

# 10<sup>TH</sup> MATHEMATICS COMPLETE

## QUESTION BANK 2024-2025

### CHAPTER - 1. RELATIONS AND FUNCTIONS

**Example 1.1** If  $A = \{1,3,5\}$  and  $B = \{2,3\}$  then (i) find  $A \times B$  and  $B \times A$ .

(ii) Is  $A \times B = B \times A$ ? If not why? (iii) Show that  $n(A \times B) = n(B \times A) = n(A) \times n(B)$

**Example 1.2** If  $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$  then find  $A$  and  $B$ .

**Example 1.3** Let  $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$ ,  $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$  and  $C = \{x \in \mathbb{N} \mid x < 3\}$ .

Then verify that (i)  $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

### Exercise 1.1

1. Find  $A \times B$ ,  $A \times A$  and  $B \times A$

(i)  $A = \{2, -2, 3\}$  and  $B = \{1, -4\}$

(ii)  $A = B = \{p, q\}$

(iii)  $A = \{m, n\}$ ;  $B = \phi$

2. Let  $A = \{1,2,3\}$  and  $B = \{x \mid x \text{ is a prime number less than } 10\}$ . Find  $A \times B$  and  $B \times A$ .

3. If  $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$  find  $A$  and  $B$ .

4. If  $A = \{5,6\}$ ,  $B = \{4,5,6\}$ ,  $C = \{5,6,7\}$ , Show that  $A \times A = (B \times B) \cap (C \times C)$ .

5. Given  $A = \{1,2,3\}$ ,  $B = \{2,3,5\}$ ,  $C = \{3,4\}$  and  $D = \{1,3,5\}$ , check if  $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$  is true?

6. Let  $A = \{x \in \mathbb{W} \mid x < 2\}$ ,  $B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$  and  $C = \{3,5\}$ . Verify that

(i)  $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii)  $(A \cup B) \times C = (A \times C) \cup (B \times C)$

7. Let  $A =$  The set of all natural numbers less than 8,  $B =$  The set of all prime numbers less than 8,  $C =$  The set of even prime number. Verify that

(i)  $(A \cap B) \times C = (A \times C) \cap (B \times C)$

(ii)  $A \times (B - C) = (A \times B) - (A \times C)$ .

**Example 1.4** Let  $A = \{3,4,7,8\}$  and  $B = \{1,7,10\}$ . Which of the following sets are relations from  $A$  to  $B$ ?

(i)  $R_1 = \{(3,7), (4,7), (7,10), (8,1)\}$

(ii)  $R_2 = \{(3,1), (4,12)\}$

(iii)  $R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$

**Example 1.5** The arrow diagram shows (Fig.1.10) a relationship between the sets  $P$  and  $Q$ . Write the relation in (i) Set builder form (ii) Roster form (iii)

What is the domain and range of  $R$ .

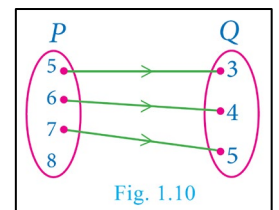


Fig. 1.10

### Exercise 1.2

1. Let  $A = \{1,2,3,7\}$  and  $B = \{3,0, -1,7\}$ , which of the following are relation from  $A$  to  $B$  ?

(i)  $R_1 = \{(2,1), (7,1)\}$

(ii)  $R_2 = \{(-1,1)\}$

(iii)  $R_3 = \{(2, -1), (7,7), (1,3)\}$

(iv)  $R_4 = \{(7, -1), (0,3), (3,3), (0,7)\}$

2. Let  $A = \{1,2,3,4, \dots, 45\}$  and  $R$  be the relation defined as "square is of a number" on  $A$ . Write  $R$  as a subset of  $A \times A$ . Also, find the domain and range of  $R$ .

3. A Relation  $R$  is given by the set  $\{(x,y)/y = x + 3, x \in \{0,1,2,3,4,5\}\}$ . Determine its domain and range.

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i)  $\{(x,y) \mid x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$

(ii)  $\{(x,y) \mid y = x + 3, x, y \text{ are natural numbers } < 10\}$

5. A company has four categories of employees given by Assistants ( $A$ ), Clerks ( $C$ ), Managers ( $M$ ) and an Executive Officer ( $E$ ). The company provide ₹10,000, ₹ 25,000 , ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories  $A, C, M$  and  $E$  respectively. If  $A_1, A_2, A_3, A_4$  and  $A_5$  were Assistants;  $C_1, C_2, C_3, C_4$  were Clerks;  $M_1, M_2, M_3$  were managers and  $E_1, E_2$  were Executive officers and if the relation  $R$  is defined by  $xRy$ , where  $x$  is the salary given to person  $y$ , express the relation  $R$  through an ordered pair and an arrow diagram.

**Example 1.6** Let  $X = \{1,2,3,4\}$  and  $Y = \{2,4,6,8,10\}$  and  $R = \{(1,2), (2,4), (3,6), (4,8)\}$ . Show that  $R$  is a function and find its domain, co-domain and range?

**Example 1.7** A relation  $f: X \rightarrow Y$  is defined by  $f(x) = x^2 - 2$  where,  $X = \{-2, -1, 0, 3\}$  and  $Y = R$ .

- (i) List the elements of  $f$                       (ii) Is  $f$  a function?

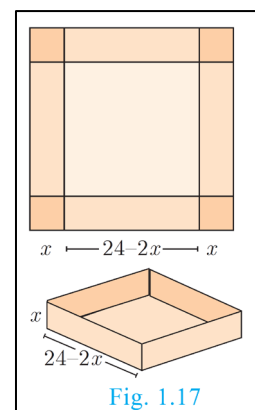
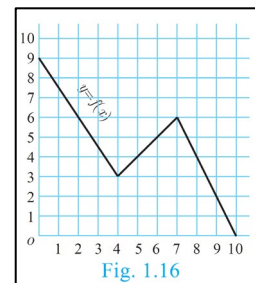
**Example 1.8** If  $X = \{-5, 1, 3, 4\}$  and  $Y = \{a, b, c\}$ , then which of the following relations are functions from  $X$  to  $Y$  ?

- (i)  $R_1 = \{(-5, a), (1, a), (3, b)\}$   
 (ii)  $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$   
 (iii)  $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

**Example 1.9** Given  $f(x) = 2x - x^2$ , find (i)  $f(1)$     (ii)  $f(x + 1)$     (iii)  $f(x) + f(1)$

### Exercise 1.3

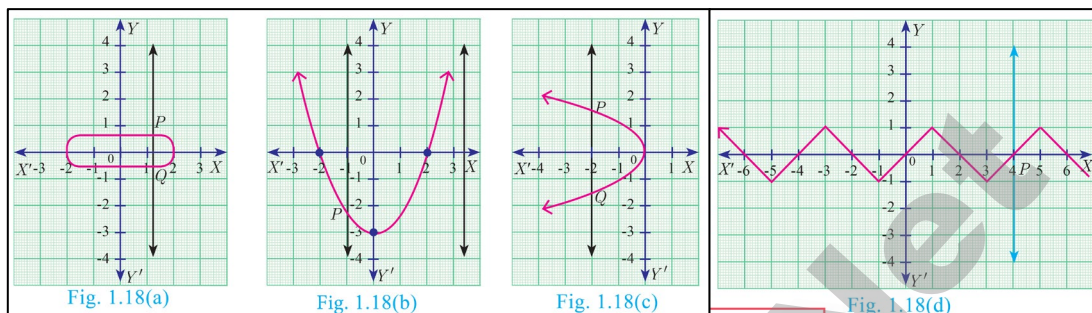
- Let  $f = \{(x, y) \mid x, y \in N \text{ and } y = 2x\}$  be a relation on  $N$ . Find the domain, co-domain and range. Is this relation a function?
- Let  $X = \{3, 4, 6, 8\}$ . Determine whether the relation  $R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$  is a function from  $X$  to  $N$  ?
- Given the function  $f: x \rightarrow x^2 - 5x + 6$ , evaluate  
 (i)  $f(-1)$                       (ii)  $f(2a)$                       (iii)  $f(2)$                       (iv)  $f(x - 1)$
- A graph representing the function  $f(x)$  is given in Figure. It is clear that  $f(9) = 2$ .  
 (i) Find the following values of the function  
 (a)  $f(0)$  (b)  $f(7)$  (c)  $f(2)$  (d)  $f(10)$   
 (ii) For what value of  $x$  is  $f(x) = 1$  ?  
 (iii) Describe the following (i) Domain (ii) Range.  
 (iv) What is the image of 6 under  $f$  ?
- Let  $f(x) = 2x + 5$ . If  $x \neq 0$  then find  $\frac{f(x+2)-f(2)}{x}$ .
- A function  $f$  is defined by  $f(x) = 2x - 3$   
 (i) find  $\frac{f(0)+f(1)}{2}$ .  
 (ii) find  $x$  such that  $f(x) = 0$ .  
 (iii) find  $x$  such that  $f(x) = x$ .  
 (iv) find  $x$  such that  $f(x) = f(1 - x)$ .
- An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown (Fig.1.17). Express the volume  $V$  of the box as a function of  $x$ .
- A function  $f$  is defined by  $f(x) = 3 - 2x$ . Find  $x$  such that  $f(x^2) = (f(x))^2$ .
- A plane is flying at a speed of 500 km per hour. Express the distance '  $d$  ' travelled by the plane as function of time  $t$  in hours.
- The data in the adjacent table depicts the length of a person forehead and their corresponding height. Based on this data, a student finds a relationship between the height ( $y$ ) and the forehead length ( $x$ ) as  $y = ax + b$ , where  $a, b$  are constants.



- (i) Check if this relation is a function.
- (ii) Find  $a$  and  $b$ .
- (iii) Find the height of a person whose forehand length is 40 cm
- (iv) Find the length of forehand of a person if the height is 53.3 inches.

Length ' $x$ ' of forehand (in cm)	Height ' $y$ ' (in inches)
35	56
45	65
50	69.5
55	74

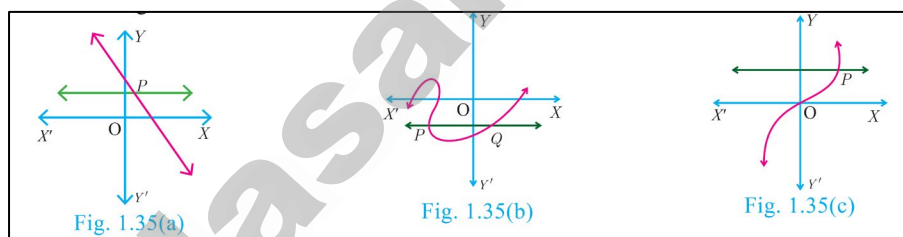
**Example 1.10** Using vertical line test, determine which of the following curves (Fig.1.18(a), 1.18(b), 1.18(c), 1.18(d)) represent a function?



**Example 1.11** Let  $A = \{1,2,3,4\}$  and  $B = \{2,5,8,11,14\}$  be two sets. Let  $f: A \rightarrow B$  be a function given by  $f(x) = 3x - 1$ . Represent this function

- (i) by arrow diagram
- (ii) in a table form
- (iii) as a set of ordered pairs
- (iv) in a graphical form

**Example 1.12** Using horizontal line test (Fig.1.35 (a), 1.35 (b), 1.35 (c)), determine which of the following functions are one - one.



**Example 1.13** Let  $A = \{1,2,3\}$ ,  $B = \{4,5,6,7\}$  and  $f = \{(1,4), (2,5), (3,6)\}$  be a function from  $A$  to  $B$ . Show that  $f$  is one - one but not onto function.

**Example 1.14** If  $A = \{-2, -1, 0, 1, 2\}$  and  $f: A \rightarrow B$  is an onto function defined by  $f(x) = x^2 + x + 1$  then find  $B$ .

**Example 1.15** Let  $f$  be a function  $f: \mathbb{N} \rightarrow \mathbb{N}$  be defined by  $f(x) = 3x + 2, x \in \mathbb{N}$

- (i) Find the images of 1,2,3
- (ii) Find the pre-images of 29,53
- (ii) Identify the type of function

**Example 1.16** Forensic scientists can determine the height (in cm ) of a person based on the length of the thigh bone. They usually do so using the function  $h(b) = 2.47b + 54.10$  where  $b$  is the length of the thigh bone.

- (i) Verify the function  $h$  is one - one or not.
- (ii) Also find the height of a person if the length of his thigh bone is 50 cm .
- (iii) Find the length of the thigh bone if the height of a person is 147.96 cm .

**Example 1.17** Let  $f$  be a function from  $\mathbb{R}$  to  $\mathbb{R}$  defined by  $f(x) = 3x - 5$ . Find the values of  $a$  and  $b$  given that  $(a, 4)$  and  $(1, b)$  belong to  $f$ .

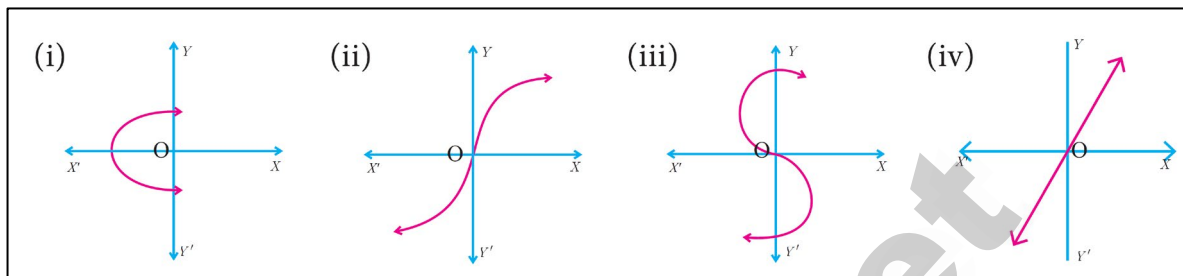
**Example 1.18** If the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \begin{cases} 2x + 7; & x < -2 \\ x^2 - 2; & -2 \leq x < 3, \\ 3x - 2; & x \geq 3 \end{cases}$

then find the values of

- (i)  $f(4)$                       (ii)  $f(-2)$                       (iii)  $f(4) + 2f(1)$                       (iv)  $\frac{f(1)-3f(4)}{f(-3)}$

**Exercise 1.4**

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



2. Let  $f: A \rightarrow B$  be a function defined by  $f(x) = \frac{x}{2} - 1$ , where  $A = \{2,4,6,10,12\}$ ,  $B = \{0,1,2,4,5,9\}$ .

Represent  $f$  by

- (i) set of ordered pairs                      (ii) a table                      (iii) an arrow diagram                      (iv) a graph

3. Represent the function  $f = \{(1,2), (2,2), (3,2), (4,3), (5,4)\}$  through

- (i) an arrow diagram                      (ii) a table form                      (iii) a graph

4. Show that the function  $f: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(x) = 2x - 1$  is one-one but not onto.

5. Show that the function  $f: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(m) = m^2 + m + 3$  is one-one function.

6. Let  $A = \{1,2,3,4\}$  and  $B = \mathbb{N}$ . Let  $f: A \rightarrow B$  be defined by  $f(x) = x^3$  then,

- (i) find the range of  $f$                       (ii) identify the type of function

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

- (i)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x + 1$                       (ii)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 3 - 4x^2$

8. Let  $A = \{-1,1\}$  and  $B = \{0,2\}$ . If the function  $f: A \rightarrow B$  defined by  $f(x) = ax + b$  is an onto function? Find  $a$  and  $b$ .

9. If the function  $f$  is defined by  $f(x) = \begin{cases} x + 2; & x > 1 \\ 2; & -1 \leq x \leq 1 \\ x - 1; & -3 < x < -1 \end{cases}$  find the values of

- (i)  $f(3)$                       (ii)  $f(0)$                       (iii)  $f(-1.5)$                       (iv)  $f(2) + f(-2)$

10. A function  $f: [-5,9] \rightarrow \mathbb{R}$  is defined as follows:

$$f(x) = \begin{cases} 6x + 1; & -5 \leq x < 2 \\ 5x^2 - 1; & 2 \leq x < 6 \\ 3x - 4; & 6 \leq x \leq 9 \end{cases}$$

- Find (i)  $f(-3) + f(2)$                       (ii)  $f(7) - f(1)$                       (iii)  $2f(4) + f(8)$                       (iv)  $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

11. The distance  $S$  an object travels under the influence of gravity in time  $t$  seconds is given by  $S(t) = \frac{1}{2}gt^2 + at + b$  where, ( $g$  is the acceleration due to gravity),  $a, b$  are constants. Verify whether the function  $S(t)$  is one-one or not.

12. The function '  $t$  ' which maps temperature in Celsius ( $C$ ) into temperature in Fahrenheit ( $F$ ) is defined by  $t(C) = F$  where  $F = \frac{9}{5}C + 32$ . Find,

- (i)  $t(0)$                       (ii)  $t(28)$                       (iii)  $t(-10)$                       (iv) the value of  $C$  when  $t(C) = 212$   
(v) the temperature when the Celsius value is equal to the Faren-heit value.

**Example 1.19** Find  $f \circ g$  and  $g \circ f$  when  $f(x) = 2x + 1$

**Example 1.20** Represent the function  $f(x) = \sqrt{2x^2 - 5x + 3}$  as a composition of two functions.

**Example 1.21** If  $f(x) = 3x - 2, g(x) = 2x + k$  and if  $f \circ g = g \circ f$ , then find the value of  $k$ .

**Example 1.22** Find  $k$  if  $f \circ f(k) = 5$  where  $f(k) = 2k - 1$ .

**Example 1.23** If  $f(x) = 2x + 3, g(x) = 1 - 2x$  and  $h(x) = 3x$ . Prove that  $f \circ (g \circ h) = (f \circ g) \circ h$

**Example 1.24** Find  $x$  if  $gff(x) = fgg(x)$ , given  $f(x) = 3x + 1$  and  $g(x) = x + 3$

### Exercise 1.5

- Using the functions  $f$  and  $g$  given below, find  $f \circ g$  and  $g \circ f$ . Check whether  $f \circ g = g \circ f$ .
  - $f(x) = x - 6, g(x) = x^2$
  - $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$
  - $f(x) = \frac{x+6}{3}, g(x) = 3 - x$
  - $f(x) = 3 + x, g(x) = x - 4$
  - $f(x) = 4x^2 - 1, g(x) = 1 + x$
- Find the value of  $k$ , such that  $f \circ g = g \circ f$ 
  - $f(x) = 3x + 2, g(x) = 6x - k$
  - $f(x) = 2x - k, g(x) = 4x + 5$
- If  $f(x) = 2x - 1, g(x) = \frac{x+1}{2}$ , show that  $f \circ g = g \circ f = x$
- If  $f(x) = x^2 - 1, g(x) = x - 2$  find  $a$ , if  $g \circ f(a) = 1$ .
- Let  $A, B, C \subseteq \mathbb{N}$  and a function  $f: A \rightarrow B$  be defined by  $f(x) = 2x + 1$  and  $g: B \rightarrow C$  be defined by  $g(x) = x^2$ . Find the range of  $f \circ g$  and  $g \circ f$ .
- Let  $f(x) = x^2 - 1$ . Find
  - $f \circ f$
  - $f \circ f \circ f$
- If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = x^5$  and  $g(x) = x^4$  then check if  $f, g$  are one-one and  $f \circ g$  is one-one?
- Consider the functions  $f(x), g(x), h(x)$  as given below. Show that  $(f \circ g) \circ h = f \circ (g \circ h)$  in each case.
  - $f(x) = x - 1, g(x) = 3x + 1$  and  $h(x) = x^2$
  - $f(x) = x^2, g(x) = 2x$  and  $h(x) = x + 4$
  - $f(x) = x - 4, g(x) = x^2$  and  $h(x) = 3x - 5$
- Let  $f = \{(-1,3), (0, -1), (2, -9)\}$  be a linear function from  $\mathbb{Z}$  into  $\mathbb{Z}$ . Find  $f(x)$ .
- In electrical circuit theory, a circuit  $C(t)$  is called a linear circuit if it satisfies the superposition principle given by  $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$ , where  $a, b$  are constants. Show that the circuit  $C(t) = 3t$  is linear.

### Exercise 1.6

#### Choose the Correct Answer.

- If  $n(A \times B) = 6$  and  $A = \{1,3\}$  then  $n(B)$  is
 

(A) 1	(B) 2	(C) 3	(D) 6
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- $A = \{a, b, p\}, B = \{2,3\}, C = \{p, q, r, s\}$  then  $n[(A \cup C) \times B]$ 

(A) 8	(B) 20	(C) 12	(D) 16
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- If  $A = \{1,2\}, B = \{1,2,3,4\}, C = \{5,6\}$  and  $D = \{5,6,7,8\}$  then state which of the following statement is true.
 

(A) $(A \times C) \subset (B \times D)$	(B) $(B \times D) \subset (A \times C)$
(C) $(A \times B) \subset (A \times D)$	(D) $(D \times A) \subset (B \times A)$
- If there are 1024 relations from a set  $A = \{1,2,3,4,5\}$  to a set  $B$ , then the number of elements in  $B$  is
 

(A) 3	(B) 2	(C) 4	(D) 8
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- The range of the relation  $R = \{(x, x^2) \mid x \text{ is a prime number less than } 13\}$  is
 

(A) $\{2,3,5,7\}$	(B) $\{2,3,5,7,11\}$	(C) $\{4,9,25,49,121\}$	(D) $\{1,4,9,25,49,121\}$
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- If the ordered pairs  $(a + 2, 4)$  and  $(5, 2a + b)$  are equal then  $(a, b)$  is
 

(A) $(2, -2)$	(B) $(5, 1)$	(C) $(2, 3)$	(D) $(3, -2)$
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7. Let  $n(A) = m$  and  $n(B) = n$  then the total number of non-empty relations that can be defined from  $A$  to  $B$  is  
 (A)  $m^n$  (B)  $n^m$  (C)  $2^{mn} - 1$  (D)  $2^{mn}$
8. If  $\{(a, 8), (6, b)\}$  represents an identity function, then the value of  $a$  and  $b$  are respectively  
 (A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)
9. Let  $A = \{1,2,3,4\}$  and  $B = \{4,8,9,10\}$ . A function  $f: A \rightarrow B$  given by  $f = \{(1,4), (2,8), (3,9), (4,10)\}$  is  
 (A) Many-one function (B) Identity function  
 (C) One-to-one function (D) Into function
10. If  $f(x) = 2x^2$  and  $g(x) = \frac{1}{3x}$ , then  $f \circ g$  is  
 (A)  $\frac{3}{2x^2}$  (B)  $\frac{2}{3x^2}$  (C)  $\frac{2}{9x^2}$  (D)  $\frac{1}{6x^2}$
11. If  $f: A \rightarrow B$  is a bijective function and if  $n(B) = 7$ , then  $n(A)$  is equal to  
 (A) 7 (B) 49 (C) 1 (D) 14
12. Let  $f$  and  $g$  be two functions given by  $f = \{(0,1), (2,0), (3, -4), (4,2), (5,7)\}$   
 $g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$  then the range of  $f \circ g$  is  
 (A)  $\{0,2,3,4,5\}$  (B)  $\{-4,1,0,2,7\}$  (C)  $\{1,2,3,4,5\}$  (D)  $\{0,1,2\}$
13. Let  $f(x) = \sqrt{1+x^2}$  then  
 (A)  $f(xy) = f(x) \cdot f(y)$  (B)  $f(xy) \geq f(x) \cdot f(y)$   
 (C)  $f(xy) \leq f(x) \cdot f(y)$  (D) None of these
14. If  $g = \{(1,1), (2,3), (3,5), (4,7)\}$  is a function given by  $g(x) = \alpha x + \beta$  then the values of  $\alpha$  and  $\beta$  are  
 (A)  $(-1,2)$  (B)  $(2, -1)$  (C)  $(-1, -2)$  (D)  $(1,2)$
15.  $f(x) = (x+1)^3 - (x-1)^3$  represents a function which is  
 (A) linear (B) cubic (C) reciprocal (D) quadratic

### Unit Exercise - 1

1. If the ordered pairs  $(x^2 - 3x, y^2 + 4y)$  and  $(-2,5)$  are equal, then find  $x$  and  $y$ .
2. The cartesian product  $A \times A$  has 9 elements among which  $(-1,0)$  and  $(0,1)$  are found. Find the set  $A$  and the remaining elements of  $A \times A$ .
3. Given that  $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$ . Find  
 (i)  $f(0)$  (ii)  $f(3)$  (iii)  $f(a+1)$  in terms of  $a$ . (Given that  $a \geq 0$ )
4. Let  $A = \{9,10,11,12,13,14,15,16,17\}$  and let  $f: A \rightarrow N$  be defined by  $f(n) =$  the highest prime factor of  $n \in A$ . Write  $f$  as a set of ordered pairs and find the range of  $f$ .
5. Find the domain of the function  $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$
6. If  $f(x) = x^2$ ,  $g(x) = 3x$  and  $h(x) = x - 2$ , Prove that  $(f \circ g) \circ h = f \circ (g \circ h)$ .
7. Let  $A = \{1,2\}$  and  $B = \{1,2,3,4\}$ ,  $C = \{5,6\}$  and  $D = \{5,6,7,8\}$ . Verify whether  $A \times C$  is a subset of  $B \times D$ ?
8. If  $f(x) = \frac{x-1}{x+1}$ ,  $x \neq -1$  show that  $f(f(x)) = -\frac{1}{x}$ , provided  $x \neq 0$ .
9. The functions  $f$  and  $g$  are defined by  $f(x) = 6x + 8$ ;  $g(x) = \frac{x-2}{3}$   
 (i) Calculate the value of  $gg\left(\frac{1}{2}\right)$  (ii) Write an expression for  $gf(x)$  in its simplest form.
10. Write the domain of the following real functions  
 (i)  $f(x) = \frac{2x+1}{x-9}$  (ii)  $p(x) = \frac{-5}{4x^2+1}$  (iii)  $g(x) = \sqrt{x-2}$  (iv)  $h(x) = x + 6$

**CHAPTER - 2. NUMBERS AND SEQUENCES**

**Example 2.1** We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked?

**Example 2.2** Find the quotient and remainder when  $a$  is divided by  $b$  in the following cases

- (i)  $a = -12, b = 5$  (ii)  $a = 17, b = -3$  (iii)  $a = -19, b = -4$

**Example 2.3** Show that the square of an odd integer is of the form  $4q + 1$ , for some integer  $q$ .

**Example 2.4** If the Highest Common Factor of 210 and 55 is expressible in the form  $55x - 325$ , find  $x$ .

**Example 2.5** Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

**Example 2.6** Find the HCF of 396, 504, 636.

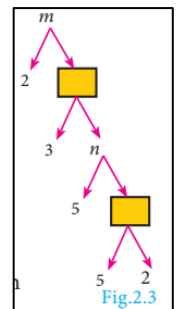
**Exercise 2.1**

- Find all positive integers, when divided by 3 leaves remainder 2 .
- A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.  
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- Prove that the product of two consecutive positive integers is divisible by 2 .
- When the positive integers  $a, b$  and  $c$  are divided by 13 , the respective remainders are 9,7 and 10 . Show that  $a + b + c$  is divisible by 13 .
- Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4 .
- Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of  
(i) 340 and 412 (ii) 867 and 255  
(iii) 10224 and 9648 (iv) 84, 90 and 120
- Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.
- If  $d$  is the Highest Common Factor of 32 and 60 , find  $x$  and  $y$  satisfying  $d = 32x + 60y$ .
- A positive integer when divided by 88 gives the remainder 61 . What will be the remainder when the same number is divided by 11 ?
- Prove that two consecutive positive integers are always coprime.

**Example 2.7:** In the given factorisation, find the numbers  $m$  and  $n$ .

**Example 2.8:** Can the number  $6^n, n$  being a natural number end with the digit 5 ? Give reason for your answer.

**Example 2.10:** ' $a$ ' and ' $b$ ' are two positive integers such that  $a^b \times b^a = 800$ . Find ' $a$ ' and ' $b$ '.



**Exercise 2.2**

- For what values of natural number  $n, 4^n$  can end with the digit 6 ?
- If  $m, n$  are natural numbers, for what values of  $m$ , does  $2^n \times 5^m$  ends in 5 ?
- Find the HCF of 252525 and 363636.
- If  $13824 = 2^a \times 3^b$  then find  $a$  and  $b$ .
- If  $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$  where  $p_1, p_2, p_3, p_4$  are primes in ascending order and  $x_1, x_2, x_3, x_4$  are integers, find the value of  $p_1, p_2, p_3, p_4$  and  $x_1, x_2, x_3, x_4$ .
- Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.
- Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36 ?
- What is the smallest number that when divided by three numbers such as 35,56 and 91 leaves remainder 7 in each case?
- Find the least number that is divisible by the first ten natural numbers.

**Example 2.11:** Find the remainders when 70004 and 778 is divided by 7 .

**Example 2.12:** Determine the value of  $d$  such that  $15 \equiv 3 \pmod{d}$ .

**Example 2.13** Find the least positive value of  $x$  such that

(i)  $67 + x \equiv 1 \pmod{4}$

(ii)  $98 \equiv (x + 4) \pmod{5}$

**Example 2.14** Solve  $8x \equiv 1 \pmod{11}$

**Example 2.15** Compute  $x$ , such that  $10^4 \equiv x \pmod{19}$

**Example 2.16** Find the number of integer solutions of  $3x \equiv 1 \pmod{15}$ .

**Example 2.17** A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

**Example 2.18** Kala and Vani are friends. Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

### Exercise 2.3

1. Find the least positive value of  $x$  such that

(i)  $71 \equiv x \pmod{8}$

(ii)  $78 + x \equiv 3 \pmod{5}$

(iii)  $89 \equiv (x + 3) \pmod{4}$

(iv)  $96 \equiv \frac{x}{7} \pmod{5}$

(v)  $5x \equiv 4 \pmod{6}$

2. If  $x$  is congruent to 13 modulo 17 then  $7x - 3$  is congruent to which number modulo 17?

3. Solve  $5x \equiv 4 \pmod{6}$

4. Solve  $3x - 2 \equiv 0 \pmod{11}$

5. What is the time 100 hours after 7 a.m.?

6. What is the time 15 hours before 11 p.m.?

7. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

8. Prove that  $2^n + 6 \times 9^n$  is always divisible by 7 for any positive integer  $n$ .

9. Find the remainder when  $2^{81}$  is divided by 17.

10. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

**Example 2.19** Find the next three terms of the sequences

(i)  $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \dots$

(ii)  $5, 2, -1, -4, \dots$

(iii)  $1, 0.1, 0.01, \dots$

**Example 2.20** Find the general term for the following sequences

(i)  $3, 6, 9, \dots$

(ii)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$

(iii)  $5, -25, 125, \dots$

**Example 2.21** The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3) & ; n \in \mathbb{N} \text{ is odd} \\ n^2 + 1 & ; n \in \mathbb{N} \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

**Example 2.22** Find the first five terms of the following sequence.

$$a_1 = 1, a_2 = 1, a_n = \frac{a_{n-1}}{a_{n-2} + 3}; n \geq 3, n \in \mathbb{N}$$

### Exercise 2.4

1. Find the next three terms of the following sequence.

(i)  $8, 24, 72, \dots$

(ii)  $5, 1, -3, \dots$

(iii)  $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \dots$

2. Find the first four terms of the sequences whose  $n^{\text{th}}$  terms are given by

(i)  $a_n = n^3 - 2$

(ii)  $a_n = (-1)^{n+1}n(n+1)$

(iii)  $a_n = 2n^2 - 6$

3. Find the  $n^{\text{th}}$  term of the following sequences

(i)  $2, 5, 10, 17, \dots$

(ii)  $0, \frac{1}{2}, \frac{2}{3}, \dots$

(iii)  $3, 8, 13, 18, \dots$



4. Find the indicated terms of the sequences whose  $n^{\text{th}}$  terms are given by

(i)  $a_n = \frac{5n}{n+2}$ ;  $a_6$  and  $a_{13}$

(ii)  $a_n = -(n^2 - 4)$ ;  $a_4$  and  $a_{11}$

5. Find  $a_8$  and  $a_{15}$  whose  $n^{\text{th}}$  term is  $a_n = \begin{cases} \frac{n^2-1}{n+3} & ; n \text{ is even, } n \in \mathbb{N} \\ \frac{n^2}{2n+1} & ; n \text{ is odd, } n \in \mathbb{N} \end{cases}$

6. If  $a_1 = 1, a_2 = 1$  and  $a_n = 2a_{n-1} + a_{n-2}, n \geq 3, n \in \mathbb{N}$ , then find the first six terms of the sequence.

**Example 2.23** Check whether the following sequences are in A.P. or not?

(i)  $x + 2, 2x + 3, 3x + 4, \dots$

(ii)  $2, 4, 8, 16, \dots$

(iii)  $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \dots$

**Example 2.24** Write an A.P. whose first term is 20 and common difference is 8.

**Example 2.25** Find the  $15^{\text{th}}, 24^{\text{th}}$  and  $n^{\text{th}}$  term (general term) of an A.P. given by 3, 15, 27, 39, ...

**Example 2.26** Find the number of terms in the A.P. 3, 6, 9, 12, ..., 111.

**Example 2.27** Determine the general term of an A.P. whose  $7^{\text{th}}$  term is -1 and  $16^{\text{th}}$  term is 17.

**Example 2.28** If  $l^{\text{th}}, m^{\text{th}}$  and  $n^{\text{th}}$  terms of an A.P. are  $x, y, z$  respectively, then show that

(i)  $x(m - n) + y(n - l) + z(l - m) = 0$

(ii)  $(x - y)n + (y - z)l + (z - x)m = 0$

**Example 2.29** In an A.P., sum of four consecutive terms is 28 and the sum of their squares is 276. Find the four numbers.

**Example 2.30** A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹ 4623. Find the amount received by each child.

## Exercise 2.5

1. Check whether the following sequences are in A.P.

(i)  $a - 3, a - 5, a - 7, \dots$

(ii)  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

(iii)  $9, 13, 17, 21, 25, \dots$

(iv)  $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \dots$

(v)  $1, -1, 1, -1, 1, -1, \dots$

2. First term  $a$  and common difference  $d$  are given below. Find the corresponding A.P.

(i)  $a = 5, d = 6$

(ii)  $a = 7, d = -5$

(iii)  $a = \frac{3}{4}, d = \frac{1}{2}$

3. Find the first term and common difference of the Arithmetic Progressions whose  $n^{\text{th}}$  terms are given below

(i)  $t_n = -3 + 2n$

(ii)  $t_n = 4 - 7n$

4. Find the  $19^{\text{th}}$  term of an A.P.  $-11, -15, -19, \dots$

5. Which term of an A.P.  $16, 11, 6, 1, \dots$  is  $-54$ ?

6. Find the middle term(s) of an A.P.  $9, 15, 21, 27, \dots, 183$ .

7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

8. If  $3 + k, 18 - k, 5k + 1$  are in A.P. then find  $k$ .

9. Find  $x, y$  and  $z$ , given that the numbers  $x, 10, y, 24, z$  are in A.P.

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

12. The ratio of  $6^{\text{th}}$  and  $8^{\text{th}}$  term of an A.P. is 7:9. Find the ratio of  $9^{\text{th}}$  term to  $13^{\text{th}}$  term.

13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is  $0^\circ\text{C}$  and the sum of the temperatures from Wednesday to Friday is  $18^\circ\text{C}$ . Find the temperature on each of the five days.

14. Priya earned ₹ 15,000 in the first month. Thereafter her salary increased by ₹ 1500 per year. Her expenses are ₹ 13,000 during the first month and the expenses increases by ₹ 900 per year. How long will it take for her to save ₹ 20,000 per month.

**Example 2.31** Find the sum of first 15 terms of the A. P.  $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \dots$

**Example 2.32** Find the sum of  $0.40 + 0.43 + 0.46 + \dots + 1$ .

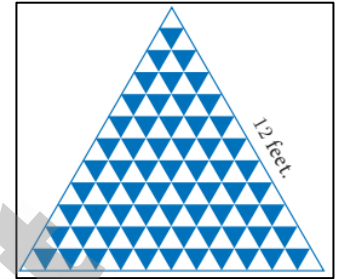
**Example 2.33** How many terms of the series  $1 + 5 + 9 + \dots$  must be taken so that their sum is 190?

**Example 2.34** The 13<sup>th</sup> term of an A.P. is 3 and the sum of first 13 terms is 234 . Find the common difference and the sum of first 21 terms.

**Example 2.35** In an A.P. the sum of first  $n$  terms is  $\frac{5n^2}{2} + \frac{3n}{2}$ . Find the 17<sup>th</sup> term.

**Example 2.36** Find the sum of all natural numbers between 300 and 600 which are divisible by 7 .

**Example 2.37** A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



**Example 2.38** The houses of a street are numbered from 1 to 49 . Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number?

**Example 2.39** The sum of first  $n, 2n$  and  $3n$  terms of an A.P. are  $S_1, S_2$  and  $S_3$  respectively. Prove that  $S_3 = 3(S_2 - S_1)$ .

## Exercise 2.6

1. Find the sum of the following

(i)  $3, 7, 11, \dots$  up to 40 terms.      (ii)  $102, 97, 92, \dots$  up to 27 terms.      (iii)  $6 + 13 + 20 + \dots + 97$

2. How many consecutive odd integers beginning with 5 will sum to 480 ?

3. Find the sum of first 28 terms of an A.P. whose  $n^{\text{th}}$  term is  $4n - 3$ .

4. The sum of first  $n$  terms of a certain series is given as  $2n^2 - 3n$ . Show that the series is an A.P.

5. The 104<sup>th</sup> term and 4<sup>th</sup> term of an A.P. are 125 and 0 . Find the sum of first 35 terms.

6. Find the sum of all odd positive integers less than 450.

7. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4 .

8. Raghu wish to buy a laptop. He can buy it by paying ₹ 40,000 cash or by giving it in 10 installments as ₹ 4800 in the first month, ₹ 4750 in the second month, ₹ 4700 in the third month and so on. If he pays the money in this fashion, find

(i) total amount paid in 10 installments.

(ii) how much extra amount that he has to pay than the cost?

9. A man repays a loan of ₹ 65,000 by paying ₹ 400 in the first month and then increasing the payment by ₹ 300 every month. How long will it take for him to clear the loan?

10. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

(i) How many bricks are required for the top most step?

(ii) How many bricks are required to build the stair case?

11. If  $S_1, S_2, S_3, \dots, S_m$  are the sums of  $n$  terms of  $m$  A.P.'s whose first terms are  $1, 2, 3, \dots, m$  and whose common differences are  $1, 3, 5, \dots, (2m - 1)$  respectively, then

$$\text{show that } S_1 + S_2 + S_3 + \dots + S_m = \frac{1}{2}mn(mn + 1).$$

12. Find the sum  $\left[ \frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \dots \text{ to 12 terms} \right]$ .

**Example 2.40** Which of the following sequences form a Geometric Progression?

(i)  $7, 14, 21, 28, \dots$

(ii)  $\frac{1}{2}, 1, 2, 4, \dots$

(iii)  $5, 25, 50, 75, \dots$

**Example 2.42** Find the 8<sup>th</sup> term of the G.P.  $9, 3, 1, \dots$

**Example 2.43** In a Geometric progression, the 4<sup>th</sup> term is  $\frac{8}{9}$  and the 7<sup>th</sup> term is  $\frac{64}{243}$ . Find the Geometric Progression.

**Example 2.44** The product of three consecutive terms of a Geometric Progression is 343 and their sum is  $\frac{91}{3}$ . Find the three terms.

**Example 2.45** The present value of a machine is ₹ 40,000 and its value depreciates each year by 10%. Find the estimated value of the machine in the 6<sup>th</sup> year.

## Exercise 2.7

1. Which of the following sequences are in G.P.?

(i) 3,9,27,81, ...      (ii) 4,44,444,4444, ...      (iii) 0.5,0.05,0.005, ...      (iv)  $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \dots$

(v) 1, -5,25, -125, ...      (vi) 120,60,30,18, ...      (vii) 16,4,1,  $\frac{1}{4}, \dots$

2. Write the first three terms of the G.P. whose first term and the common ratio are given below.

(i)  $a = 6, r = 3$       (ii)  $a = \sqrt{2}, r = \sqrt{2}$       (iii)  $a = 1000, r = \frac{2}{5}$

3. In a G.P. 729,243,81, ... find  $t_7$ .

4. Find  $x$  so that  $x + 6, x + 12$  and  $x + 15$  are consecutive terms of a Geometric Progression.

5. Find the number of terms in the following G.P.

(i) 4,8,16, ...,8192 ?      (ii)  $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, \frac{1}{2187}$

6. In a G.P. the 9<sup>th</sup> term is 32805 and 6<sup>th</sup> term is 1215 . Find the 12<sup>th</sup> term.

7. Find the 10<sup>th</sup> term of a G.P. whose 8<sup>th</sup> term is 768 and the common ratio is 2 .

8. If  $a, b, c$  are in A.P. then show that  $3^a, 3^b, 3^c$  are in G.P.

9. In a G.P. the product of three consecutive terms is 27 and the sum of the product of two terms taken at a time is  $\frac{57}{2}$ . Find the three terms.

10. A man joined a company as Assistant Manager. The company gave him a starting salary of ₹ 60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?

11. Sivamani is attending an interview for a job and the company gave two offers to him. Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years. Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years. What is his salary in the 4<sup>th</sup> year with respect to the offers A and B ?

12. If  $a, b, c$  are three consecutive terms of an A.P. and  $x, y, z$  are three consecutive terms of a G.P. then prove that  $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$ .

**Example 2.46** Find the sum of 8 terms of the G.P. 1, -3,9, -27 ...

**Example 2.47** Find the first term of a G.P. in which  $S_6 = 4095$  and  $r = 4$ .

**Example 2.48** How many terms of the series  $1 + 4 + 16 + \dots$  make the sum 1365 ?

**Example 2.49** Find the sum  $3 + 1 + \frac{1}{3} + \dots \infty$

**Example 2.50** Find the rational form of the number 0.6666 ...

**Example 2.51** Find the sum to  $n$  terms of the series  $5 + 55 + 555 + \dots$

**Example 2.52** Find the least positive integer  $n$  such that  $1 + 6 + 6^2 + \dots + 6^n > 5000$ .

**Example 2.53** A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year?

## Exercise 2.8

1. Find the sum of first  $n$  terms of the G.P.      (i)  $5, -3, \frac{9}{5}, -\frac{27}{25}, \dots$       (ii) 256,64,16, ...

2. Find the sum of first six terms of the G.P. 5,15,45, ...

3. Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872 .

- Find the sum to infinity of (i)  $9 + 3 + 1 + \dots$  (ii)  $21 + 14 + \frac{28}{3} + \dots$
- If the first term of an infinite G.P. is 8 and its sum to infinity is  $\frac{32}{3}$  then find the common ratio.
- Find the sum to  $n$  terms of the series  
 (i)  $0.4 + 0.44 + 0.444 + \dots$  to  $n$  terms (ii)  $3 + 33 + 333 + \dots$  to  $n$  terms
- Find the sum of the Geometric series  $3 + 6 + 12 + \dots + 1536$ .
- Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹2 to mail one letter, find the amount spent on postage when 8<sup>th</sup> set of letters is mailed.
- Find the rational form of the number  $0.\overline{123}$ .
- If  $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots$   $n$  terms then prove that

$$(x - y)S_n = \left[ \frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1} \right]$$

**Example 2.54** Find the value of (i)  $1 + 2 + 3 + \dots + 50$  (ii)  $16 + 17 + 18 + \dots + 75$

**Example 2.55** Find the sum of (i)  $1 + 3 + 5 + \dots$  to 40 terms (ii)  $2 + 4 + 6 + \dots + 80$   
 (iii)  $1 + 3 + 5 + \dots + 55$

**Example 2.56** Find the sum of (i)  $1^2 + 2^2 + \dots + 19^2$  (ii)  $5^2 + 10^2 + 15^2 + \dots + 105^2$   
 (iii)  $15^2 + 16^2 + 17^2 + \dots + 28^2$

**Example 2.57** Find the sum of (i)  $1^3 + 2^3 + 3^3 + \dots + 16^3$  (ii)  $9^3 + 10^3 + \dots + 21^3$

**Example 2.58** If  $1 + 2 + 3 + \dots + n = 666$  then find  $n$ .

## Exercise 2.9

- Find the sum of the following series  
 (i)  $1 + 2 + 3 + \dots + 60$  (ii)  $3 + 6 + 9 + \dots + 96$  (iii)  $51 + 52 + 53 + \dots + 92$   
 (iv)  $1 + 4 + 9 + 16 + \dots + 225$  (v)  $6^2 + 7^2 + 8^2 + \dots + 21^2$   
 (vi)  $10^3 + 11^3 + 12^3 + \dots + 20^3$  (vii)  $1 + 3 + 5 + \dots + 71$
- If  $1 + 2 + 3 + \dots + k = 325$ , then find  $1^3 + 2^3 + 3^3 + \dots + k^3$ .
- If  $1^3 + 2^3 + 3^3 + \dots + k^3 = 44100$  then find  $1 + 2 + 3 + \dots + k$ .
- How many terms of the series  $1^3 + 2^3 + 3^3 + \dots$  should be taken to get the sum 14400 ?
- The sum of the cubes of the first  $n$  natural numbers is 2025, then find the value of  $n$ .
- Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, ..., 24 cm. How much area can be decorated with these colour papers?
- Find the sum of the series  $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + \dots$  to  
 (i)  $n$  terms (ii) 8 terms

## Exercise 2.10

### Multiple choice questions

- Euclid's division lemma states that for positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  such that  $a = bq + r$ , where  $r$  must satisfy.  
 (A)  $1 < r < b$  (B)  $0 < r < b$  (C)  $0 \leq r < b$  (D)  $0 < r \leq b$
- Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are  
 (A) 0,1,8 (B) 1,4,8 (C) 0,1,3 (D) 1,3,5
- If the HCF of 65 and 117 is expressible in the form of  $65m - 117$ , then the value of  $m$  is  
 (A) 4 (B) 2 (C) 1 (D) 3
- The sum of the exponents of the prime factors in the prime factorization of 1729 is  
 (A) 1 (B) 2 (C) 3 (D) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is  
 (A) 2025 (B) 5220 (C) 5025 (D) 2520
6.  $7^{4k} \equiv \text{_____} \pmod{100}$   
 (A) 1 (B) 2 (C) 3 (D) 4
7. Given  $F_1 = 1, F_2 = 3$  and  $F_n = F_{n-1} + F_{n-2}$  then  $F_5$  is  
 (A) 3 (B) 5 (C) 8 (D) 11
8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.  
 (A) 4551 (B) 10091 (C) 7881 (D) 13531
9. If 6 times of 6<sup>th</sup> term of an A.P. is equal to 7 times the 7<sup>th</sup> term, then the 13<sup>th</sup> term of the A.P. is  
 (A) 0 (B) 6 (C) 7 (D) 13
10. An A.P. consists of 31 terms. If its 16<sup>th</sup> term is  $m$ , then the sum of all the terms of this A.P. is  
 (A) 16  $m$  (B) 62  $m$  (C) 31  $m$  (D)  $\frac{31}{2} m$
11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?  
 (A) 6 (B) 7 (C) 8 (D) 9
12. If  $A = 2^{65}$  and  $B = 2^{64} + 2^{63} + 2^{62} + \dots + 2^0$  which of the following is true?  
 (A)  $B$  is  $2^{64}$  more than  $A$  (B)  $A$  and  $B$  are equal  
 (C)  $B$  is larger than  $A$  by 1 (D)  $A$  is larger than  $B$  by 1
13. The next term of the sequence  $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \dots$  is  
 (A)  $\frac{1}{24}$  (B)  $\frac{1}{27}$  (C)  $\frac{2}{3}$  (D)  $\frac{1}{81}$
14. If the sequence  $t_1, t_2, t_3, \dots$  are in A.P. then the sequence  $t_6, t_{12}, t_{18}, \dots$  is  
 (A) a Geometric Progression (B) an Arithmetic Progression  
 (C) neither an Arithmetic Progression nor a Geometric Progression  
 (D) a constant sequence
15. The value of  $(1^3 + 2^3 + 3^3 + \dots + 15^3) - (1 + 2 + 3 + \dots + 15)$  is  
 (A) 14400 (B) 14200 (C) 14280 (D) 14520

## Unit Exercise - 2

1. Prove that  $n^2 - n$  divisible by 2 for every positive integer  $n$ .
2. A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.
3. When the positive integers  $a, b$  and  $c$  are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when  $a + 2b + 3c$  is divided by 13.
4. Show that 107 is of the form  $4q + 3$  for any integer  $q$ .
5. If  $(m + 1)^{\text{th}}$  term of an A.P. is twice the  $(n + 1)^{\text{th}}$  term, then prove that  $(3m + 1)^{\text{th}}$  term is twice the  $(m + n + 1)^{\text{th}}$  term.
6. Find the 12<sup>th</sup> term from the last term of the A. P  $-2, -4, -6, \dots - 100$ .
7. Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their 10<sup>th</sup> terms is the same as the difference between their 21<sup>st</sup> terms, which is the same as the difference between any two corresponding terms.
8. A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?
9. Find the G.P. in which the 2<sup>nd</sup> term is  $\sqrt{6}$  and the 6<sup>th</sup> term is  $9\sqrt{6}$ .
10. The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000?

**CHAPTER - 3. ALGEBRA**

**Example 3.1** The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

**Example 3.2** Solve  $2x - 3y = 6, x + y = 1$

**Example 3.3** Solve the following system of linear equations in three variables

$$3x - 2y + z = 2, 2x + 3y - z = 5, x + y + z = 6.$$

**Example 3.4** In an interschool athletic meet, with total of 24 individual prizes, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

**Example 3.5** Solve  $x + 2y - z = 5; x - y + z = -2; -5x - 4y + z = -11$

**Example 3.6** Solve  $3x + y - 3z = 1; -2x - y + 2z = 1; -x - y + z = 2.$

**Example 3.7** Solve  $\frac{x}{2} - 1 = \frac{y}{6} + 1 = \frac{z}{7} + 2; \frac{y}{3} + \frac{z}{2} = 13$

**Example 3.8** Solve :  $\frac{1}{2x} + \frac{1}{4y} - \frac{1}{3z} = \frac{1}{4}; \frac{1}{x} = \frac{1}{3y}; \frac{1}{x} - \frac{1}{5y} + \frac{4}{z} = 2 \frac{2}{15}$

**Example 3.9** The sum of thrice the first number, second number and twice the third number is 5 . If thrice the second number is subtracted from the sum of first number and thrice the third we get 2 . If the third number is subtracted from the sum of twice the first, thrice the second, we get 1 . Find the numbers.

**Exercise 3.1**

- Solve the following system of linear equations in three variables
  - $x + y + z = 5; 2x - y + z = 9; x - 2y + 3z = 16$
  - $\frac{1}{x} - \frac{2}{y} + 4 = 0; \frac{1}{y} - \frac{1}{z} + 1 = 0; \frac{2}{z} + \frac{3}{x} = 14$
  - $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$
- Discuss the nature of solutions of the following system of equations
  - $x + 2y - z = 6; -3x - 2y + 5z = -12; x - 2z = 3$
  - $2y + z = 3(-x + 1); -x + 3y - z = -4; 3x + 2y + z = -\frac{1}{2}$
  - $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2}; x + y + z = 27$
- Vani, her father and her grand father have an average age of 53 . One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now?
- The sum of the digits of a three-digit number is 11 . If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number?
- There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹ 105 . When first 2 sorts are interchanged in their numbers its value will be increased by ₹ 20 . Find the number of currencies in each sort.

**Example 3.10** Find the GCD of the polynomials  $x^3 + x^2 - x + 2$  and  $2x^3 - 5x^2 + 5x - 3.$

**Example 3.11** Find the GCD of  $6x^3 - 30x^2 + 60x - 48$  and  $3x^3 - 12x^2 + 21x - 18.$

**Example 3.12** Find the LCM of the following

(i)  $8x^4y^2, 48x^2y^4$

(ii)  $5x - 10, 5x^2 - 20$

(iii)  $x^4 - 1, x^2 - 2x + 1$

(iv)  $x^3 - 27, (x - 3)^2, x^2 - 9$

## Exercise 3.2

- Find the GCD of the given polynomials
  - $x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$
  - $x^4 - 1, x^3 - 11x^2 + x - 11$
  - $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$
  - $3x^3 + 3x^2 + 3x + 3, 6x^3 + 12x^2 + 6x + 12$
- Find the LCM of the given expressions.
  - $4x^2y, 8x^3y^2$  (ii)  $9a^3b^2, 12a^2b^2c$  (iii)  $16m, 12m^2n^2, 8n^2$  (iv)  $p^2 - 3p + 2, p^2 - 4$
  - $2x^2 - 5x - 3, 4x^2 - 36$  (vi)  $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

## Exercise 3.3

- Find the LCM and GCD for the following and verify that  $f(x) \times g(x) = LCM \times GCD$ 
  - $21x^2y, 35xy^2$  (ii)  $(x^3 - 1)(x + 1), (x^3 + 1)$  (iii)  $(x^2y + xy^2), (x^2 + xy)$
- Find the LCM of each pair of the following polynomials
  - $a^2 + 4a - 12, a^2 - 5a + 6$  whose GCD is  $a - 2$
  - $x^4 - 27a^3x, (x - 3a)^2$  whose GCD is  $(x - 3a)$
- Find the GCD of each pair of the following polynomials
  - $12(x^4 - x^3), 8(x^4 - 3x^3 + 2x^2)$  whose LCM is  $24x^3(x - 1)(x - 2)$
  - $(x^3 + y^3), (x^4 + x^2y^2 + y^4)$  whose LCM is  $(x^3 + y^3)(x^2 + xy + y^2)$
- Given the LCM and GCD of the two polynomials  $p(x)$  and  $q(x)$  find the unknown polynomial in the following table

S.No.	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3 - 10a^2 + 11a + 70$	$a - 7$	$a^2 - 12a + 35$	
(ii)	$(x^4 - y^4)(x^4 + x^2y^2 + y^4)$	$(x^2 - y^2)$		$(x^4 - y^4)(x^2 + y^2 - xy)$

**Example 3.13** Reduce the rational expressions to its lowest form

(i)  $\frac{x-3}{x^2-9}$  (ii)  $\frac{x^2-16}{x^2+8x+16}$

**Example 3.14** Find the excluded values of the following expressions (if any).

(i)  $\frac{x+10}{8x}$  (ii)  $\frac{7p+2}{8p^2+13p+5}$  (iii)  $\frac{x}{x^2+1}$

## Exercise 3.4

- Reduce each of the following rational expressions to its lowest form.
  - $\frac{x^2-1}{x^2+x}$  (ii)  $\frac{x^2-11x+18}{x^2-4x+4}$  (iii)  $\frac{9x^2+81x}{x^3+8x^2-9x}$  (iv)  $\frac{p^2-3p-40}{2p^3-24p^2+64p}$
- Find the excluded values, if any of the following expressions.

(i)  $\frac{y}{y^2-25}$  (ii)  $\frac{t}{t^2-5t+6}$  (iii)  $\frac{x^2+6x+8}{x^2+x-2}$  (iv)  $\frac{x^3-27}{x^3+x^2-6x}$

**Example 3.15** (i) Multiply  $\frac{x^3}{9y^2}$  by  $\frac{27y}{x^5}$  (ii) Multiply  $\frac{x^4b^2}{x-1}$  by  $\frac{x^2-1}{a^4b^3}$

**Example 3.16** Find

(i)  $\frac{14x^4}{y} \sqrt{\frac{7x}{3y^4}}$  (ii)  $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$  (iii)  $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

## Exercise 3.5

- Simplify
  - $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$  (ii)  $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$  (iii)  $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

2. Simplify

(i)  $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

(ii)  $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$

3. Simplify

(i)  $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$

(ii)  $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$

(iii)  $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$

(iv)  $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

4. If  $x = \frac{a^2+3a-4}{3a^2-3}$  and  $y = \frac{a^2+2a-8}{2a^2-2a-4}$  find the value of  $x^2y^{-2}$ .

5. If a polynomial  $p(x) = x^2 - 5x - 14$  is divided by another polynomial  $q(x)$  we get  $\frac{x-7}{x+2}$ , find  $q(x)$ .

**Example 3.17** Find  $\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28}$

**Example 3.18** Simplify  $\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$

### Exercise 3.6

1. Simplify (i)  $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$  (ii)  $\frac{x+2}{x+3} + \frac{x-1}{x-2}$  (iii)  $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

2. Simplify (i)  $\frac{(2x+1)(x-2)}{x-4} - \frac{(2x^2-5x+2)}{x-4}$  (ii)  $\frac{4x}{x^2-1} - \frac{x+1}{x-1}$

3. Subtract  $\frac{1}{x^2+2}$  from  $\frac{2x^3+x^2+3}{(x^2+2)^2}$

4. Which rational expression should be subtracted from  $\frac{x^2+6x+8}{x^3+8}$  to get  $\frac{3}{x^2-2x+4}$

5. If  $A = \frac{2x+1}{2x-1}$ ,  $B = \frac{2x-1}{2x+1}$  find  $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$

6. If  $A = \frac{x}{x+1}$ ,  $B = \frac{1}{x+1}$ , prove that  $\frac{(A+B)^2+(A-B)^2}{A+B} = \frac{2(x^2+1)}{x(x+1)^2}$

7. Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?

8. Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy?

**Example 3.19** Find the square root of the following expressions

(i)  $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

(ii)  $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

**Example 3.20** Find the square root of the following expressions

(i)  $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$  (ii)  $(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$

(iii)  $[\sqrt{15}x^2 + (\sqrt{3} + \sqrt{10})x + \sqrt{2}][\sqrt{5}x^2 + (2\sqrt{5} + 1)x + 2][\sqrt{3}x^2 + (\sqrt{2} + 2\sqrt{3})x + 2\sqrt{2}]$

### Exercise 3.7

1. Find the square root of the following rational expressions.

(i)  $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

(ii)  $\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$

(iii)  $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

2. Find the square root of the following

(i)  $4x^2 + 20x + 25$

(ii)  $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$



(iii)  $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

(iv)  $\left(2x^2 + \frac{17}{6}x + 1\right)\left(\frac{3}{2}x^2 + 4x + 2\right)\left(\frac{4}{3}x^2 + \frac{11}{3}x + 2\right)$

**Example 3.21** Find the square root of  $64x^4 - 16x^3 + 17x^2 - 2x + 1$

**Example 3.22** If  $9x^4 + 12x^3 + 28x^2 + ax + b$  is a perfect square, find the values of  $a$  and  $b$ .

### Exercise 3.8

1. Find the square root of the following polynomials by division method

(i)  $x^4 - 12x^3 + 42x^2 - 36x + 9$

(ii)  $37x^2 - 28x^3 + 4x^4 + 42x + 9$

(iii)  $16x^4 + 8x^2 + 1$

(iv)  $121x^4 - 198x^3 - 183x^2 + 216x + 144$

2. Find the values of  $a$  and  $b$  if the following polynomials are perfect squares

(i)  $4x^4 - 12x^3 + 37x^2 + bx + a$

(ii)  $ax^4 + bx^3 + 361x^2 + 220x + 100$

3. Find the values of  $m$  and  $n$  if the following polynomials are perfect squares

(i)  $36x^4 - 60x^3 + 61x^2 - mx + n$

(ii)  $x^4 - 8x^3 + mx^2 + nx + 16$

**Example 3.23** Find the zeroes of the quadratic expression  $x^2 + 8x + 12$ .

**Example 3.24** Write down the quadratic equation in general form for which sum and product of the roots are given below.

(i) 9, 14

(ii)  $-\frac{7}{2}, \frac{5}{2}$

(iii)  $-\frac{3}{5}, -\frac{1}{2}$

**Example 3.25** Find the sum and product of the roots for each of the following quadratic equations:

(i)  $x^2 + 8x - 65 = 0$

(ii)  $2x^2 + 5x + 7 = 0$

(iii)  $kx^2 - k^2x - 2k^3 = 0$

### Exercise 3.9

1. Determine the quadratic equations, whose sum and product of roots are

(i) -9, 20

(ii)  $\frac{5}{3}, 4$

(iii)  $\frac{-3}{2}, -1$

(iv)  $-(2 - a)^2, (a + 5)^2$

2. Find the sum and product of the roots for each of the following quadratic equations

(i)  $x^2 + 3x - 28 = 0$

(ii)  $x^2 + 3x = 0$

(iii)  $3 + \frac{1}{a} = \frac{10}{a^2}$

(iv)  $3y^2 - y - 4 = 0$

**Example 3.26** Solve  $2x^2 - 2\sqrt{6}x + 3 = 0$

**Example 3.27** Solve  $2m^2 + 19m + 30 = 0$

**Example 3.28** Solve  $x^4 - 13x^2 + 42 = 0$

**Example 3.29** Solve  $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

### Exercise 3.10

1. Solve the following quadratic equations by factorization method

(i)  $4x^2 - 7x - 2 = 0$

(ii)  $3(p^2 - 6) = p(p + 5)$

(iii)  $\sqrt{a(a-7)} = 3\sqrt{2}$

(iv)  $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(v)  $2x^2 - x + \frac{1}{8} = 0$

2. The number of volleyball games that must be scheduled in a league with  $n$  teams is given by

$G(n) = \frac{n^2 - n}{2}$  where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

**Example 3.30** Solve  $x^2 - 3x - 2 = 0$

**Example 3.31** Solve  $2x^2 - x - 1 = 0$

**Example 3.32** Solve  $x^2 + 2x - 2 = 0$  by formula method

**Example 3.33** Solve  $2x^2 - 3x - 3 = 0$  by formula method.

**Example 3.34** Solve  $3p^2 + 2\sqrt{5}p - 5 = 0$  by formula method.

**Example 3.35** Solve  $pqx^2 - (p + q)^2x + (p + q)^2 = 0$

## Exercise 3.11

- Solve the following quadratic equations by completing the square method
  - $9x^2 - 12x + 4 = 0$
  - $\frac{5x+7}{x-1} = 3x + 2$
- Solve the following quadratic equations by formula method
  - $2x^2 - 5x + 2 = 0$
  - $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$
  - $3y^2 - 20y - 23 = 0$
  - $36y^2 - 12ay + (a^2 - b^2) = 0$
- A ball rolls down a slope and travels a distance  $d = t^2 - 0.75t$  feet in  $t$  seconds. Find the time when the distance travelled by the ball is 11.25 feet.

**Example 3.36** The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

**Example 3.37** A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets if the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?

**Example 3.38** A flock of swans contained  $x^2$  members. As the clouds gathered,  $10x$  went to a lake and one-eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

**Example 3.39** A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of the express train is more than that of the passenger train by 20 km per hour. Find the average speed of both the trains.

## Exercise 3.12

- If the difference between a number and its reciprocal is  $\frac{24}{5}$ , find the number.
- A garden measuring 12 m by 16 m is to have a pedestrian pathway that is 'w' meters wide installed all the way around so that it increases the total area to 285 m<sup>2</sup>. What is the width of the pathway?
- A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.
- A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.
- A pole has to be erected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4 m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?
- From a group of  $2x^2$  black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?
- Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m. Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).
- There is a square field whose side is 10 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square metre respectively is ₹ 364. Find the width of the gravel path.

9. The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm . Find the length of the smallest side.

**Example 3.40** Determine the nature of roots for the following quadratic equations

(i)  $x^2 - x - 20 = 0$  (ii)  $9x^2 - 24x + 16 = 0$  (iii)  $2x^2 - 2x + 9 = 0$

**Example 3.41** (i) Find the values of ' k ', for which the quadratic equation  $kx^2 - (8k + 4)x + 81 = 0$  has real and equal roots?

(ii) Find the values of ' k ' such that quadratic equation  $(k + 9)x^2 + (k + 1)x + 1 = 0$  has no real roots?

**Example 3.42** Prove that the equation  $x^2(p^2 + q^2) + 2x(pr + qs) + r^2 + s^2 = 0$  has no real roots. If  $ps = qr$ , then show that the roots are real and equal.

### Exercise 3.13

1. Determine the nature of the roots for the following quadratic equations

(i)  $15x^2 + 11x + 2 = 0$  (ii)  $x^2 - x - 1 = 0$  (iii)  $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

(iv)  $9y^2 - 6\sqrt{2}y + 2 = 0$  (v)  $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$

2. Find the value(s) of ' k ' for which the roots of the following equations are real and equal.

(i)  $(5k - 6)x^2 + 2kx + 1 = 0$  (ii)  $kx^2 + (6k + 2)x + 16 = 0$

3. If the roots of  $(a - b)x^2 + (b - c)x + (c - a) = 0$  are real and equal, then prove that  $b, a, c$  are in arithmetic progression.

4. If  $a, b$  are real then show that the roots of the equation  $(a - b)x^2 - 6(a + b)x - 9(a - b) = 0$  are real and unequal.

5. If the roots of the equation  $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$  are real and equal prove that either  $a = 0$  (or)  $a^3 + b^3 + c^3 = 3abc$

**Example 3.43** If the difference between the roots of the equation  $x^2 - 13x + k = 0$  is 17 find  $k$ .

**Example 3.44** If  $\alpha$  and  $\beta$  are the roots of  $x^2 + 7x + 10 = 0$  find the values of

(i)  $(\alpha - \beta)$  (ii)  $\alpha^2 + \beta^2$  (iii)  $\alpha^3 - \beta^3$

(iv)  $\alpha^4 + \beta^4$  (v)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (vi)  $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

**Example 3.45** If  $\alpha, \beta$  are the roots of the equation  $3x^2 + 7x - 2 = 0$ , find the values of

(i)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (ii)  $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

**Example 3.46** If  $\alpha, \beta$  are the roots of the equation  $2x^2 - x - 1 = 0$ , then form the equation whose roots

are (i)  $\frac{1}{\alpha}, \frac{1}{\beta}$  (ii)  $\alpha^2\beta, \beta^2\alpha$  (iii)  $2\alpha + \beta, 2\beta + \alpha$

### Exercise 3.14

1. Write each of the following expression in terms of  $\alpha + \beta$  and  $\alpha\beta$ .

(i)  $\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha}$  (ii)  $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$  (iii)  $(3\alpha - 1)(3\beta - 1)$  (iv)  $\frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha}$

2. The roots of the equation  $2x^2 - 7x + 5 = 0$  are  $\alpha$  and  $\beta$ . Without solving for the roots, find

(i)  $\frac{1}{\alpha} + \frac{1}{\beta}$  (ii)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (iii)  $\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2}$

3. The roots of the equation  $x^2 + 6x - 4 = 0$  are  $\alpha, \beta$ . Find the quadratic equation whose roots are

(i)  $\alpha^2$  and  $\beta^2$  (ii)  $\frac{2}{\alpha}$  and  $\frac{2}{\beta}$  (iii)  $\alpha^2\beta$  and  $\beta^2\alpha$

4. If  $\alpha, \beta$  are the roots of  $7x^2 + ax + 2 = 0$  and if  $\beta - \alpha = \frac{-13}{7}$ . Find the values of  $a$ .

5. If one root of the equation  $2y^2 - ay + 64 = 0$  is twice the other then find the values of  $a$ .

6. If one root of the equation  $3x^2 + kx + 81 = 0$  (having real roots) is the square of the other then find  $k$ .

**Example 3.47** Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm .

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

**Example 3.48** A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

- (i) the constant of variation
- (ii) how far will it travel in 90 minutes?
- (iii) the time required to cover a distance of 300 km from the graph..

**Example 3.49** A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

- (i) Graph the above data and identify the type of variation.
- (ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers?
- (iii) If the work has to be completed by 200 days, how many workers are required?

**Example 3.50** Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2hrs, 3hrs, 4hrs and 6 hours respectively.

Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

### Exercise 3.15

1. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find
  - (i) the marked price when a customer gets a discount of ₹3250 (from graph)
  - (ii) the discount when the marked price is ₹2500
2. Draw the graph of  $xy = 24, x, y > 0$ . Using the graph find,
  - (i) y when  $x = 3$  and
  - (ii) x when  $y = 6$ .
3. Graph the following linear function  $y = \frac{1}{2}x$ . Identify the constant of variation and verify it with the graph. Also
  - (i) find y when  $x = 9$
  - (ii) find x when  $y = 7.5$ .
4. The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

- (i) find the time taken to fill the tank when five pipes are used
- (ii) Find the number of pipes when the time is 9 minutes.

5. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

6. A two wheeler parking zone near bus stand charges as below.

Time (in hours) ( $x$ )	4	8	12	24
Amount ₹ ( $y$ )	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr ; (ii) find the parking duration when the amount paid is ₹ 150 .

**Example 3.51** Discuss the nature of solutions of the following quadratic equations.

(i)  $x^2 + x - 12 = 0$       (ii)  $x^2 - 8x + 16 = 0$       (iii)  $x^2 + 2x + 5 = 0$

**Example 3.52** Draw the graph of  $y = 2x^2$  and hence solve  $2x^2 - x - 6 = 0$

**Example 3.53** Draw the graph of  $y = x^2 + 4x + 3$  and hence find the roots of  $x^2 + x + 1 = 0$

**Example 3.54** Draw the graph of  $y = x^2 + x - 2$  and hence solve  $x^2 + x - 2 = 0$ .

**Example 3.55** Draw the graph of  $y = x^2 - 4x + 3$  and use it to solve  $x^2 - 6x + 9 = 0$

### Exercise 3.16

1. Graph the following quadratic equations and state their nature of solutions.

(i)  $x^2 - 9x + 20 = 0$       (ii)  $x^2 - 4x + 4 = 0$       (iii)  $x^2 + x + 7 = 0$   
 (iv)  $x^2 - 9 = 0$       (v)  $x^2 - 6x + 9 = 0$       (vi)  $(2x - 3)(x + 2) = 0$

2. Draw the graph of  $y = x^2 - 4$  and hence solve  $x^2 - x - 12 = 0$

3. Draw the graph of  $y = x^2 + x$  and hence solve  $x^2 + 1 = 0$

4. Draw the graph of  $y = x^2 + 3x + 2$  and use it to solve  $x^2 + 2x + 1 = 0$

5. Draw the graph of  $y = x^2 + 3x - 4$  and hence use it to solve  $x^2 + 3x - 4 = 0$

6. Draw the graph of  $y = x^2 - 5x - 6$  and hence solve  $x^2 - 5x - 14 = 0$

7. Draw the graph of  $y = 2x^2 - 3x - 5$  and hence solve  $2x^2 - 4x - 6 = 0$

8. Draw the graph of  $y = (x - 1)(x + 3)$  and hence solve  $x^2 - x - 6 = 0$

**Example 3.56** Consider the following information regarding the number of men and women workers in three factories I, II and III.

Factory	Men	Women
I	23	18
II	47	36
III	15	16

**Example 3.57** If a matrix has 16 elements, what are the possible orders it can have?

**Example 3.58** Construct a  $3 \times 3$  matrix whose elements are  $a_{ij} = i^2 j^2$

**Example 3.59** Find the value of  $a, b, c, d$  from the equation  $\begin{pmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$

## Exercise 3.17

1. In the matrix  $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$ , write
  - (i) The number of elements
  - (ii) The order of the matrix
  - (iii) Write the elements  $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$ .
2. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?
3. Construct a  $3 \times 3$  matrix whose elements are given by
  - (i)  $a_{ij} = |i - 2j|$
  - (ii)  $a_{ij} = \frac{(i+j)^3}{3}$
4. If  $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$  then find the transpose of  $A$ .
5. If  $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$  then find the transpose of  $-A$ .
6. If  $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$  then verify  $(A^T)^T = A$
7. Find the values of  $x, y$  and  $z$  from the following equations
  - (i)  $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$
  - (ii)  $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$
  - (iii)  $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

**Example 3.60** If  $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$ , find  $A + B$ .

**Example 3.61** Two examinations were conducted for three groups of students namely group 1, group 2, group 3 and their data on average of marks for the subjects Tamil, English, Science and Mathematics are given below in the form of matrices  $A$  and  $B$ . Find the total marks of both the examinations for all the three groups.

$$A = \begin{matrix} & \begin{matrix} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{matrix} \\ \begin{matrix} \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{matrix} & \begin{pmatrix} 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{pmatrix} \end{matrix}$$

$$B = \begin{matrix} & \begin{matrix} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{matrix} \\ \begin{matrix} \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{matrix} & \begin{pmatrix} 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{pmatrix} \end{matrix}$$

**Example 3.62** If  $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$ , find  $A + B$ .

**Example 3.63** If  $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}, B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$  then Find  $2A + B$ .

**Example 3.64** If  $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}, B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ , find  $4A - 3B$ .

**Example 3.65** Find the value of  $a, b, c, d$  from the following matrix equation.

$$\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$$

**Example 3.66** If  $A = \begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}, B = \begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$  compute the following :

(i)  $3A + 2B - C$

(ii)  $\frac{1}{2}A - \frac{3}{2}B$

### Exercise 3.18

1. If  $A = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}, B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$  then verify that

(i)  $A + B = B + A$

(ii)  $A + (-A) = (-A) + A = O$ .

2. If  $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$  and  $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$  then verify that

$$A + (B + C) = (A + B) + C.$$

3. Find  $X$  and  $Y$  if  $X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$  and  $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

4. If  $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$  find the value of (i)  $B - 5A$  (ii)  $3A - 9B$

5. Find the values of  $x, y, z$  if (i)  $\begin{pmatrix} x - 3 & 3x - z \\ x + y + 7 & x + y + z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

(ii)  $\begin{pmatrix} x & y - z & z + 3 \\ y & 4 & 3 \end{pmatrix} + \begin{pmatrix} y & 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 16 \end{pmatrix}$

6. Find  $x$  and  $y$  if  $x \begin{pmatrix} 4 \\ -3 \end{pmatrix} + y \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

7. Find the non-zero values of  $x$  satisfying the matrix equation

$$x \begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2 \begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2 \begin{pmatrix} x^2 + 8 & 24 \\ 10 & 6x \end{pmatrix}$$

8. Solve for  $x, y: \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2 \begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

**Example 3.67** If  $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ , find  $AB$ .

**Example 3.68** If  $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$  find  $AB$  and  $BA$ . Verify  $AB = BA$  ?

**Example 3.69** If  $A = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$

Show that  $A$  and  $B$  satisfy commutative property with respect to matrix multiplication.

**Example 3.70** Solve  $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

**Example 3.71** If  $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$  and  $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$  show that  $(AB)C = A(BC)$ .

**Example 3.72** If  $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}, C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$  verify that  $A(B + C) = AB + AC$ .

**Example 3.73** If  $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$  show that  $(AB)^T = B^T A^T$

### Exercise 3.19

- Find the order of the product matrix  $AB$  if
- If  $A$  is of order  $p \times q$  and  $B$  is of order  $q \times r$  what is the order of  $AB$  and  $BA$  ?

	(i)	(ii)	(iii)	(iv)	(v)
Orders of A	$3 \times 3$	$4 \times 3$	$4 \times 2$	$4 \times 5$	$1 \times 1$
Orders of B	$3 \times 3$	$3 \times 2$	$2 \times 2$	$5 \times 1$	$1 \times 3$

- $A$  has ' $a$ ' rows and ' $a + 3$ ' columns.  $B$  has ' $b$ ' rows and ' $17 - b$ ' columns, and if both products  $AB$  and  $BA$  exist, find  $a, b$  ?
- If  $A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$  find  $AB, BA$  and verify  $AB = BA$  ?
- Given that  $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$  verify that  $A(B + C) = AB + AC$ .
- Show that the matrices  $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$  satisfy commutative property  $AB = BA$
- Let  $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$  Show that (i)  $A(BC) = (AB)C$   
(ii)  $(A - B)C = AC - BC$  (iii)  $(A - B)^T = A^T - B^T$
- If  $A = \begin{pmatrix} \cos \theta & 0 \\ 0 & \cos \theta \end{pmatrix}, B = \begin{pmatrix} \sin \theta & 0 \\ 0 & \sin \theta \end{pmatrix}$  then show that  $A^2 + B^2 = I$ .
- If  $A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$  prove that  $AA^T = I$ .
- Verify that  $A^2 = I$  when  $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$
- If  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  show that  $A^2 - (a + d)A = (bc - ad)I_2$
- If  $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$  verify that  $(AB)^T = B^T A^T$
- If  $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$  show that  $A^2 - 5A + 7I_2 = 0$

## Exercise 3.20

### Multiple choice questions

- A system of three linear equations in three variables is inconsistent if their planes  
(A) intersect only at a point (B) intersect in a line  
(C) coincides with each other (D) do not intersect
- The solution of the system  $x + y - 3z = -6, -7y + 7z = 7, 3z = 9$  is  
(A)  $x = 1, y = 2, z = 3$  (B)  $x = -1, y = 2, z = 3$   
(C)  $x = -1, y = -2, z = 3$  (D)  $x = 1, y = -2, z = 3$
- If  $(x - 6)$  is the HCF of  $x^2 - 2x - 24$  and  $x^2 - kx - 6$  then the value of  $k$  is  
(A) 3 (B) 5 (C) 6 (D) 8
- $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$  is  
(A)  $\frac{9y}{7}$  (B)  $\frac{9y^3}{(21y-21)}$  (C)  $\frac{21y^2-42y+21}{3y^3}$  (D)  $\frac{7(y^2-2y+1)}{y^2}$
- $y^2 + \frac{1}{y^2}$  is not equal to  
(A)  $\frac{y^4+1}{y^2}$  (B)  $\left(y + \frac{1}{y}\right)^2$  (C)  $\left(y - \frac{1}{y}\right)^2 + 2$  (D)  $\left(y + \frac{1}{y}\right)^2 - 2$
- $\frac{x}{x^2-25} - \frac{8}{x^2+6x+5}$  gives  
(A)  $\frac{x^2-7x+40}{(x-5)(x+5)}$  (B)  $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$  (C)  $\frac{x^2-7x+40}{(x^2-25)(x+1)}$  (D)  $\frac{x^2+10}{(x^2-25)(x+1)}$
- The square root of  $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$  is equal to  
(A)  $\frac{16}{5} \left| \frac{x^2z^4}{y^2} \right|$  (B)  $16 \left| \frac{y^2}{x^2z^4} \right|$  (C)  $\frac{16}{5} \left| \frac{y}{xz^2} \right|$  (D)  $\frac{16}{5} \left| \frac{xz^2}{y} \right|$
- Which of the following should be added to make  $x^4 + 64$  a perfect square  
(A)  $4x^2$  (B)  $16x^2$  (C)  $8x^2$  (D)  $-8x^2$
- The solution of  $(2x - 1)^2 = 9$  is equal to  
(A) -1 (B) 2 (C) -1,2 (D) None of these



10. The values of  $a$  and  $b$  if  $4x^4 - 24x^3 + 76x^2 + ax + b$  is a perfect square are  
 (A) 100,120 (B) 10,12 (C) -120,100 (D) 12,10
11. If the roots of the equation  $q^2x^2 + p^2x + r^2 = 0$  are the squares of the roots of the equation  $qx^2 + px + r = 0$ , then  $q, p, r$  are in  
 (A) A.P (B) G.P (C) Both A.P and G.P (D) none of these
12. Graph of a linear equation is a  
 (A) straight line (B) circle (C) parabola (D) hyperbola
13. The number of points of intersection of the quadratic polynomial  $x^2 + 4x + 4$  with the  $X$  axis is  
 (A) 0 (B) 1 (C) 0 or 1 (D) 2
14. For the given matrix  $A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$  the order of the matrix  $A^T$  is  
 (A)  $2 \times 3$  (B)  $3 \times 2$  (C)  $3 \times 4$  (D)  $4 \times 3$
15. If  $A$  is a  $2 \times 3$  matrix and  $B$  is a  $3 \times 4$  matrix, how many columns does  $AB$  have  
 (A) 3 (B) 4 (C) 2 (D) 5
16. If number of columns and rows are not equal in a matrix then it is said to be a  
 (A) diagonal matrix (B) rectangular matrix  
 (C) square matrix (D) identity matrix
17. Transpose of a column matrix is  
 (A) unit matrix (B) diagonal matrix (C) column matrix (D) row matrix
18. Find the matrix  $X$  if  $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$   
 (A)  $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$  (B)  $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$  (C)  $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$  (D)  $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$
19. Which of the following can be calculated from the given matrices  
 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ , (i)  $A^2$  (ii)  $B^2$  (iii)  $AB$  (iv)  $BA$   
 (A) (i) and (ii) only (B) (ii) and (iii) only  
 (C) (ii) and (iv) only (D) all of these
20. If  $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$  and  $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$ . Which of the following statements are correct?  
 (i)  $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$  (ii)  $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$  (iii)  $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$  (iv)  $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$   
 (A) (i) and (ii) only (B) (ii) and (iii) only  
 (C) (iii) and (iv) only (D) all of these

### Unit Exercise - 3

- Solve  $\frac{1}{3}(x + y - 5) = y - z = 2x - 11 = 9 - (x + 2z)$
- One hundred and fifty students are admitted to a school. They are distributed over three sections  $A, B$  and  $C$ . If 6 students are shifted from section  $A$  to section  $C$ , the sections will have equal number of students. If 4 times of students of section  $C$  exceeds the number of students of section  $A$  by the number of students in section  $B$ , find the number of students in the three sections.
- In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.
- Find the least common multiple of  $xy(k^2 + 1) + k(x^2 + y^2)$  and  $xy(k^2 - 1) + k(x^2 - y^2)$

5. Find the GCD of the following by division algorithm  
 $2x^4 + 13x^3 + 27x^2 + 23x + 7, x^3 + 3x^2 + 3x + 1, x^2 + 2x + 1$
6. Reduce the given Rational expressions to its lowest form
  - (i)  $\frac{x^{3a}-8}{x^{2a}+2x^a+4}$
  - (ii)  $\frac{10x^3-25x^2+4x-10}{-4-10x^2}$
7. Simplify  $\frac{\frac{1}{p} + \frac{1}{q+r}}{\frac{1}{1} - \frac{1}{q+r}} \times \left(1 + \frac{q^2+r^2-p^2}{2qr}\right)$
8. Arul, Madan and Ram working together can clean a store in 6 hours. Working alone, Madan takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?
9. Find the square root of  $289x^4 - 612x^3 + 970x^2 - 684x + 361$ .
10. Solve  $\sqrt{y+1} + \sqrt{2y-5} = 3$
11. A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr, what is the speed of the boat in still water?
12. Is it possible to design a rectangular park of perimeter 320 m and area  $4800 \text{ m}^2$ ? If so find its length and breadth.
13. At  $t$  minutes past 2 pm, the time needed to 3 pm is 3 minutes less than  $\frac{t^2}{4}$ . Find  $t$ .
14. The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.
15. If  $\alpha$  and  $\beta$  are the roots of the polynomial  $f(x) = x^2 - 2x + 3$ , find the polynomial whose roots are (i)  $\alpha + 2, \beta + 2$  (ii)  $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$ .
16. If -4 is a root of the equation  $x^2 + px - 4 = 0$  and if the equation  $x^2 + px + q = 0$  has equal roots, find the values of  $p$  and  $q$ .
17. Two farmers Thilagan and Kausigan cultivates three varieties of grains namely rice, wheat and ragi. If the sale (in ₹) of three varieties of grains by both the farmers in the month of April is given by the matrix and the May month sale (in ₹) is exactly twice as that of the April month sale for each variety.
 

April sale in ₹			
rice	wheat	ragi	
500	1000	1500	Thilagan
2500	1500	500	Kausigan

  - (i) What is the average sales of the months April and May.
  - (ii) If the sales continues to increase in the same way in the successive months, what will be sales in the month of August?
18. If  $\cos \theta \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} + \sin \theta \begin{pmatrix} x & -\cos \theta \\ \cos \theta & x \end{pmatrix} = I_2$ , find  $x$ .
19. Given  $A = \begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$  and if  $BA = C^2$ , find  $p$  and  $q$ .
20.  $A = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix}, C = \begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix}$  find the matrix  $D$ , such that  $CD - AB = 0$

**CHAPTER - 4.GEOMETRY**

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