10

ACHIEVE WITH EASE

MATHEMATICS

CONTENTS : -

Total = 42 Marks

Additional printed pages you will get minimum 25 marks

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ACHIEVE WITH EASE (MASE)

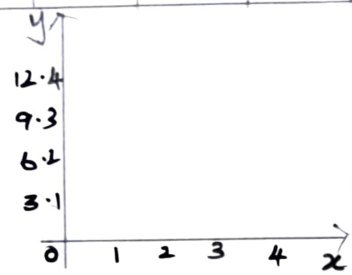
DIRECT VARIATION

1) Varshika:- (Table given)

x	1	2	3	4	5	(6)*
y	3.1	6.2	9.3	12.4	15.5	(18.6)

* x → 1 unit
y → 3.1 units

* K = 3.1

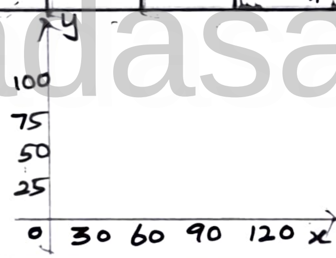


2) Bus:- (Table not given)

x	60	(90)*	120	180	240	(360)
y	50	(75)	100	150	200	(300)*

* x → 30 units.
y → 25 units.

* $K = \frac{5}{6}$

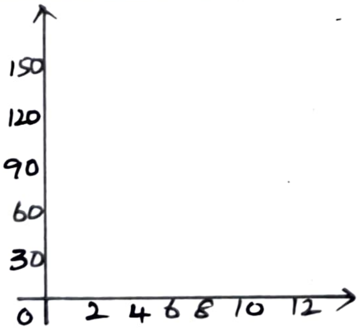


3) Two wheeler:- (Table given)

x	4	(6)*	8	(10)	12	24
y	60	(90)	120	(150)*	180	360

* x → 2 units
y → 30 units

* K = 15

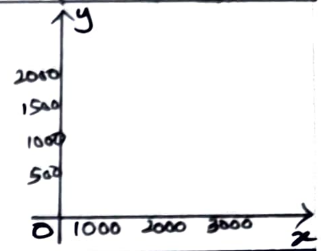


4) Garment Shop:- (Table not given)

x	1000	2000	*(2500)	3000	4000	5000	(6500)
y	500	1000	(1250)	1500	2000	2500	*(3250)

*) x → 1000 units
y → 500 units.

* $K = \frac{1}{2}$

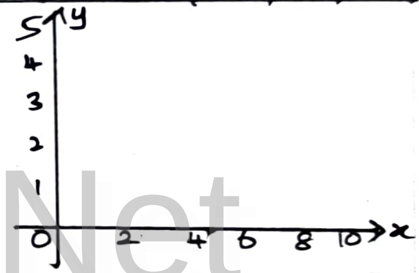


5) $y = \frac{1}{2}x$ (Table not given)

x	2	4	6	8	(9)*	10	(15)
y	1	2	3	4	(4.5)	5	(7.5)*

*) x → 2 units
y → 1 unit

$K = \frac{1}{2}$



Inverse Variation.

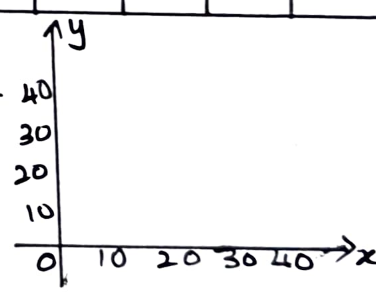
6) Company:- (Table given)

x	(30)*	40	50	60	75	(120)*
y	(200)	150	120	100	80	(50)

x → 10 units.
y → 10 units.

k = xy

K = 6000



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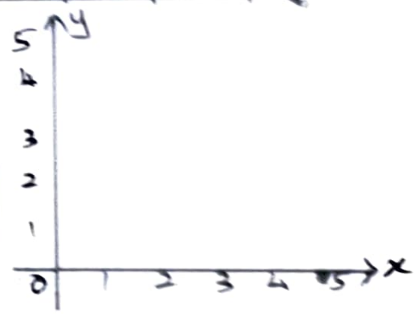
7) Nishanth :- (Table not given)

x	12	6	4	3	(2·4)*	2
y	1	2	3	4	(5)	6

x → 1 unit

y → 1 unit

K = xy
= 12



8) Tank: (Pipe):- (Table given)

x	2	3	*(5)	6	9	(10)
y	45	30	(18)	15	10	(9)*

x → 1 unit

y → 2 units

K = xy
= 90

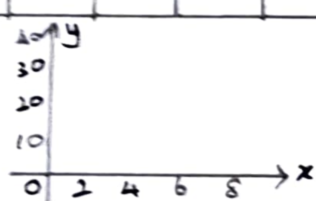
9) School :- (Table given)

x	2	4	6	8	10	12
y	180	90	60	45	36	30

x → 2 unit

y → 10 units.

K = 360



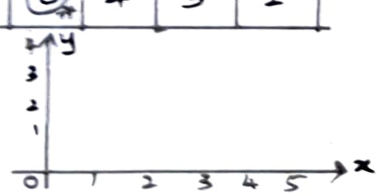
10) xy = 24 (Table not given)

x	2	(3)*	(4)	6	8	12
	12	(8)	(6)*	4	3	2

x → 1 unit

y → 1 unit

K = 24



Geometry : Tangent

1) d = 6cm, away 8cm

$\therefore r = \frac{6}{2} = 3cm$

length of the tangent PA = PB = 7·4cm

2) r = 5cm, away 10cm

length of the tgt PA = PB = 8·7cm

3) r = 4cm, away 11cm

length of the tgt PA = PB = 10·3cm

4) d = 6cm, away 5cm

$\therefore r = \frac{6}{2} = 3cm$ ↓

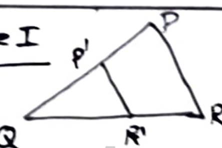
PA = PB = 4cm

5) r = 3·6cm, distance 7·2cm

PA = PB = 6·2cm.

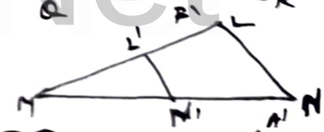
Similar Δ :- Type I

1) $\frac{3}{5}$ (PQR) ⇒



2) $\frac{2}{3}$ (PQR) ⇒

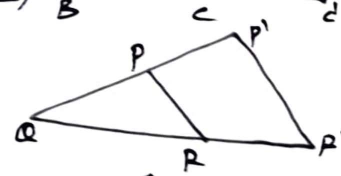
3) $\frac{4}{5}$ (LMN) ⇒



Type-II

4) $\frac{6}{5}$ (ABC) ⇒

5) $\frac{7}{3}$ (PQR) ⇒



6) $\frac{7}{4}$ (PQR) ⇒ ← above fig. →

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5 marks:- Quadrilateral:

1) (8,6), (5,11), (-5,12), (-4,3)

Area of the Quadrilateral $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ sq. units

$= \frac{1}{2}\begin{vmatrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 11 & 11 & 3 & 6 \end{vmatrix} = 79$ sq. units

2) (-9,2) ④, (-8,-4) ①, (2,2) ③, (1,-3) ②

Re Arrange the order, → 1 mark

(-8,-4), (1,-3), (2,2), (-9,2)

Area of the Quadrilateral $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ sq. units

$= \frac{1}{2}\begin{vmatrix} -8 & 1 & 2 & -9 & -8 \\ -4 & -3 & 2 & 2 & -4 \end{vmatrix} = 35$ sq. units

3) (-9,0) ④, (-8,6) ③, (-1,-2) ②, (-6,-3) ①

Re Arrange the order → 1 mark

(-6,-3), (-1,-2), (-8,6), (-9,0)

Area of the Quadrilateral $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ sq. units

$= \frac{1}{2}\begin{vmatrix} -6 & -1 & -8 & -9 \\ -3 & -2 & 6 & 0 \end{vmatrix} = 34$ sq. units

4) (-4,-2), (-3,k), (3,-2), and (2,3) .... area 28 sq. units. find 'k'.

Area of the Quadrilateral = 28 sq. units

$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} = 28$

$\Rightarrow \frac{1}{2}\begin{vmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{vmatrix} = 28$

$\therefore$ $k = -5$

5) (-5,7), (-4,k), (-1,-6), (4,5) area = 72 sq. units. find 'k'.

Area of the quadrilateral = 72 sq. units

$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} = 72$

$\frac{1}{2}\begin{vmatrix} -5 & -4 & -1 & 4 & -5 \\ 7 & k & -6 & 5 & 7 \end{vmatrix} = 72$

$\Rightarrow$ $k = -5$

5 marks → Matrices.

1) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Show that $A^2 - (a+d)A = (bc - ad)I_2$

③ If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ S.T. $A^2 - 5A + 7I_2 = 0$

Using above sum (formula)

a = 3, c = -1, b = 1, d = 2 | a+d = 3+2 = 5, bc = 1×-1 = -1, ad = 3×2 = 6

$\therefore A^2 - (a+d)A = (bc - ad)I_2$

$A^2 - 5A = (-1-6)I_2 = -7I_2$

$\Rightarrow A^2 - 5A + 7I_2 = 0$

—x—

④ If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ P.T. $A^2 - 4A + 5I_2 = 0$

a = 1, b = -1, c = 2, d = 3 | a+d = 1+3 = 4, bc = -1×2 = -2, ad = 1×3 = 3

$\therefore A^2 - (a+d)A = (bc - ad)I_2$

$A^2 - 4A = (-2-3)I_2 = -5I_2$

$\Rightarrow A^2 - 4A + 5I_2 = 0$

Hence.

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1) $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$

LHS :- $(AB)^T$

$= \left[\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}\right]^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$

RHS: $B^T A^T$

$= \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$

LHS = RHS ∴ Proved.

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2) $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

LHS: $(AB)^T$

$= \left[\begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}\right]^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$

RHS: $B^T A^T$

$= \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix} = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$

Verified.

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3) $A = \begin{pmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{pmatrix}$, $B = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$

$A^2 + B^2 = \begin{pmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix} + \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= I$

Hence.

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2 marks → Matrices

1) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$x = 3$ | $z = 3$

$y = 12$

2) Verify $A^2 = I$ when $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$

$A^2 = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

3) $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ find transpose of A

$A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

4) $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ find transpose of $-A$

$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

5) $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ Verify $(A^T)^T = A$

$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$

$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & 5/2 \\ 8 & 3 & 1 \end{pmatrix} = A.$

6) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$

$A + B = \begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$

7) If a matrix has 16 elements.

1×16	4×4	2×8
16×1		8×2

8) Matrix has 18 elements

1×18	2×9	3×6
18×1	9×2	6×3

9) $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Rightarrow AA^T = I$

$A \cdot A^T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$

5 marks – Relation & function

1) $f(x) = \begin{cases} 2x+7 & , x<2 \\ x^2-2 & , -2 \le x < 3 \\ 3x-2 & , x \geq 3 \end{cases}$

— x —

i) $f(4) = 10$

ii) $f(-2) = 2$

iii) $f(4) + 2f(1) = 10 - 2 = 8$

iv) $\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1-30}{1} = -31$

2) $f(x) = \begin{cases} x+2 & , x > 1 \\ 2 & , -1 \le x \le 1 \\ x-1 & , -3 < x < -1 \end{cases}$

— x —

i) $f(3) = 5$ ii) $f(0) = 2$

iii) $f(-1.5) = -2.5$ iv) $f(2) + f(-2) = 4 - 3 = 1$

3) $f(x) = \begin{cases} 6x+1 & -5 \le x < 2 \\ 5x^2-1 & 2 \le x < 6 \\ 3x-4 & 6 \le x \le 9 \end{cases}$

i) $f(-3) + f(2) = -17 + 19 = 2$

ii) $f(7) - f(1) = 17 - 7 = 10$

iii) $2f(4) + f(8) = 158 + 20 = 178$

iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{-22-14}{79-11} = \frac{-9}{17}$

2 marks

1) $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$

$A = \{3,5\}$; $B = \{2,4\}$

2) $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$

$B = \{-2, 0, 3\}$; $A = \{3,4\}$

3) $A = \{1,2,3, \ldots 45\}$, "is Square of a number".

Domain $= \{1,2,3,4,5,6\}$

Range $= \{1,4,9,16,25,36\}$

4) $\{(x,y) \mid y = x+3\}$, $x \in \{0,1,2,3,4,5\}$

Domain $= \{0,1,2,3,4,5\}$

Range $= \{3,4,5,6,7,8\}$

5) let $A = \{1,2,3,4\}$, $f(x) = x^3$

Range $= \{1, 8, 27, 64\}$

One-one and into function.

6) $f(x) = 3x - 5$, given $(a,4)$ and $(1,b)$.

$a = 3$; $b = -2$

7) $f(x) = x^2 - 1$, $g(x) = x - 2$

find 'a'. if $g \circ f(a) = 1$

$a = \pm 2$

8) $f: N \to N$, $f(x) = 3x+2$

i) Image of 1,2,3 ⇒ 5, 8, 11 (+3) (+3)

ii) pre-image of $29 = 9$

pre image of $53 = 17$

iii) f is one-one and into function.

— x —

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(6)

2 marks: Numbers & Seq

1) $1+3+5+\dots+55 \Rightarrow = \left(\frac{55+1}{2}\right)^2 = \left(\frac{56}{2}\right)^2$

$= 28^2 = 784$

2) $1+3+5+\dots$ to 40 terms

$= n^2 = (40)^2 = 1600$

3) $2+4+6+\dots+80$

$= 2[1+2+\dots+40] = 2\left[\frac{40\times41}{2}\right]$

$= 1640$

4) $1+2+3+\dots+60$

$= \frac{n(n+1)}{2} = \frac{60\times61}{2} = 1830$

5) $3+6+9+\dots+96$

$= 3(1+2+3+\dots+32)$

$= 3\left(\frac{32\times33}{2}\right) = 3\times328$

$= 1584$

6) $1+2+3+\dots+n = 666$ find 'n'

$\frac{n(n+1)}{2} = 666$

$n = 36$

7) $1+2+3+\dots+k = 325$. find $1^3+2^3+\dots+k^3 = ?$

$= (325)^2 = 105625$

8) $1^3+2^3+3^3+\dots+k^3 = 44100$

find $1+2+3+\dots+k = ?$

$\left[\frac{k(k+1)}{2}\right]^2 = 44100 = (210)^2$

$\Rightarrow 1+2+3+\dots+k = 210$

9) Aman has 532 flower pots. row 21, how many flower pots are left over

$532 = 21\times25+7$

rows $= 25$

left over flower pots $= 7$ Pots

10) $a^b \times b^a = 800$

2	800
2	400
2	200
2	100
2	50
5	25
	5

$\therefore a^b \times b^a = 2^5 \times 5^2$

$\therefore a=2, b=5$

(or) $a=5, b=2$.

11) $p^2\times q^1\times r^4\times s^3 = 3,15,000$

$= 3^2\times7^1\times5^4\times2^3$

$\therefore p=3, q=7, r=5, s=2$

12) $13824 = 2^a\times3^b$

$a=9, b=3$

13) $7\times5\times3\times2+3$

Sum $= 213 = 2+1+3 = 6$

$6\div3$ and $213\div3$

$\therefore$ 213 not a prime number.

It is a composite number.

14) find 19th term of an A.P $-11, -15, -19, \dots$

$a=-11$, $d = -15-(-11) = -15+11 = -4$

$t_n = a+(n-1)d$

$t_{19} = -11+18(-4) = -11-72 = -83$

15) Which term of an A.P, $16, 11, 6, 1, \dots$ is -54?

$a=16$

$d = 11-16 = -5$

$l = -54$

$n = \frac{l-a}{d}+1$

$n = 15$

2 Marks: Algebra:

1) find the square root of $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

—x—

$= \frac{12}{9}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

2) find the square root of $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$

3) Determine the QE, whose sum and product of roots are

i, −9, 20 ii, $\frac{5}{3}$, 4 iii, $-\frac{3}{2}$, −1

i, $\alpha+\beta = -9$

$\alpha\beta = 20$

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta=0$

$\therefore x^2+9x+20=0$

ii, $\frac{5}{3}$, 4

$\Rightarrow x^2-\frac{5}{3}x+4=0$

$(\times)3 \Rightarrow 3x^2-5x+12=0$

iii, $-\frac{3}{2}$, −1

$\Rightarrow x^2+\frac{3}{2}x-1=0$

$(\times)2 \Rightarrow 2x^2+3x-2=0$

4) find the sum and product of the roots.

i, $x^2+3x-28=0$

ii, $x^2+3x=0$

—x—

i, $a=1, b=3, c=-28$

$\alpha+\beta = -\frac{b}{a} = \frac{-3}{1} = -3$

$\alpha\beta = \frac{c}{a} = \frac{-28}{1} = -28$

ii, $a=1, b=3, c=0$

$\alpha+\beta = \frac{-3}{1} = -3$

$\alpha\beta = \frac{0}{1} = 0$

5) Determine the nature of the roots,

i, $15x^2+11x+2=0$

$a=15, b=11, c=2$

$b^2-4ac = (+11)^2-4(15)(2)$

$= 121-120 = 1 > 0$

$\therefore$ The roots are real and unequal

ii, $x^2-x-1=0$ $a=1, b=-1, c=-1$

$b^2-4ac = (-1)^2-4(1)(-1)$

$= 1+4$

$= 5$

$\therefore$ The roots are real & unequal.

iii, $9x^2-24x+16=0$ | $a=9$, $b=-24$, $c=16$

$b^2-4ac = (-24)^2-4(9)(16)$

$= 576-576 = 0$

$\therefore$ The roots are real and Equal

iv, $2x^2-2x+9=0$

$a=2, b=-2, c=9$

$b^2-4ac = (-2)^2-4(2)(9)$

$= 4-72 = -68 < 0$

6) i, Multiply $\frac{x^3}{9y^2} \times \frac{27y}{x^5}$

$= \frac{27}{9x^2y} = \frac{3}{x^2y}$

ii, $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$

$= \frac{(x+4)(x-4)}{(x+4)} \times \frac{(x+4)}{(x-4)} = (x+4)$

iii, $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

$= \frac{5t^3}{4(t-2)} \times \frac{6(t-2)}{10t} = \frac{3t^2}{4}$

iv, $\frac{14x^4}{y} \div \frac{7x}{3y^4}$

$= \frac{14x^4}{y} \times \frac{3y^4}{7x} = 6x^3y^3$

v, $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

$= \frac{3x^3z}{5y^3}$

vi, $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

$= \frac{x^3}{x-y} - \frac{y^3}{x-y}$

$= \frac{x^3-y^3}{x-y} = \frac{(x-y)(x^2+xy+y^2)}{(x-y)}$

$= (x^2+xy+y^2)$

7) Find the excluded values

i, $\frac{7p+2}{8p^2+13p+5}$

ii, $\frac{y}{y^2-25}$ iii, $\frac{t}{t^2-5t+6}$

i, $8p^2+13p+5=0 \Rightarrow (8p+5)(p+1)=0$

∴ The Excluded Values are $-\frac{5}{8}$ and -1

ii, $y^2-25=0$

$y=\pm 5$

∴ The Excluded Values are -5 and 5

iii, $t^2-5t+6=0$

$(t-2)(t-3)=0$

∴ The Excluded value are 2 and 3

8) Find square root $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

$= 2\left|\frac{y^4z^6}{x^2}\right|$

9) If the difference between a number and its reciprocal is $\frac{24}{5}$. Find the number.

Let x = Required number

$\frac{1}{x}$ = Reciprocal

Given: $x - \frac{1}{x} = \frac{24}{5}$

$\Rightarrow \frac{x^2-1}{x} = \frac{24}{5}$

$5x^2-5=24x$

$5x^2-24x-5=0$

$x = 5$ & $-\frac{1}{5}$

2 Marks :- Probability :-

1) A bag contains 5 blue, 4 Green ball. —*—

$S = \{5B + 4G\}$

$n(S) = 9$

∴ i) let A = Getting blue ball

$n(A) = 5$; $P(A) = 5/9$

ii) let B = Getting not blue ball

$n(B) = 4$

$P(B) = \frac{4}{9}$.

---

2) Two coins are tossed together.

$S = \{(HH), (HT), (TH), (TT)\}$

$n(S) = 4$

let A = Getting different faces.

$n(A) = 2$; $P(A) = 2/4$.

---

3) What is the probability that a leap year selected at random will contain 53 saturday.

leap Year = 366 days = 52 weeks + 2 days

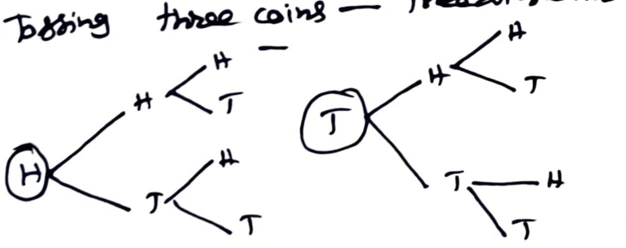
$n(S) = 7$ (1 week)

let A = Getting 53^{rd} Saturday

$n(A) = 2$; $P(A) = 2/7$.

---

4) Tossing three coins — Tree diagrams



$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

$n(S) = 8$.

---

5) A die is rolled and a coin is tossed. Die shows an odd no and coin is head —*—

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$n(S) = 12$

let A = Getting odd no. and a head

$A = \{1H, 3H, 5H\}$; $n(A) = 3$

$P(A) = 3/12 = 1/4$

---

6) A coin is tossed twice.

$n(S) = 8$

let A = Getting two consecutive tails

$A = \{(HTT), (TTH), (TTT)\}$

$n(A) = 3$; $P(A) = \frac{3}{8}$

---

7) $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.37 + 0.42 - 0.09$

$= 0.7$

---

8) $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \text{ and } B) = \frac{1}{8}$

i) $P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$

$= \frac{5}{8}$

ii) $P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

---

9) $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$

$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{10+6-5}{15} = \frac{11}{15}$

---

10) $P(A) = 0.42$, $P(B) = 0.48$, $P(A \cap B) = 0.16$

i) $P(\text{not } A) = 0.58$

ii) $P(\text{not } B) = 0.52$

iii) $P(A \text{ or } B) = 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$= 0.74$

$1.00 - .42 = .58$

$1.00 - .48 = .52$

11) In a two children family, find the prob. that there is atleast one girl in a family.

$S = \{BB, BG, GB, GG\}$ $\therefore n(S) = 4$

Let A = Atleast one girl

$n(A) = 3$ $\therefore P(A) = 3/4$ //

12) A bag contains 5 white and some black balls.

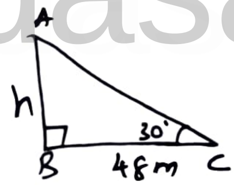
Given: $P(B) = 2 \times P(W)$

$= 2 \times 5$

$= 10$ balls.

2 Marks – Trigonometry

1) A tower stands vertically on the ground. From a point on the ground .... angle of elevation top of tower is 30°. Find height of tower.



$\tan 30° = \frac{h}{48}$

$\frac{1}{\sqrt{3}} = \frac{h}{48}$

$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3} = 16\sqrt{3}$ m

2) A kite:

$\sin 60° = \frac{75}{AC}$

$\frac{\sqrt{3}}{2} = \frac{75}{AC}$

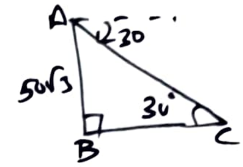
$AC = \frac{75 \times 2}{\sqrt{3}} = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 15\sqrt{3}$ m

3) Find the angle of elevation.

$\tan \theta = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30°$

5) From the top of a rock $50\sqrt{3}$

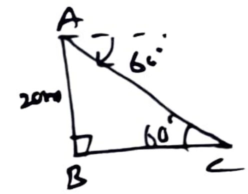


$\tan 30° = \frac{50\sqrt{3}}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$

$BC = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 = 150$ m

6) A player sitting



$\tan 60° = \frac{20}{BC}$

$\sqrt{3} = \frac{20}{BC}$

$\Rightarrow BC = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} = 11.55$ m.

7) P.T. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

LHS: $\sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$

$= \frac{1+\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$ = RHS

8) P.T. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

LHS: $\sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

= RHS.

Let A = $\{x \in N \mid 1 < x < 4\}$, B = $\{x \in W \mid 0 \leq x < 2\}$ and C = $\{x \in N \mid x < 3\}$. Then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:

Given $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$,

$B = \{x \in W \mid 0 \leq x < 2\} = \{0, 1\}$,

$C = \{x \in N \mid x < 3\} = \{1, 2\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0,1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C)$

$= \{2, 3\} \times \{0, 1, 2\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= (2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C)$

$= \{(2,0), (2,1), (3, 0), (3,1)\} \cup \{(2,1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3,0), (3, 1), (3, 2)\}$ ..... (2)

From (1) = (2).

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\}$

$= \{(2, 1), (3, 1)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C)$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$ ..... (2)

(1) = (2)

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence it is Verified

If A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}, Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution:

Given A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}

LHS:

$A \times A = \{5, 6\} \times \{5, 6\}$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (1)

RHS = $(B \times B) \cap (C \times C)$.

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B \times B) \cap (C \times C)$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (2)

$\therefore$ From (1) and (2). LHS = RHS

Given A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution:

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$

$A \cap C = \{3\}$,

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D)$

$= \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ ..... (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1,5), (2, 2), (2, 3), (2, 5),(3, 2), (3, 3), (3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ..... (2)

(1), (2) are equal.

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

Hence it is verified.

Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \le 4\}$ and $C = \{3, 5\}$. Verify that
(i) $A\times(B \cup C) = (A\times B)\cup(A\times C)$
(ii) $A\times(B \cap C)=(A\times B) \cap (A\times C)$
(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution:

Given:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \le 4\}$

$\Rightarrow B = \{2, 3, 4\}$; $C = \{3, 5\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$A\times(B\cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$=\{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C)$

$= \{(0, 2), (0, 3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ..... (2)

$\therefore$(1) = (2) Hence Verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ..... (1

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)$

$A \times C = \{0,1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ..... (2

$\therefore$ (1) = (2). Hence Proved.

iii. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$

$= \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3$
$(3,5), (4,3), (4,5)\}$ ..... (1

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\}$

$= \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5$

$\therefore (A \times C) \cup (B \times C)$

$= \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ..... (2

$\therefore$ From (1) and (2) LHS = RHS.

Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that
(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$
(ii) $A \times (B - C) = (A \times B) - (A \times C)$

Solution:

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$ $C = \{2\}$

To verify $(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\}$

$= \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$

$\therefore (A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ .....(1)

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\}$

$= \{(2,2), (3,2), (5,2), (7,2)\}$

$(A \times C) \cap (B \times C)$

$= \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ..... (2)

$\therefore$ From (1) and (2), LHS = RHS

ii. To verify $A \times (B - C)=(A \times B) - (A \times C)$

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$

$A\times(B - C) = \{1, 2, 3, 4, 5, 6, 7\}\times\{3, 5, 7\}$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3, 7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (1)

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5),(6,7), (7,2), (7,3), (7,5), (7,7)\}$

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$(A \times B) - (A \times C)$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (2)

(1), (2) are equal.

$\therefore A \times (B - C) = (A \times B) - (A \times C)$.

Hence it is verified.

Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

(ii) $\{(x, y) | y = x + 3$, x, y are natural numbers $< 10\}$

Solution:

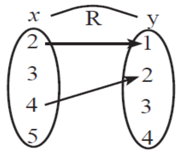
i. $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

$x = 2y$

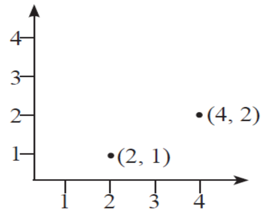
$f(x) = \frac{x}{2}$; $f(2) = \frac{2}{2} = 1$; $f(3) = \frac{3}{2}$;

$f(4) = \frac{4}{2} = 2$; $f(5) = \frac{5}{2}$

a) An Arrow diagram



b) Graph



c) Roster Form

{(2, 1), (4, 2)}

ii. $\{(x, y) | y = x + 3$, x, y are natural numbers $< 10\}$

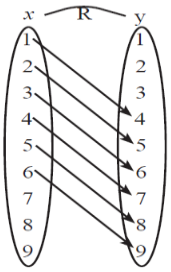
Solution:

$f(x) = x + 3$;

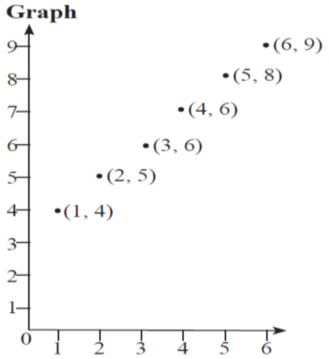
$f(1) = 4$; $f(2) = 5$; $f(3) = 6$;

$f(4) = 7$; $f(5) = 8$; $f(6) = 9$

a) An Arrow diagram



b) Graph



c) Roster Form

(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f : A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function

(i) by arrow diagram

(ii) in a table form

(iii) as a set of ordered pairs

(iv) in a graphical form

Solution:

A = {1, 2, 3, 4}, B = {2, 5, 8, 11, 14}

$f(x) = 3x - 1$

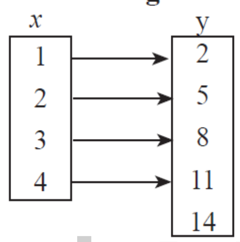
$f(1) = 3(1) - 1 = 3 - 1 = 2$;

$f(2) = 3(2) - 1 = 6 - 1 = 5$ $f(3) = 3(3) - 1$ $= 9 - 1 = 8$;

$f(4) = 3(4) - 1 = 12 - 1 = 11$.

R = {(1, 2), (2, 5), (3, 8), (4, 11)}

i) Arrow Diagram



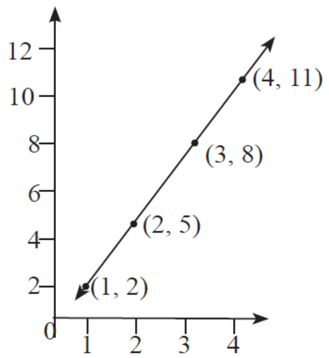
ii) Table

x	1	2	3	4
y	2	5	8	11

iii) Set of Ordered pairs

{(1, 2), (2, 5), (3, 8), (4, 11)}

iv) Graphical Form



Let f : A → B be a function defined by $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9} . Represent f by

i) set of ordered pairs ii) a table
iii) an arrow diagram iv) a graph

Solution:

Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0$

$x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2$

$x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

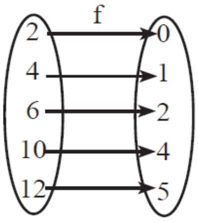
i) Set of Ordered Pairs:

f = {(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

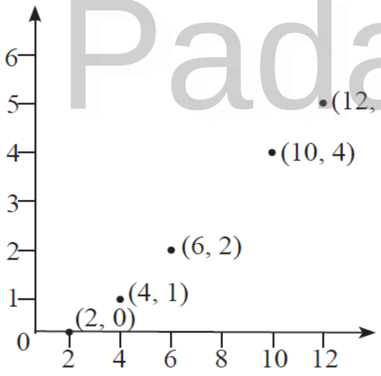
ii) Table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

iii) Arrow Diagram



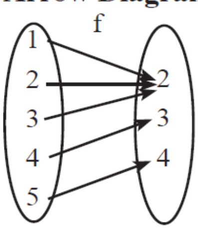
iv) Graph



Represent the function
f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} throug
(i) an arrow diagram
(ii) a table form
(iii) a graph

Solution:

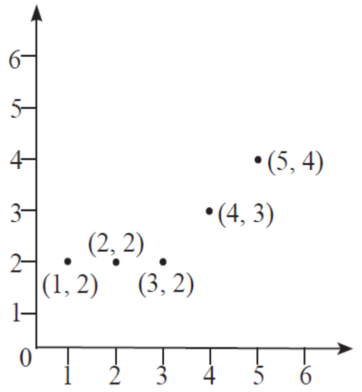
i) Arrow Diagram



ii) Table Form:

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

iii) Graph



If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Solution:

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore P_1 = 2, \ P_2 = 3, \ P_3 = 5, \ P_4 = 7$

$x_1 = 3, \ x_2 = 4, \ x_3 = 2, \ x_4 = 1$

Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution:

A.P. $\Rightarrow x, 10, y, 24, z$

That is $y = \frac{10 + 24}{2} = \frac{34}{2} = 17$

$\therefore$ A.P $= x, 10, 17, 24, z$

Here we know that $d = 17 - 10 = 7$

$\therefore x = 10 - 7 = 3$

$z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31.$

Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution:

$S_n = 5 + 55 + 555 + \ldots + n$ terms

$= 5[1 + 11 + 111 + \ldots + n \text{ terms}]$

$= \frac{5}{9}[9 + 99 + 999 + \ldots + n \text{ terms}]$

$= \frac{5}{9}[10 - 1 + 100 - 1 + 1000 - 1 + \ldots n \text{ terms}]$

$= \frac{5}{9}[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$= \frac{50}{81}\left[(10^n - 1) - \frac{5}{9}n\right]$

3 + 33 + 333 + ... n

$= 3(1 + 11 + 111 + \ldots + n \text{ terms})$

$= \frac{3}{9}(9 + 99 + 999 + \ldots + n \text{ terms})$

$= \frac{3}{9}((10-1) + (100-1) + (1000-1) + \ldots + n \text{ terms})$

$= \frac{3}{9}(10 + 100 + 1000 + \ldots + n \text{ terms}) - (1 + 11 + 111 + \ldots + n \text{ terms})$

$= \frac{3}{9}\left(10\left(\frac{10^n - 1}{9}\right) - n\right)$

$= \frac{30}{81}(10n - 1) - \frac{3n}{9}$

Find the sum of $9^3 + 10^3 + \ldots + 21^3$

Solution:

$9^3 + 10^3 + \ldots + 21^3$

$= (1^3 + 2^3 + 3^3 \ldots + 21^3) - (1^3 + 2^3 + 3^3 \ldots + 8^3)$

$= \left[\frac{21\times(21+1)}{2}\right]^2 - \left[\frac{8\times(8+1)}{2}\right]^2$

$= (231)^2 - (36)^2$

$= 52065$

Find the sum of the following series

(i) $6^2 + 7^2 + 8^2 + \ldots + 21^2$

(ii) $10^3 + 11^3 + 12^3 + \ldots + 20^3$

i. $6^2 + 7^2 + 8^2 + \ldots + 21^2$

$= (1^2 + 2^2 + 3^2 \ldots + 21^2) - (1^2 + 2^2 + 3^2 + \ldots + 5^2)$

$= \frac{21\times(21+1)(42+1)}{6} - \frac{5\times(5+1)(10+1)}{6}$

$= \frac{21\times 22\times 43}{6} - \frac{5\times 6\times 11}{6}$

$= 3311 - 55 = 3256$

ii. $10^3 + 11^3 + 12^3 + \ldots + 20^3$

$= 1^3 + 2^3 + 3^3 + \ldots + 20^3 - 1^3 + 2^3 + 3^3 + \ldots + 9^3$

$= \left[\frac{20\times 21}{6}\right]^2 - \left[\frac{9\times 10}{3}\right]^2$

$= [210]^2 - (45)^2$

$= 44100 - 2025 = 42075$

Find the sum of $15^2 + 16^2 + 17^2 + \ldots + 28^2$

$15^2 + 16^2 + 17^2 + \ldots + 28^2$

$= (1^2 + 2^2 + 3^2 \ldots + 28^2) - (1^2 + 2^2 + 3^2 \ldots + 14^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{28\times 29\times 57}{2\times 3} - \frac{14\times 15\times 29}{2\times 3}$

$= 14 \times 29 \times 19 - 7 \times 5 \times 29$

$= 7714 - 1015 = 6699$

Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, …, 24 cm. How much area can be decorated with these colour papers?

Solution:

The Required Area

$= 10^2 + 11^2 + 12^2 + \ldots + 24^2$

Area $= (1^2 + 2^2 + 3^2 + \ldots + 24^2) - (1^2 + 2^2 + \ldots + 9^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{24\times 25\times 49}{6} - \frac{9\times 10\times 19}{6}$

$= 4900 - 285 = 4615 \text{ cm}^2$

Therefore Rekha has 4615 cm^2 colour paper.

5 Marks

Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Solution:

	8 −1 1
8	64 −16 17 −2 1
	(−) 64
16 −1	−16 17
	(+) −16 (−) 1
16 −2	16 −2 1
1	(−) 16 (+) −2 (−) 1
	0

Required Square root = $|8x^2 - x + 1|$

If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution:

	3 2 4
3	9 12 28 a b
	(−) 9
6 2	12 28
	(−) 12 (−) 4
6 4 4	24 a b
	(−) 24 (−) 16 (−) 16
	a = 16, b = 16

Find the square root of the following polynomials by division method

Solution:

i. $x^4 - 12x^3 + 42x^2 - 36x + 9$

	1 −6 3
1	1 −12 42 −36 9
	(−) 1
2 −6	−12 42
	(+) −12 (−) 36
2 −12	6 −36 9
3	(−) 6 (+) −36 (−) 9
	0

Required Square root = $|x^2 - 6x + 3|$

ii. $37x^2 - 28x^3 + 4x^4 + 42x + 9$

	2 −7 −3
2	4 −28 37 42 9
	(−) 4
4 −7	−28 37
	(+) −28 (−) 49
4 −14	−12 42 9
−3	(+) −12 (−) 42 (−) 9
	0

Required Square root = $|2x^2 - 7x - 3|$

iii. $16x^4 + 8x^2 + 1$

	4 0 1
4	16 0 8 0 1
	(−) 16
8 0	0 8
	0 0
8 0 1	8 0 1
	(−) 8 (−)0 (−)1
	0

Required Square root = $|4x^2 + 1|$

iv. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

	11 −9 −12
11	121 −198 −183 216 144
	(−)121
22 −9	− 198 −183
	(+) −198 (−)81
22 −18 −12	−264 216 144
	(+) −264 (−)216 (−)144
	0

Required Square root = $|11x^2 - 9x - 12|$

Find the values of a and b if the following polynomials are perfect squares

i. $4x^4 - 12x^3 + 37x^2 + bx + a$

Solution:

	2 −3 7
2	4 −12 37 b a
	(−) 4
4 −3	−12 37
	(+) −12 (−) 9
4 −6 7	28 b a
	(−) 28 (+)− 42 (−) 49
	a = 49 , b = −42

ii. $ax^4 + bx^3 + 361x^2 + 220x + 100$

Solution:

	10 11 12
10	100 220 361 b a
	(–)100
20 11	220 361
	(–) 220 (–)121
20 22 12	240 b a
	(–) 240 (–)264 (–)144
	a = 144 , b = 264

Find the values of m and n if the followi polynomials are perfect sqaures

i. $36x^4 - 60x^3 + 61x^2 - mx + n$

Solution:

	6 –5 3
6	36 –60 61 –m n
	(–) 36
12 –5	–60 61
	(+) –60 (–)25
12 –10 3	36 –m n
	(–) 36 (+)– 30 (–) 9
	–m = –30, m = 30
	n = 9

ii. $x^4 - 8x^3 + mx^2 + nx + 16$

Solution:

	1 –4 4
1	1 –8 m n 16
	(–) 1
2 –4	–8 m
	(+) –8 (–)16
2 –8 4	m–16 n 16
	(–) 8 (+)– 32 (–) 16
	0

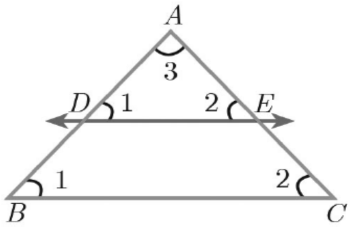
$\frac{m-16}{2} = 4$

m – 16 = 8, n = –32

m = 8 + 16

m = 24

State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem.



Statement:

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given:

In ΔABC, D is a point on AB and E is a point on AC

To Prove:

$\frac{AD}{DB} = \frac{AE}{EC}$

State and Prove Angle Bisector Theorem.

Statement:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

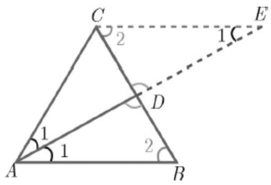
Proof

Given:

In ΔABC, AD is the internal bisector

To Prove:

$\frac{AB}{AC} = \frac{BD}{CD}$.



Construction:

State and Prove Pythagoras Theorem.

Statement:

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

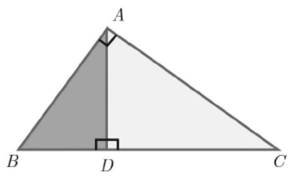
Proof

Given:

In ΔABC, ∠A = 90°

To Prove:

$AB^2 + AC^2 = BC^2$



Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution:

When we roll two dice, the sample space is given by

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) };

n (S) = 36

Let A = Getting sum equal to 4

A = {(1,3) , (2,2),(3,1)} ∴n(A) = 3

P(A) = 3/36 = 1/12

Let B = Getting greater than 10

B = { (5,6),(6,5),(6,6) ∴ n(B) = 3

P(B) = 3/36 = 1/12

Let C = Getting out come less than 13

∴n(c) = 36 ∴ P(C) = 36/36 = 1

Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

B = {(1,3), (2,2), (3,1)}

$\therefore A \cap B = \{(2,2)\}$

Then, n(A) = 6 , n(B) = 3 , $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1

Solution:

n(S) = 36

i) A = Probability of getting Doublets (Equal numbers on both dice)

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

$n(A) = 6; P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

B = Probability of getting the product of the prime number

ii) B = {(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)}

$n(B) = 6; P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

C = Probability of getting sum of the prime number.

iii) C = {(1,1), (2,1), (1,2), (1,4), (4,1), (1,6), (6,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5)}

$n(C) = 14; P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

iv) D = Probability of getting the sum as 1

$n(D) = 0; P(D) = \frac{n(D)}{n(S)} = 0$

6. Three fair coins are tossed together. Find the probability of getting
(i) all heads
(ii) atleast one tail
(iii) atmost one head
(iv) atmost two tails

Solution:

Possible Outcomes = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No.of possible outcomes,

$n(S) = 2 \times 2 \times 2 = 8$

i) A = Probability of getting all heads

A = {HHH} n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

ii) B = Probability of getting atleast one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(B) = 7 \quad P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

iii) C = Probability of getting atmost one hea

C = {TTT, TTH, THT, HTT}

$n(C) = 4 \quad P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

iv) D = Probability of getting atmost two tai

D = {TTH, THT, HTT, HHT, HTH, THH
HHH}

$n(D) = 7 \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

A bag contains 5 red balls, 6 white balls green balls, 8 black balls. One ball is dra at random from the bag. Find the probabil that the ball drawn is

(i) white

(ii) black or red

(iii) not white

(iv) neither white nor black

Solution:

S = {5 Red, 6 White, 7 Green, 8 Black}

n(S) = 26

i) A – probability of getting white balls

$n(A) = 6; \quad P(A) = \frac{6}{26} = \frac{3}{13}$

ii) B – Pabability of getting black (or) red balls

$n(B) = 8 + 5 = 13; \; P(B) = \frac{13}{26} = \frac{1}{2}$

iii) C – Probability of not getting white balls

$n(C) = 20; \quad P(C) = \frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting of neither wh nor black

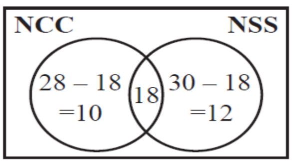
$n(D) = 12; \quad P(D) = \frac{12}{26} = \frac{6}{13}$

15. In a class of 50 students, 28 opted for NC 30 opted for NSS and 18 opted both N(and NSS. One of the students is selected random. Find the probability that

(i) The student opted for NCC but not N!

(ii) The student opted for NSS but not NC

(iii) The student opted for exactly one them.



Solution:

Total number of students n(S) = 50

i. A : A : opted only NCC but not NSS

$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50} = \frac{1}{5}$

ii. B : opted only NSS but not NCC

$P(B) = \frac{n(B)}{n(S)} = \frac{12}{50} = \frac{6}{25}$

iii. C : opted only one

$P(C) = \frac{n(C)}{n(S)} = \frac{(10+12)}{50} = \frac{22}{50} = \frac{11}{25}$

Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) =36

A = Probability of getting an even number in the first die.

A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6))}

$n(A) = 18; \quad P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

B = Probability of getting a total face sum is 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

$n(B) = 5; \quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B = \{(2,6), (4, 4), (6, 2)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36} = \frac{5}{9}$

A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution:

S = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Let A = Multiple of 7

A= {7, 21, 35}, n(A) = 3

$P(A) = \frac{3}{18}$

Let B = a Prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

n(B) = 11; $P(B) = \frac{11}{18}$

$A \cap B = \{7\} \Rightarrow n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{18}$

P(Either A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Probability of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

n(A) = 7 $P(A) = \frac{7}{8}$

B = Probability of getting atmost 2 heads

B = {HHT, HTH, THH, HHH}

n(B) = 4 $P(B) = \frac{4}{8}$

$P(A \cap B) = \frac{4}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

A flower is selected at random from a basket containing 80 yellow, 70 red and 50 white flowers. Find the probability of selecting a yellow or red flower?

Solution:

Total number of flowers

n(S) = 80 + 70 + 50 = 200

No. of yellow flowers n(Y) = 80

$\therefore P(Y) = \frac{n(Y)}{n(S)} = \frac{80}{200}$

No. of red flowers n(R) = 70

$\therefore P(R) = \frac{n(R)}{n(S)} = \frac{70}{200}$

Y and R are mutually exclusive

$P(Y \cup R) = P(Y) + P(R)$

Probability of drawing either a yellow or red flower

$P(Y \cup R) = \frac{80}{200} + \frac{70}{200} = \frac{150}{200} = \frac{3}{4}$

A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Exactly 2 Heads

A = {HHT, HTH, THH}

n(A) = 3

$\Rightarrow P(A) = \frac{3}{8}$

B = Atleaset one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

n(B) = 7

$\Rightarrow P(B) = \frac{7}{8}$

C = Consecutively 2 heads

C = {HHH, HHT,THH}

$n(C) = 3 \Rightarrow P(C) = \frac{3}{8}$

$P(A \cap B) = \frac{3}{8}$; $P(B \cap C) = \frac{2}{8}$

$P(A \cap C) = \frac{2}{8}$; $P(A \cap B \cap C) = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$

$= \frac{15-7}{8} = \frac{8}{8} = 1$

Geometry 2 Marks

If ΔABC is similar to ΔDEF such that BC = 3 cm, EF= 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

gives $$\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

$$\text{Area } (\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm² and BC = 2.1 cm. Find the length of EF.

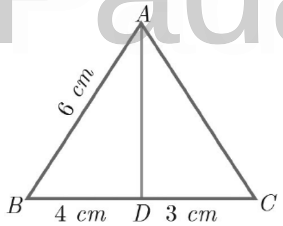
$$\frac{Area\ of(\Delta ABC)}{Area\ of(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow EF^2 = (2.1)^2 \times \frac{16}{9}$$

$$\Rightarrow EF = 2.1 \times \frac{4}{3} = 2.8 \text{ cm}$$

. **In the Figure, AD is the bisector of ∠A. If BD = 4 cm, DC = 3 cm and AB = 6 cm. find AC.**



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{4}{3} = \frac{6}{AC} \text{ gives } 4AC = 18$$

$$\text{Hence } AC = \frac{9}{2} = 4.5 \text{ cm}$$

In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{3}{4} = \frac{x}{15 - x}$$

$$3(15 - x) = 4x$$

$$45 - 3x = 4x$$

$$45 = 7x$$

$$x = \frac{45}{7} = 6.43 \text{ cm}$$

What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution:

Let x be the length of the ladder.

BC = 4 ft.

AC = 7 ft.

By Pythagoras Theorem we have,

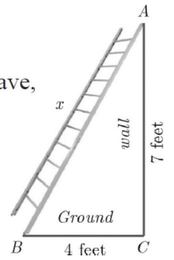
$AB^2 = AC^2 + BC^2$

$x^2 = 7^2 + 4^2$ gives $x^2 = 49 + 16$

$x^2 = 65$

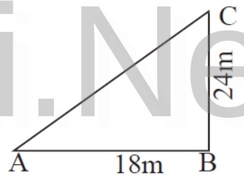
Hence $x = \sqrt{65}$

$= 8.1$



. **A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?**

Solution:



In ΔABC, $AC^2 = AB^2 + BC^2$

$AC^2 = (18)^2 + (24)^2 = 324 + 576$

$AC^2 = 900$

$AC = \sqrt{900}$

$AC = 30$ m

∴ The distance from the starting point is 30 m

Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

$OP^2 = OT^2 + PT^2$

(By Phythagorous Theorem)

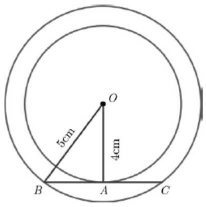
$5^2 = 3^2 + PT^2$

$PT^2 = 25 - 9 = 16$

Length of the tangent PT = 4 cm.

If radii of two concentric circles are 4cm and 5cm then find the length of the chord of one circle which is a tangent to the other circle.

Solution:



OA = 4 cm,

OB = 5 cm,

also OA ⊥ BC

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$

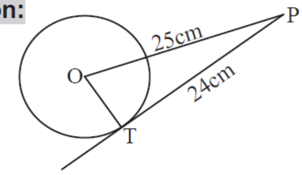
$AB^2 = 25 - 16 = 9$

Therefore AB = 3 cm, BC = 2AB

hence, BC = 2 × 3 = 6 cm

. **The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?**

Solution:



From the figure, $r = \sqrt{OP^2 - AP^2}$

$= \sqrt{25^2 - 24^2}$

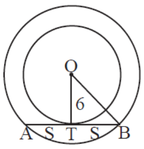
$= \sqrt{625 - 576}$

$= \sqrt{49}$

r = 7cm

23. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Solution:



AB = 16 cm and OC = 6 cm

then, AC = CB = 8cm

To find OB.(OB is radius of larger circle)

By Pythagoras,

$OB = \sqrt{OC^2 + BC^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

OB = 10 cm

Coordinate Geometry 2 Marks

Find the slope of a line joining the given points (i) (–6, 1) and (–3, 2) (ii) (14, 10) and (14, –6)

i) (–6, 1) and (–3, 2)

Slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-1}{-3-(-6)}$

$= \dfrac{2-1}{-3+6}$

∴ Slope, $m = \dfrac{1}{3}$

ii) (14, 10) and (14, –6)

Slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-10}{14-14} = \dfrac{-16}{0}$

∴ Slope, $m = \dfrac{-16}{0}$

The slope is undefined.

The line r passes through the points (–2, 2) and (5, 8) and the line s passes through the points (–8, 7) and (–2, 0). Is the line r perpendicular to s?

Solution:

The slope of line r is $m_1 = \dfrac{8-2}{5+2} = \dfrac{6}{7}$

The slope of line r is $m_2 = \dfrac{0-7}{-2+8} = \dfrac{-7}{6}$

The product of slopes $= \dfrac{6}{7} \times \dfrac{-7}{6} = -1$

That is, $m_1 m_2 = -1$

Therefore, the line r is perpendicular to line s.

The line p passes through the points (3, –2), (12, 4) and the line q passes through the points (6, –2) and (12, 2) . Is p parallel to q?

Solution:

The slope of line p is $m_1 = \dfrac{4+2}{12-3} = \dfrac{6}{9} = \dfrac{2}{3}$

The slope of line q is $m_2 = \dfrac{2+2}{12-6} = \dfrac{4}{6} = \dfrac{2}{3}$

Thus, slope of line p = slope of line q.

Therefore, line p is parallel to the line q.

Find the slope of a line joining the points
(i) (5, $\sqrt{5}$) with the origin
(ii) (sin θ, –cosθ) and (–sinθ, cosθ)

Solution:

i) Given points are (5, $\sqrt{5}$) and (0, 0)

Slope = m

$= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-\sqrt{5}}{0-5}$

$= \dfrac{\sqrt{5}}{5} = \dfrac{1}{\sqrt{5}}$

ii) Given points are (sinθ, –cosθ) and (–sinθ, cosθ)

Slope = m

$= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{\cos\theta - (-\cos\theta)}{-\sin\theta - \sin\theta}$

$= \dfrac{2\cos\theta}{-2\sin\theta} = -\cot\theta$

Find the equation of a straight line whose
(i) Slope is 5 and y intercept is –9
(ii) Inclination is 45° and y intercept is 11

Solution:

i) Given Slope, m = 5, y intercept, c = –9

Therefore, equation of a straight line is,

$$y = mx + c$$
$$y = 5x - 9$$
$$0 = 5x - y - 9$$

∴ Required equation is $5x - y - 9 = 0$

ii) Given, θ=45°, y intercept, c = 11

Slope, m = tan θ

m = tan 45°

Slope, m = 1

y intercept, C = 11

Therefore, equation of a straight line is,

$$y = mx + C$$
$$y = 1x + 11$$
$$0 = x - y + 11$$

∴ Required equation is $x - y + 11 = 0$

Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$

Solution:

$$8x - 7y + 6 = 0$$
$$8x + 6 = 7y$$
$$(\div 7)\quad \frac{8}{7}x + \frac{6}{7} = \frac{7}{7}y$$
$$\frac{8}{7}x + \frac{6}{7} = y$$

Comparing $y = mx + C$

Slope, $m = \frac{8}{7}$

y intercept, $C = \frac{6}{7}$

Find the equation of a line passing through the point (3, –4) and having slope $\frac{-5}{7}$

Solution:

$(x_1, y_1) = (3, -4)$

Slope, $m = -\frac{5}{7}$

Equation of the straight line

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{5}{7}(x - 3)$$
$$7(y + 4) = -5(x - 3)$$
$$7y + 28 = -5x + 15$$
$$5x + 7y + 28 - 15 = 0$$
$$5x + 7y + 13 = 0$$

Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.

x intercept a = –9;

my intercept to b = 4

Find the equation of a straight line which has slope –5/4 and passing through the point (–1, 2).

Solution:

Given a point (–1, 2) and slope, $\frac{-5}{4}$

The required equation, $y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - 2 = \frac{-5}{4}(x - (-1))$$
$$\Rightarrow \quad 4y - 8 = -5x - 5$$
$$\Rightarrow \quad 5x + 4y - 3 = 0$$

Find the equation of a line whose intercepts on the x and y axes are given below.
(i) 4, –6 (ii) –5, $\frac{3}{4}$

Solution:

x intercept, a = 4, y intercept, b = –6

Equation of the straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\frac{x}{4} + \frac{y}{-6} = 1$$
$$\frac{x}{4} - \frac{y}{6} = 1$$
$$\frac{6x - 4y}{24} = 1$$
$$\frac{2(3x - 2y)}{24} = 1$$
$$\frac{3x - 2y}{12} = 1$$
$$3x - 2y = 12$$
$$3x - 2y - 12 = 0$$

x intercept, a = –5, y intercept, b = $\frac{3}{4}$

Equation of the straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\frac{x}{-5} + \frac{y}{\frac{3}{4}} = 1$$
$$\frac{x}{-5} + \frac{4y}{3} = 1$$
$$\frac{3x - 20y}{-15} = 1$$
$$3x - 20y = -15$$
$$3x - 20y + 15 = 0$$

Find the intercepts made by the following lines on the coordinate axes.

(i) $3x - 2y - 6 = 0$ (ii) $4x + 3y + 12 = 0$

Solution:

Intercepts form : $\frac{x}{a} + \frac{y}{b} = 1$

$\because$ a - x intercepts, b - y intercepts

$3x - 2y - 6 = 0$

$\Rightarrow 3x - 2y = 6 \Rightarrow \frac{3x}{6} - \frac{2y}{6} = 1$

$\Rightarrow \frac{x}{2} + \frac{y}{-3} = 1$

$\Rightarrow \therefore$ a = 2, b = –3

i) $4x + 3y + 12 = 0$

$4x + 3y = -12 \ (\div -12)$

$\Rightarrow \frac{4x}{-12} + \frac{3y}{-12} = 1$

$\Rightarrow \frac{x}{-3} + \frac{y}{-4} = 1$

$\Rightarrow \therefore$ a = –3, b = –4

Find the slope of the straight line $6x + 8y + 7 = 0$.

Solution:

Given $6x + 8y + 7 = 0$

Slope m = $-\frac{\text{coefficient of } x}{\text{coefficient of } y} = -\frac{6}{8} = -\frac{3}{4}$

Therefore, the slope of the straight line is $-\frac{3}{4}$.

Find the slope of the line which is

(i) parallel to $3x - 7y = 11$

(ii) perpendicular to $2x - 3y + 8 = 0$

Solution:

Given straight line is $3x - 7y = 11$

gives, $3x - 7y - 11 = 0$

Slope, m = $\frac{-3}{-7} = \frac{3}{7}$

Since parallel line have same slopes, slope of any line parallel to

$3x - 7y = 11$ is $\frac{3}{7}$

i) Given straight line is $2x - 3y + 8 = 0$

Slope, m = $\frac{-2}{-3} = \frac{2}{3}$

Some product of slope is –1 for perpendicular lines, slope of any line perpendicular to

$2x - 3y + 8 = 0$ is $\frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution:

Slope of the straight line $2x + 3y - 8 = 0$

$m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{2}{3}$

Slope of the straight line $4x + 6y + 18 = 0$

$m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{2}{3}$

Here $m_1 = m_2$

That is, slopes are equal.

Hence, the two straight lines are parallel.

. Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution:

Slope of the straight line $x - 2y + 3 = 0$

$m_1 = -\left(\frac{a}{b}\right) = -\left(\frac{1}{-2}\right) = \frac{1}{2}$

Slope of the straight line $6x + 3y + 8 = 0$

$m_2 = -\left(\frac{a}{b}\right) = -\left(\frac{6}{3}\right) = -2$

$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

Product of the slopes = –1

Hence, the two straight lines are perpendicular.

Find the slope of the following straight lines (i) $5y - 3 = 0$ (ii) $7x - \frac{3}{17} = 0$

Solution:

i) $5y - 3 = 0$

$\therefore$ Slope, m = $\frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$= \frac{0}{5} = 0$

ii) $7x - \frac{3}{17} = 0$

$\Rightarrow 7x = \frac{3}{17}$

$\Rightarrow 0y + 7x + \frac{3}{17}$

$\therefore$ Slope , m = $\frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$m = \frac{-7}{0}$

$\therefore m = \infty$ (undefined)

. Find the slope of the line which is

(i) parallel to $y = 0.7x - 11$

(ii) perpendicular to the line $x = -11$

i) $y = 0.7x - 11$

This line parallel to $y = 0.7x - k$

$\therefore$ The slope of the required line is 0.7

ii) $x = -11$

$\Rightarrow x + 0y + 11 = 0$

(1) line perpendicular to $0x - y + k = 0$

$\Rightarrow y = 0x + k$

$\therefore$ The Slope of the required line is 0.

8. Statistics and Probability

2 Marks

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:

Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

Coefficient of range $= \frac{L-S}{L+S}$

Coefficient of range $= \frac{67-18}{67+18} = \frac{49}{85} = 0.576$

2. Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution:

Here

Largest value, L = 28

Smallest Value, S = 18

Range R = L − S

R = 28 − 18 = 10 Years.

3. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution:

Range R = 13.67

Largest value L = 70.08

Range R = L − S

13.67 = 70.08 − S

S = 70.08 − 13.67 = 56.41

Therefore, the smallest value is 56.41

4. Find the range and coefficient of range of following data

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

i. 63, 89, 98, 125, 79, 108, 117, 68

L = 125, S = 63

Range, R = L − S = 125 − 63 = 62

Coefficient of Range $= \frac{L-S}{L+S}$

$= \frac{125-63}{125+63} = \frac{62}{188} = 0.33$

ii. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4, S = 13.6

Range, R = L − S = 61.4 − 13.6 = 47.8

Coefficient of Range $= \frac{L-S}{L+S}$

$= \frac{47.8}{61.4+13.6} = \frac{47.8}{75.0} = 0.64$

5. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution:

Range, R = 36.8

Smallest Value, S = 13.4

Largest Value, L = R + S

= 36.8 + 13.4 = 50.2

6. Calculate the range of the following data.

Income	400-450	450-500	500-550
Number of workers	8	12	30
Income	550-600	600-650	
Number of workers	21	6	

Solution:

Given: Largest Value, L = 650

Smallest Value,, S = 400

∴ Range = L − S = 650 − 400 = 250

7. Find the standard deviation of first 21 natural numbers.

Solution:

Standard Deviation of first 21 natural numbers,

$\sigma = \sqrt{\frac{n^2-1}{12}}$

$= \sqrt{\frac{(21)^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$

$= \sqrt{36.66} = 6.05$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution:

standard deviation of a data, σ = 4.5

each value of the data decreased by 5,

the new standard deviation does not change and it is also 4.5.

If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

The new standard deviation = 3.6/3 = 1.2

The new variance $= 1.2^2 = 1.44$