{3}$

$x = 20\sqrt{3} \times 0.7813$

$x = 20 \times 1.732 \times 0.7813$

$x = 27.064m$

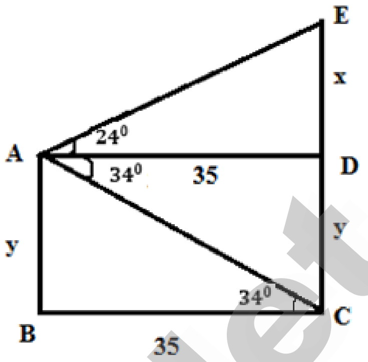
Height of the lamp post

$h = 60 - x$

$= 60 - 27.064$

$\boldsymbol{h = 32.93m}$

4. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of top of the statue is 34°. Find the height of the statue. $(\tan 24^\circ = 0.4452, \tan 34^\circ = 0.6745)$ PTA-4



$AB = \text{Building} = y$

$CE = \text{State} = x + y$

$BC = AD = 35m$

In right angle $\Delta\, ADE$

$$\tan 24^0 = \frac{ED}{AD} = 0.4452$$

$$\frac{x}{35} = 0.4452$$

$$x = 35 \times 0.4452$$

$$x = 15.582$$

In right angle ΔABC,

$$\tan 34^\circ = \frac{AB}{BC} = 0.6745$$

$$\frac{y}{35} = 0.6745$$

$$y = 0.6745 \times 35$$

$$= 23.6075$$

Height of the statues

$$CE = x + y$$

$$= 15.582 + 23.608$$

$$= 39.190$$

$$CE = 39.19m$$

Height of the statue $= \mathbf{39.19}\boldsymbol{m}$

7. Mensuration

1 mark Questions

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(A) $60\pi\ cm^2$ (B) $68\pi\ cm^2$ (C) $120\pi\ cm^2$ **(D) $136\pi\ cm^2$**

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is MAY-22

(A) $4\pi r^2$ sq. units (B) $6\pi r^2$ sq. units (C) $3\pi r^2$ sq. units (D) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be SEP-21

(A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is JUL-22

(A) 1 : 2 **(B) 1 : 4** (C) 1 : 6 (D) 1 : 8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is PTA-1

(A) $\frac{9\pi h^2}{8}$ sq. units (B) $24\pi h^2$ sq. units **(C) $\frac{8\pi h^2}{9}$ sq. units** (D) $\frac{56\pi h^2}{9}$ sq. units

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is PTA-4

(A) $5600\pi\ cm^3$ **(B) $1120\pi\ cm^3$** (C) $56\pi\ cm^3$ (D) $3600\pi\ cm^3$

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is

(A) made 6 times **(B) made 18 times** (C) made 12 times (D) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.

(A) π (B) 4π **(C) 3π** (D) 2π PTA-3, SEP-21, JUL-22

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

(A) $3x\ cm$ (B) $x\ cm$ **(C) $4x\ cm$** (D) $2x\ cm$

10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. Then, the volume of the frustum is

(A) $3328\pi\ cm^3$ (B) $3228\pi\ cm^3$ (C) $3240\pi\ cm^3$ (D) $3340\pi\ cm^3$

11. A shuttle cock used for playing badminton has the shape of the combination of

(A) a cylinder and a sphere (B) a hemisphere and a cone

(C) a sphere and a cone **(D) frustum of a cone and a hemisphere**

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is PTA-6, SEP-20

(A) 2: 1 (B) 1: 2 (C) 4: 1 (D) 1: 4

13. The volume (in cm^3) of the greatest sphere that ca be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

(A) $\boldsymbol{\frac{4}{3}\pi}$ (B) $\frac{10}{3}\pi$ (C) 5π (D) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2:h_1 = 1:2$ then $r_2:r_1$ is PTA-2

(A) $1:3$ **(B) $\boldsymbol{1:2}$** (C) $2:1$ (D) $3:1$

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is PTA-5

(A) $1:2:3$ (B) $2:1:3$ (C) $1:3:2$ **(D) $\boldsymbol{3:1:2}$**

2 mark Questions

1. **The radius and height of a cylinder in the ratio 5:7 and its curved surface area is $\boldsymbol{5500}$sq. cm Find its radius and height.** JUL-22

$$\frac{\text{Radius}}{\text{Height}} = \frac{r}{h} = \frac{5}{7} \quad \Rightarrow r = \frac{5h}{7} \text{ ....(1)}$$

CSA of the cylinder $= 2\pi rh = 5500$

$$2 \times \frac{22}{7} \times \frac{5h}{7} \times h = 5500$$

$$h^2 = \frac{\overset{\cancel{500}\ \cancel{100}\ \cancel{50}\ 25}{\cancel{5500}}\times 7\times 7}{\cancel{2}\times\underset{\cancel{2}}{\cancel{22}}\times\cancel{5}}$$

$$= 5 \times 5 \times 7 \times 7$$

$$h = 35\text{cm}$$

Substitute h=35 in (1), $r = \frac{5(35)}{7} \Rightarrow r = 25$cm.

$\boldsymbol{r = 25}$ **cm,** $\boldsymbol{h = 35}$**cm**

2. **The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.** PTA-2

Smaller cone:	**Large cone:**
$r_1 \to r$	$r_2 \to 3r$
$h_1 \to 3r$	$h_2 \to 3r$
$l_1 = \sqrt{(3r)^2 + r^2} = \sqrt{10r^2} = r\sqrt{10}$	$l_2 = \sqrt{(3r)^2 + (3r)^2} = \sqrt{18r^2} = \sqrt{9\times 2}(r) = 3r\sqrt{2}$

CSA of small cone : CSA of large cone

$$\pi r_1 l_1 : \pi r_2 l_2$$

$$r \times r\sqrt{10} : 3r \times 3r\sqrt{2}$$

$$\sqrt{5}\,\sqrt{2} : 9\sqrt{2}$$

$$\sqrt{5} : 9$$

Ratio of the CSA is $\sqrt{5}:9$

3. **A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?**

SEP-20

Volume of water raised in cylindrical glass

= Volume of cylindrical metal immersed

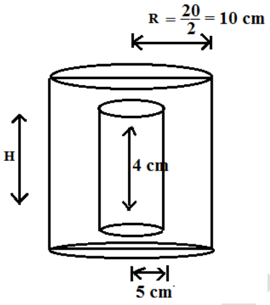
$$\pi R^2 H = \pi r^2 h$$

$$\pi \times 10 \times 10 \times h = \pi \times 5 \times 5 \times 4$$

$$h = \frac{5\times5\times4}{10\times10}$$

$$= 1$$

The raise of the water in the glass **= 1 cm**



4. **The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights.**

PTA-4, MAY-22

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone 1 : Volume of cone 2 $= 3600 : 5040$

$$\frac{1}{3}\pi r^2 \times h_1 : \frac{1}{3}\pi r^2 \times h_2 = 180 : 252$$

$$h_1 : h_2 = 45 : 63$$

$$h_1 : h_2 = 5 : 7$$

5. **A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3} : 4$.**

PTA-6

TSA of sphere = TSA of hemisphere

$$\cancel{4}\pi r_1^2 = \cancel{3}\pi r_2^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

$$\frac{\text{volume of sphere}}{\text{volume of hemisphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3}$$

$$= 2\left(\frac{r_1}{r_2}\right)^3$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{2\times3\sqrt{3}}{8}$$

$$= \frac{3\sqrt{3}}{4}$$

∴ Ratio of the volume **$3\sqrt{3} : 4$**

6. **Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.** PTA-1

$$\text{Number of coins} = \frac{\text{volume of cylinder } (\pi r^2 h)}{\text{volume of a coin } (\pi r^2 h)}$$

$$= \frac{\pi\times45\times45\times10\times10\times2\times10\times2\times10}{2\times10\times2\times10\times\pi\times15\times15\times2}$$

Number of coins to be melted = **450 coins**

5 mark Questions

1. **A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre.** MAY-22

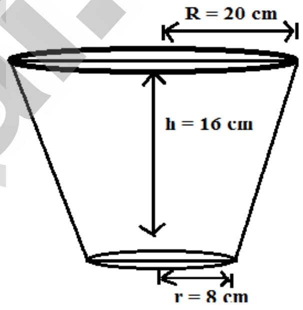
$$\text{Volume of frustum} = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 16\big(20^2 + 8^2 + (20\times 8)\big)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 16\times 624$$

$$= \frac{73216}{7}$$

$$= 10459.4\ cm^3$$



Volume of frustum = 10.4594 litres

$\therefore 1000\ cm^3 = 1 litre$

Required cost = 10.4594 × 40

= ₹ 418.376

Cost of the milk which can completely fill the container ≅ ₹ **418.38**

2. **Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.** MAY-22

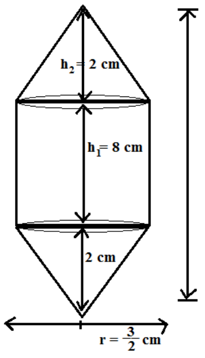
Volume of the model = Volume of cylinder + Volume of cone × 2

$$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 \times 2$$

$$= \pi r^2\left[h_1 + \frac{2}{3}h_2\right]$$

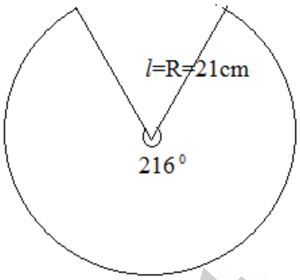
$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times\left[8 + \frac{2}{3}(2)\right]$$

$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times\frac{28}{3}$$



Volume of the model = **66 cm^3**

3. A metallic sheet in the form of a sector of a circle of radius $21\ cm$ has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed. PTA-2



Arc length $L = \frac{2\pi R}{360} \times 216$

$L = \frac{2\pi \times 21 \times 3}{5}$

Circum of base of the cone = Arc length

i.e, $2\pi r = \frac{2\pi \times 21 \times 3}{5}$

$= \frac{63}{5}$

$r = 12.6\ cm$

$h = \sqrt{l^2 - r^2}$

$= \sqrt{21^2 - 12.6^2}$

$= \sqrt{441 - 158.76}$

$= \sqrt{282.24}$

$h = 16.8\ cm$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$

$= 2794.176\ \ cm^3$

Volume of the cone formed $= \mathbf{2794.176\ cm^3}$

8. Statistics and Probability

1 mark Questions

1. Which of the following is not a measure of dispersion? PTA-6
(A) Range (B) Standard deviation **(C) Arithmetic mean** (D) Variance

2. The range of the data 8,8,8,8,8,…8 is
(A) 0 (B) 1 (C) 8 (D) 3

3. The sum of all deviations of the data from its mean is
(A) Always positive (B) Always negative **(C) zero** (D) non- zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all observations is SEP-20
(A) 40000 **(B) 160900** (C) 160000 (D) 30000

5. Variance of first 20 natural numbers is PTA-5
(A) 32.25 (B) 44.25 **(C) 33.25** (D) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
(A) 3 (B) 15 (C) 5 **(D) 225**

7. If the standard deviation of x, y, z is p then the standard deviation of $3x+5, 3y+5, 3z+5$ is
(A) $3p+5$ **(B) $3p$** (C) $p+5$ (D) $9p+15$

8. If the mean and coefficient of variation of a data are 4 *and* 87.5% then the standard deviation is
(A) 3.5 (B) 3 (C) 4.5 (D) 2.5

9. Which of the following is incorrect? PTA-1, 4, 5
(A) $P(A) > 1$ (B) $0 \le P(A) \le 1$ (C) $P(\emptyset) = 0$ (D) $P(A) + P(\bar{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is
(A) $\frac{q}{p+q+r}$ **(B) $\frac{p}{p+q+r}$** (C) $\frac{p+q}{p+q+r}$ (D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is SEP-21, JUL-22
(A) $\frac{3}{10}$ **(B) $\frac{7}{10}$** (C) $\frac{3}{9}$ (D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is MAY-22
(A) 2 **(B) 1** (C) 3 (D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of kamalam winning is $\frac{1}{9}$, then the number of tickets bought by kamalam is
(A) 5 (B) 10 **(C) 15** (D) 20

14. If a letter is chosen at random from the English alphabets $\{a, b, \dots., z\}$, then the probability that the letter chosen precedes x. SEP-20
(A) $\frac{12}{13}$ (B) $\frac{1}{13}$ **(C) $\frac{23}{26}$** (D) $\frac{3}{26}$

15. A purse contains 10 notes of ₹ 2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹ 500 note or ₹200 note?
(A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{3}$ **(D) $\frac{4}{5}$**

2 mark Questions

1. **Find the range and coefficient of range of the following data.**

(i) $\mathbf{63, 89, 98, 125, 79, 108, 117, 68}$ SEP-20

Arrange in Ascending order:

63,68,79,89,98,108,117,125

$Range = L - S = 125 - 63 = 62$

Coefficient of Range $= \frac{L-S}{L+S}$

$= \frac{125-63}{125+63}$

$= \frac{62}{188}$

$= 0.3297$

$= \mathbf{0.33}$

2. **Find the standard deviation of first 21 natural numbers.** PTA-6

Standard deviation of first 21

natural numbers.

$\sigma = \sqrt{\frac{n^2-1}{12}}; \ n = 21$

$= \sqrt{\frac{(21)^2-1}{12}}$

$= \sqrt{\frac{441-1}{12}}$

$= \sqrt{\frac{440}{12}}$

$= \sqrt{36.67}$

$\sigma = 6.049$

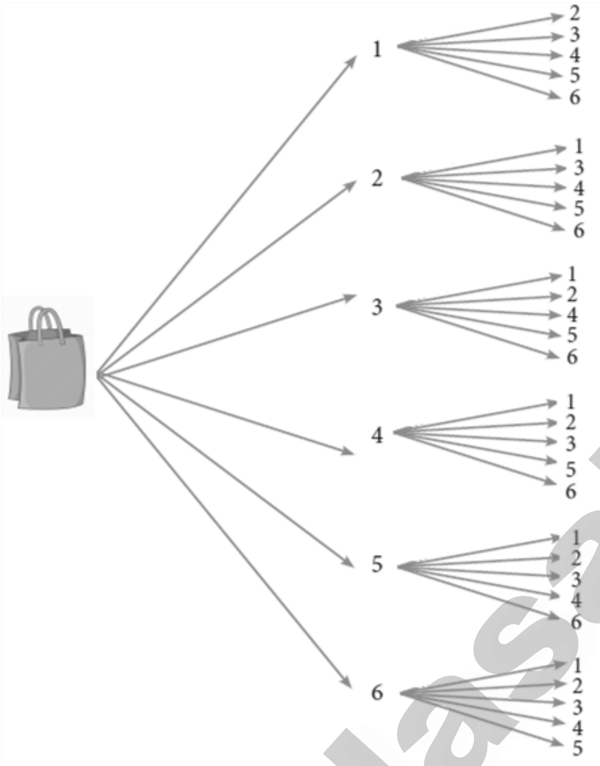
$\boldsymbol{\sigma = 6.05}$

3. **Write the sample space for selecting two balls at a time from a bag containing 6 balls numbered 1 to 6 (using tree diagram).**

PTA-4

When we select two balls from a bag containing 6 balls numbered 1,2,3.4,5,6.

Tree diagram:



Hence the sample space can be written as,

$$S = \{(1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5)\}$$

5 mark Questions

1. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

PTA-5

Time taken (sec)	8.5 -9.5	9.5 -10.5	10.5 -11.5	11.5 -12.5	12.5 -13.5
Number of students	6	8	17	10	9

$A = 11, c = 1$

Time taken (sec)	Mid value x_i	No.of students f_i	$d = X_i - A$	d^2	$f_i d_i$	$f_i d_i^2$
8.5 - 9.5	9	6	-2	4	-12	24
9.5 - 10.5	10	8	-1	1	-8	8
10.5 - 11.5	11	17	0	0	0	0
11.5 - 12.5	12	10	1	1	10	10
12.5 - 13.5	13	9	2	4	18	36
		$N = 50$	$\Sigma d_i = 0$		$\Sigma f_i d_i = -8$	$\Sigma f_i d_i^2 = 78$

$$\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{-8}{50}\right)^2}$$

$$= \sqrt{1.56 - (-0.16)^2}$$

$$= \sqrt{1.56 - 0.0256}$$

$$= \sqrt{1.5344}$$

$$= 1.238$$

$\boldsymbol{\sigma \cong 1.24}$

2. If A is an event of a random experiment such that $P(A):P(\bar{A}) = 17:15$ and $n(S) = 640$ then find (i) $P(\bar{A})$ (ii) $n(A)$

PTA-3

$P(A):P(\bar{A}) = 17:15,$
$n(S) = 640$

(i) $P(\bar{A}) =?$

$$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$$

$$\frac{P(A)}{1-P(A)} = \frac{17}{15}$$

$$15P(A) = 17(1 - P(A))$$

$$15P(A) = 17 - 17P(A)$$

$$15P(A) + 17P(A) = 17$$

$$32P(A) = 17$$

$$P(A) = \frac{17}{32};$$

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - \frac{17}{32}$$

$$P(\bar{A}) = \frac{32-17}{32}$$

$$P(\bar{A}) = \mathbf{\frac{15}{32}}$$

(ii) $n(A)$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) \times n(S) = n(A)$$

$$n(A) = \frac{17}{32} \times 640$$

$$n(A) = 340$$

3. Two unbiased dice are rolled once. Find the probability of getting SEP-20, JUL-22

(i) a doublet (equal numbers on both dice) (ii) the product as a prime number

(iii) the sum as a prime number (iv) the sum as 1 5M

Two unbiased dice are rolled once.

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(S) = 36$

(i) Let the A be event of getting a doublet.

$A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$n(A) = 6,$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \mathbf{\frac{1}{6}}$$

(ii) Let B be the event of getting the product as a prime number.

$B = \{(1,2),(1,3),(1,5),(2,1),(3,1),(5,1)\}$

$n(B) = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \mathbf{\frac{1}{6}}$$

(iii) Let C be the event of getting the sum as a prime number.

$$C = \{(1,1),(1,2),(1,4),(1,6),(2,1),\\(2,3),(2,5),(3,2),(3,4),(4,1),\\(4,3),(5,2),(5,6),(6,1),(6,5)\}$$

$n(C) = 15$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \mathbf{\frac{5}{12}}$$

(iv) Let D be the event of getting the sum as 1.

$D = \{\quad\}$

$n(D) = 0$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{0}{36} = 0$$

$\therefore P(D) = \mathbf{0}$

4. Three fair coins are tossed together. Find the probability of getting PTA-5

(ii) atleast one tail (iii) atmost one head

Three fair coins are tossed together.

$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$

$n(S) = 8.$

i) Let A be the event of getting all heads.

$A = \{HHH\}; n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{1}{8}}$$

ii) Let B be the event of getting atleast one tail.

$B = \{HHT, HTH, HTT, TTT, TTH, THT, THH\}$

$n(B) = 7$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{7}{8}}$$

iii) Let C be the event of getting atmost one head.

$C = \{HTT, TTH, THT, TTT\}$

$n(C) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \mathbf{\frac{4}{8}}$$

$$\therefore P(C) = \frac{1}{2}$$

iv) Let D be the event of getting atmost 2 tails.

$D = \{HHH, HHT, HTH, HTT, TTH, THT, THH\}$

n (D)=7

$$P(D) = \frac{n(D)}{n(S)} \Rightarrow \quad \therefore P(D) = \mathbf{\frac{7}{8}}$$

5. **A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is.**

(i) white (ii) black or red (iii) not white (iv) neither white nor black

$n(R) = 5,\quad n(W) = 6,\quad n(G) = 7,\quad n(B) = 8$

JUL-22

$n(S) = 5 + 6 + 7 + 8 \quad = 26$

i) Let A be the event of drawn white ball $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{6}{26} = \mathbf{\frac{3}{13}}$$

ii) Let B & R be the event of drawn black or red ball.

$$P(B) = \frac{8}{26}$$

$$P(R) = \frac{5}{26}$$

$$P(B \cup R) = P(B) + P(R) \quad = \frac{8}{26} + \frac{5}{26}$$

$$\therefore\ P(B \cup R) = \frac{13}{26} = \mathbf{\frac{1}{2}}$$

iii) Let $\bar{A}$ be the event of getting not white ball.

$$P(A) = \frac{3}{13}$$

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{3}{13} = \frac{13-3}{13}$$

$$\therefore P(\bar{A}) = \frac{10}{13}$$

iv) Let C be the event of neither white nor black.

$$n(C) = 26 - (6 + 8) \quad = 26 - 14 \quad = 12$$

$$P(\text{neither white not black}) = \mathrm{P}(C) = \frac{n(C)}{n(S)} = \frac{12}{26} = \mathbf{\frac{6}{13}}$$

6. If $\boldsymbol{P(A)=\frac{2}{3}, P(B)=\frac{2}{5}, P(A\cup B)=\frac{1}{3}}$ then find $\boldsymbol{P(A\cap B)}$. PTA-1

$P(A)=\frac{2}{3},$

$P(B)=\frac{2}{5},$

$P(A\cup B)=\frac{1}{3}$

$P(A\cap B)=?$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{1}{3}=\frac{2}{3}+\frac{2}{5}-P(A\cap B)$

$P(A\cap B)=\frac{2}{3}+\frac{2}{5}-\frac{1}{3}$

$=\frac{1}{3}+\frac{2}{5}$

$=\frac{5+6}{15}$

$=\boldsymbol{\frac{11}{15}}$

7. If $\boldsymbol{A\ and\ B}$ are two mutually exclusive events of a random experiment and $\boldsymbol{P(notA)=0.45}$, $\boldsymbol{P(A\cup B)=0.65}$, then find $\boldsymbol{P(B)}$.

PTA-2

$P(not\ A)=0.45=P(\bar{A}), P(A\cup B)=0.65$

$P(B)=?$

$P(A)=1-P(\bar{A})$

$=1-0.45$

$=0.55$

$P(A\cup B)=P(A)+P(B)$

$0.65=0.55+P(B)$

$0.65-0.55=P(B)$

$0.10=P(B)$

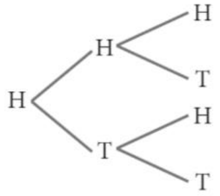
$\therefore P(B)=\boldsymbol{0.1}$

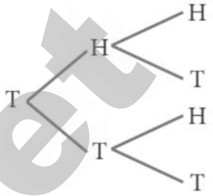
8. **A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or consecutive two heads.**

A coin is tossed thrice,

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$





i) Let A be the event of getting exactly two heads.

$A = \{HHT, HTH, THH\}$

$n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{3}{8}}$

ii) Let B be the event of the getting atleast one tail.

$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(B) = 7$

$\therefore P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{7}{8}}$

iii) Let C be the event of getting consecutive two heads.

$C = \{HHH, HHT, THH\}$

$n(C) = 3$

$\therefore P(C) = \frac{n(C)}{n(S)} = \mathbf{\frac{3}{8}}$

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3, \ P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{3}{8}$

$B \cap C = \{HHT, THH\}, \ n(B \cap C) = 2$

$\therefore P(B \cap C) = \frac{n(B\cap C)}{n(S)} = \frac{2}{8}$

$A \cap C = \{HHT, THH\}, \ n(A \cap C) = 2$

$P(A \cap C) = \frac{n(A\cap C)}{n(S)} = \frac{2}{8}$

$A \cap B \cap C = \{HHT, THH\}$

$n(A \cap B \cap C) = 2,$

$P(A \cap B \cap C) = \frac{n(A\cap B\cap C)}{n(S)} = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + n(A \cap B \cap C)$

$P(A \cup B \cup C) = \cancel{\frac{3}{8}} + \frac{7}{8} + \frac{3}{8} - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}}$

$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$

$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$

$= \frac{10-2}{8} = \frac{8}{8} = \mathbf{1}$