11th STD

PUBLIC EXAMINATION - MARCH 2025

Reg. No.

PART - III

Business Mathematics And Statistics (with answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. The angle between the pair of straight lines $x^2 - 7xy + 4y^2 = 0$ is :

(a) $\tan^{-1}\left(\frac{\sqrt{33}}{5}\right)$ (b) $\tan^{-1}\left(\frac{1}{3}\right)$

(c) $\tan^{-1}\left(\frac{5}{\sqrt{33}}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

2. $\lim_{x \to 0} \frac{e^x - 1}{x} =$

(a) 1 (b) e (c) 0 (d) nx^{n-1}

3. If demand and the cost function of a firm are $p = 2 - x$ and $C = -2x^2 + 2x + 7$, then its profit function is :

(a) $-x^2 + 7$ (b) $x^2 + 7$

(c) $-x^2 - 7$ (d) $x^2 - 7$

4. If $|A| = 5$, then the value of $|A^{-1}|$ is _______

(a) 1 (b) 5

(c) does not exist (d) $\frac{1}{5}$

5. When one regression coefficient is negative, the other would be :

(a) Positive (b) Zero

(c) Negative (d) None of them

6. The maximum value of the objective function $Z = 3x + 5y$ subject to the constraints $x \geq 0, y \geq 0$ and $2x + 5y \leq 10$ is :

(a) 25 (b) 6 (c) 31 (d) 15

7. The term regression was introduced by :

(a) Karl Pearson (b) R.A Fisher

(c) Croxton and Cowden (d) Sir Francis Galton

8. The correct relationship among A.M.,G.M., and H.M., is :

(a) H.M. ≥ G.M. ≥ A.M.

(b) A.M.< G.M. < H.M.

(c) A.M. ≥ G.M. ≥ H.M

(d) G.M. ≥ A.M. ≥ H.M

9. What is the amount realised on selling 8% stock of 200 shares of face value ₹100 at ₹50?

(a) ₹7,000 (b) ₹16,000

(c) ₹9,000 (d) ₹10,000

10. If $Q_1 = 30$ and $Q_3 = 50$, the coefficient of Quartile deviation is :

(a) 10 (b) 20 (c) 0.25 (d) 40

11. Example of contingent annuity is :

(a) An endowment fund to give scholarships to students

(b) Personal loan from a bank

(c) Installments of payment for a plot of land

(d) All the above

12. If $u = e^{x^2}$, then $\frac{\partial u}{\partial x}$ is equal to :

(a) $2e^{x^2}$ (b) $2xe^{x^2}$ (c) 0 (d) e^{x^2}

13. If $f(x) = \frac{1-x}{1+x}$, $x > 1$, then $f(-x)$ is equal to :

(a) $-\frac{1}{f(x)}$ (b) $-f(x)$

(c) $f(x)$ (d) $\frac{1}{f(x)}$

14. Range of $\sin^{-1}x$ is _______.

(a) $[0, \pi]$ (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(c) $\mathbb{R}$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

15. The equation of the circle with centre on the x - axis and passing through the origin is :

(a) $x^2 + y^2 = a^2$ (b) $x^2 - 2ax + y^2 = 0$

(c) $x^2 - 2ay + y^2 = 0$ (d) $y^2 - 2ay + x^2 = 0$

16. $\lim\limits_{\theta \to 0} \frac{\sin 2\theta}{2\theta}$ = _______.

(a) 1 (b) ∞ (c) 2 (d) 0

17. Thirteen guests have participated in a dinner. The number of handshakes that happened in the dinner is :

(a) 286 (b) 715 (c) 13 (d) 78

18. The value of $4\cos^3 40° - 3\cos 40°$ is :

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2}$

19. The number of ways 8 identical flowers can be arranged in a ring is _______.

(a) $\frac{7}{2}$ (b) $8!$ (c) $\frac{7!}{2}$ (d) $\frac{8!}{2}$

20. The inverse matrix of $\begin{pmatrix} \frac{4}{5} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{1}{2} \end{pmatrix}$ is

(a) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$ (b) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$

(c) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{4}{5} \end{pmatrix}$ (d) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

22. What is the present value of an annuity due of ₹1,500 for 16 years at 8% per annum? $[(1.08)^{-16} = 0.2919]$

23. Calculate the correlation co-efficient from the following data. N = 9, ΣX = 45, ΣY = 108, $\Sigma X^2 = 285$, $\Sigma Y^2 = 1356$, ΣXY = 597.

24. An aeroplane flies, along the four sides of a square at speeds of 100, 200, 300 and 400 kilometres per hour respectively. What is the average speed of the plane in its flight around the square?

25. For the given demand function $p = 40 - x$, find the output when $\eta_d = 1$.

26. Solve : $\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$

27. Find the rank of the word 'RANK' in dictionary.

28. If A = 30° then prove that $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

29. How many five digits telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 with no digit appearing more than once?

30. If (–2, –7) is one extremity of a diameter of the circle $x^2 + y^2 - 2x + 6y - 15 = 0$, find the other extremity.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory**. **7 × 3 = 21**

31. Resolve into partial fractions. $\frac{2x-1}{x^2-5x+6}$

32. The rank of 10 students of same batch in two subjects A and B are given below. Calculate the rank correlation coefficient.

Rank of A	1	2	3	4	5	6	7	8	9	10
Rank of B	6	7	5	10	3	9	4	1	8	2

33. If $u = x^2(y - x) + y^2(x - y)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -2(x - y)^2$.

34. Solve by matrix inversion method. $2x + 3y - 5 = 0$; $x - 2y + 1 = 0$.

35. Find the co-ordinates of the focus, vertex, equation of the directrix of the parabola $x^2 = 8y$.

36. Construct the network for the projects consisting of various activities and their precedence relationships are as given below: A,B,C can start simultaneously A < F, E ; B < D, C ; E, D < G

37. If $y = 500e^{7x} + 600e^{-7x}$, then show that $y_2 - 49y = 0$.

38. Prove that $\frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} + \frac{\sin(A-B)}{\cos A\cos B} = 0$.

39. Find the value of k so that the line $3x + 4y - k = 0$ is a tangent to $x^2 + y^2 - 64 = 0$.

40. Evaluate : $\lim_{x\to 0} \frac{\log(1+x^4)}{\tan^4 x}$

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) You are given the following transaction matrix for a two sector economy.

Sector	Sales 1	Sales 2	Final demand	Gross output
1	4	3	13	20
2	5	4	3	12

i) Write the technology matrix.

ii) Determine the output when the final demand for the output sector 1 alone increases to 23 units.

(OR)

(b) The demand for a quantity A is $q = 13 - 2p_1 - 3p_2^2$. Find the partial elasticities $\frac{Eq}{Ep_1}$ and $\frac{Eq}{Ep_2}$ when $p_1 = p_2 = 2$

42. (a) Three boxes B_1 , B_2 , B_3 contain Lamp bulbs some of which are defective. The defective proportions in box B_1, box B_2 and box B_3 are respectively $\frac{1}{2}, \frac{1}{8}$ and $\frac{3}{4}$. A box is selected at random and a bulb drawn from it. If the selected bulb is found to be defective, what is the probability that the selected bulb is from the box B_1?

(OR)

(b) Using Mathematical Induction Method, prove that $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$, for all $n \in \mathbb{N}$.

43. (a) Show that the pair of straight lines $4x^2 + 12xy + 9y^2 - 6x - 9y + 2 = 0$ represents two parallel straight lines and also find the separate equations of the straight lines.

(OR)

(b) Find the means of X and Y variables and the coefficient of correlation between them from the following two regression equations :

$4X - 5Y + 33 = 0$

$20X - 9Y - 107 = 0$

44. (a) Solve the following LPP

Maximize $Z = 2x_1 + 5x_2$ subject to the conditions $x_1 + 4x_2 \le 24$, $3x_1 + x_2 \le 21$, $x_1 + x_2 \le 9$ and $x_1, x_2 \ge 0$.

(OR)

(b) Prove that the term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5....(2n-1)2^n}{n!}$.

45. (a) Prove that $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$.

(OR)

(b) The following table gives the annual demand and unit price of item A.

Item	Annual Demand	Unit Price
A	800	0.02

Ordering cost is ₹ 5 per order and annual holding cost is 10% of unit price.

Determine the following:

(i) EOQ in units

(ii) Minimum inventory cost

(iii) EOQ in rupees

(iv) EOQ in years of supply

(v) Number of orders per year

46. (a) If $x^m.y^n = (x + y)^{m+n}$, then show that $\frac{dy}{dx} = \frac{y}{x}$.

(OR)

(b) Sundar bought ₹4,500, 12% of ₹10 shares at par. He sold them when the price rose to ₹23 and invested the proceeds in ₹25 shares paying 10% per annum at ₹18. Find the change in his income.

47. (a) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ then, show that $(AB)^{-1} = B^{-1}A^{-1}$.

(OR)

(b) Compute the coefficient of quartile deviation from the following data.

Marks	10	20	30	40	50	60
No. of students	5	8	10	8	7	2

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ANSWERS

Part - I

1. (a) $\tan^{-1}\left(\frac{\sqrt{33}}{5}\right)$
2. (a) 1
3. (d) $x^2 - 7$
4. (d) $\frac{1}{5}$
5. (c) Negative
6. (d) 15
7. (d) Sir Francis Galton
8. (c) A.M. ≥ G.M. ≥ H.M
9. (d) ₹10,000
10. (c) 0.25
11. (a) An endowment fund to give scholarships to students
12. (b) $2xe^{x^2}$
13. (d) $\frac{1}{f(x)}$
14. (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
15. (b) $x^2 - 2ax + y^2 = 0$
16. (a) 1
17. (d) 78
18. (d) $\frac{-1}{2}$
19. (c) $\frac{7!}{2}$
20. (a) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$

Part - II

21. Given that $f(x) = g(x)$

$$\Rightarrow 2x^2 - 1 = 1 - 3x$$
$$2x^2 + 3x - 2 = 0$$
$$(x+2)(2x-1) = 0$$
$$\therefore x = -2, x = \frac{1}{2}$$

Domain is $\left\{-2, \frac{1}{2}\right\}$

22. Given $a = ₹\ 1500$

$i = 8\% = 0.08$

$n = 16$

$$P = \frac{a(1+i)}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$
$$= \frac{1500(1.08)}{0.08}\left[1 - \frac{1}{(1.08)^{16}}\right]$$
$$= \frac{1500(1.08)}{0.08}\left[1 - (1.08)^{-16}\right]$$
$$= \frac{1500(1.08)}{0.08}[1 - 0.2919]$$
$$= \frac{1500(1.08)}{0.08} \times 0.7081$$
$$P = ₹\ 14,339$$

23. We know that correlation coefficent

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$
$$= \frac{9(597) - (45 - 108)}{\sqrt{9(285) - (45)^2} \times \sqrt{9(1356) - (108^2)}}$$
$$r = +0.95$$

24. Since we are given speed per hour, harmonic mean will give the correct answer.

$$\text{Harmonic mean} = \frac{n}{1/a + 1/b + 1/c + 1/d}$$
$$= \frac{4}{1/100 + 1/200 + 1/300 + 1/400} = \frac{4}{\frac{12+6+4+3}{1200}}$$
$$= \frac{4}{25/1200} = \frac{4 \times 1200}{25} = 4 \times 48$$
$$HM = 192 \text{ km/hr}$$

25. $p = 40 - x \Rightarrow x = 40 - p$

$$\frac{dx}{dp} = -1$$

Elasticity of demand : $\eta_d = -\frac{p}{x}\cdot\frac{dx}{dp} = \frac{40-x}{x}$

Given that $\eta_d = 1$

$$\therefore \frac{40-x}{x} = 1 \Rightarrow x = 20$$

$\therefore$ Output (x) = 20 units.

26. Expanding the given determinant along R_1 we get

$$7\begin{vmatrix} 5 & x \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} -3 & x \\ -x & 1 \end{vmatrix} + 11\begin{vmatrix} -3 & 5 \\ -x & 3 \end{vmatrix} = 0$$

$\Rightarrow 7(5 - 3x) - 4(-3 + x^2) + 11(-9 + 5x) = 0$

$\Rightarrow 35 - 21x + 12 - 4x^2 - 99 + 55x = 0$

$\Rightarrow -4x^2 + 34x - 52 = 0$

Dividing by – 2 we get,

$2x^2 - 17x + 26 = 0$

Using factorization, the factors are $2x^2 - 17x + 26 = 0$

$2x^2 - 4x - 13x + 26 = 0$

$2x(x - 2) - 13(x - 2) = 0$

$(x - 2)(2x - 13) = 0 \Rightarrow x = 2$ or $x = \frac{13}{2}$

The values of x are 2, $\frac{13}{2}$

27. Here maximum number of the word in dictionary formed by using the letters of the word 'RANK' is 4!

The letters of the word RANK in alphabetical order are A, K, N, R

Number of words starting with A = 3! = 6
Number of words starting with K = 3! = 6
Number of words starting with N = 3! = 6
Number of words starting with RAK = 1! = 1
Number of words starting with RANK = 0! = 1

$\therefore$ Rank of the word RANK is 6 + 6 + 6 + 1+1= 20

28. LHS = sin 2A

Putting A = 30° in LHS and RHS, we get

$$\text{LHS} = \sin 2 \times 30° = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\text{RHS} = \frac{2 \times \tan 30°}{1 + \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\cancel{2} \times 3}{\sqrt{3} \times \cancel{4}_{2}} = \frac{\sqrt{3}}{2}$$

Hence, LHS = RHS. It is proved.

29.

6	7			

For a five digit number, the number starts with 67. So we have to fill the unit place, tens place hundreds place.

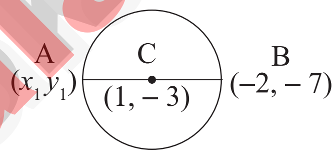
- Unit place can be filled up in 8 ways using numbers 0, 1, 2, 3, 4, 5, 8, 9
- Tens place can be filled up in 7 ways since repetition is not allowed.
- Hundreds place can be filled up in 6 ways.

$\therefore$ By fundamental principle of multiplication, the total number of 5 digits telephone numbers.

= 1 × 1 × 8 × 7 × 6 = 336

30. Given : (–2, –7)

$$x^2 + y^2 - 2x + 6y - 15 = 0$$



Find the centre

Let $x^2 + y^2 - 2x + 6y - 15 = 0$

We know that $x^2 + y^2 - 2gx + 2fy + C = 0$

Then the centre is

$$O = \left(\frac{-2}{2}, \frac{6}{-2}\right)$$

$$O = (1, -3)$$

Find the other extremity of the diameter.

Let the other end of the diameter be M (x_1, y_1)

We know that (–2, –7)

Substitute the centre O(1, –3) and (–2, –7)

$$\frac{x_1 - 2}{2} = 1$$

$$x_1 - 2 = 2$$

$$x_1 = 4$$

$$\frac{y_1 - 7}{2} = -3$$

$$y_1 - 7 = -6$$

$$y_1 = -6 + 7$$

$$\boxed{y_1 = 1}$$

The other extremity of the diameter is (4, 1)

Part - III

31. Write the denominator into the product of linear factors.

$$\text{Here } x^2 - 5x + 6 = (x-2)\ (x-3)$$

$$\text{Let } \frac{2x-1}{x^2-5x+6} = \frac{A}{(x-2)} + \frac{B}{(x-3)} \quad \ldots (1)$$

Multiplying both the sides of (1) by $(x-2)\ (x-3)$, we get

$$2x - 1 = A\,(x-2) + B(x-3) \quad \ldots (2)$$

Put $x = 3$ in (2) we get

$6 - 1 = B(1)$

$\Rightarrow\ B = 5$

Put $x = 2$ in (2) we get

$4 - 1 = A\,(-1)$

$\Rightarrow\ 3 = A\,(-1)$

$\Rightarrow\ A = -3$

Substituting the value of A and B in (1) we get

$$\frac{2x-1}{x^2-5x+6} = \frac{-3}{(x-2)} + \frac{5}{(x-3)}$$

32.

R_A	R_B	$d = R_A - R_B$	d^2
1	6	– 5	25
2	7	– 5	25
3	5	– 2	4
4	10	– 6	36
5	3	2	4
6	9	– 3	9
7	4	3	9
8	1	7	49
9	8	1	1
10	2	8	64
			$\Sigma d^2 = 226$

Rank Correlation Co-efficient is given by

$$\rho = 1 - \frac{6\Sigma d^2}{N(N^2-1)} = 1 - \frac{6(226)}{10(10^2-1)}$$

$$= 1 - \frac{6(226)}{10(99)} = 1 - \frac{1356}{990} = 1 - 1.37$$

$$\rho = -0.37$$

33. $u = x^2 y - x^3 + xy^2 - y^3$

$$\frac{\partial u}{\partial x} = 2xy - 3x^2 + y^2 \ ; \ \frac{\partial u}{\partial y} = x^2 + 2xy - 3y^2$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -2x^2 - 2y^2 + 4xy = -2(x^2 + y^2 - 2xy)$$

$$= -2\,(x-y)^2$$

34. Given equations are $2x + 3y = 5,\ x - 2y = -1$. Writing the given equations in matrix form we get,

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \quad \Rightarrow AX = B$$

$$\text{Where } A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\Rightarrow X = A^{-1} B.$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}$$

$$= -4 - 3 = -7 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$\text{adj } A = \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

[Interchange the places of leading diagonal elements and change the sign of off diagonal elements]

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 10-3 \\ 5+2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ 7 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Rightarrow\ x = 1$ and $y = 1$

35. Given equation of the parabola is $x^2 = 8y$

Comparing this with $x^2 = 4ay$, we get

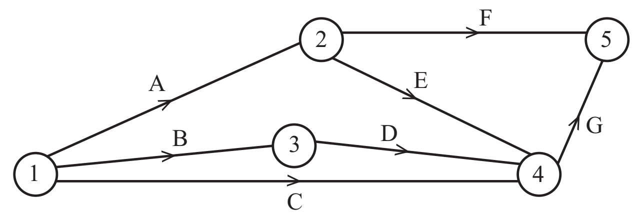
$\Rightarrow 4a = 8 \Rightarrow a = 2$

(i) Focus is $(0,a) \Rightarrow F\ (0,2)$

(ii) Vertex is (0,0)

(iii) Equation of the directrix is $y = -a \Rightarrow y = -2$

36.



37. Given $y = 500\, e^{7x} + 600\, e^{-7x}$ ... (1)

Differentiating with respect to 'x' we get,

$$y_1 = \frac{dy}{dx} = 500\,(e^{7x})\,(7) + 600\,(e^{-7x})\,(-7)$$
$$= 7\,(500\, e^{7x} - 600\, e^{-7x})$$

Differentiating again with respect to 'x' we get,

$$y_2 = 7(500\, e^{7x}(7) - 600\,(e^{-7x})\,(-7))$$
$$= 49\,(500\, e^{7x} + 600\, e^{-7x}) = 49y \quad \text{[using (1)]}$$
$$\therefore y_2 - 49y = 0$$

38. $$\text{LHS} = \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} + \frac{\sin(A-B)}{\cos A\cos B} \quad [\because \sin(A-B) = \sin A.\cos B - \cos A \sin B]$$

$$= \frac{\sin B\cos C - \cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A - \cos C\sin A}{\cos C\cos A} + \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}$$

$$= \frac{\sin B \cancel{\cos C}}{\cos B \cancel{\cos C}} - \frac{\cancel{\cos B}\sin C}{\cancel{\cos B}\cos C} + \frac{\sin C \cancel{\cos A}}{\cos C \cancel{\cos A}} - \frac{\cancel{\cos C}\sin A}{\cancel{\cos C}\cos A} + \frac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} - \frac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}$$

$$= \cancel{\tan B} - \cancel{\tan C} + \cancel{\tan C} - \cancel{\tan A} + \cancel{\tan A} - \cancel{\tan B} \quad \left[\because \frac{\sin A}{\cos A} = \tan A\right]$$

$= 0 =$ RHS. Hence Proved.

39. The given equations are $x^2 + y^2 - 64 = 0$ and $3x + 4y - k = 0$

The condition for the tangency is $c^2 = a^2(1 + m^2)$

Here $a^2 = 64$, $m = \frac{-3}{4}$ and $c = \frac{k}{4}$

$$c^2 = a^2(1 + m^2)$$
$$\Rightarrow \frac{k^2}{16} = 64\left(1 + \frac{9}{16}\right)$$
$$k^2 = 64 \times 25$$
$$k = \pm 40$$

40. Given the limit $\lim\limits_{x\to 0} \frac{\log(1+x^4)}{\tan^4 x}$

Rewrite the given limit $\lim\limits_{x\to 0} \frac{\log(1+x^4)}{x^4} \cdot \frac{x^4}{\tan^4 x}$

$$\lim_{x\to 0} \frac{\log(1+x^4)}{x^4} \cdot \left(\frac{x}{\tan x}\right)^4$$

As $x \to 0$, $x^4 \to 0$, thus

$$\lim_{x\to 0} \frac{\log(1+x^4)}{4} = 1 \text{ and } \lim_{x\to 0}\left(\frac{x}{\tan x}\right)^4 = \left(\lim_{x\to 0} \frac{x}{\tan x}\right)^4 = 1^4 = 1$$

Part - IV

41. (a) $a_{11} = 4, a_{12} = 3, x_1 = 20$

$a_{21} = 5, a_{22} = 4, x_2 = 12$

$b_{11} = \frac{a_{11}}{x_1} = \frac{4}{20} = \frac{1}{5}; \; b_{12} = \frac{a_{12}}{x_2} = \frac{3}{12} = \frac{1}{4}$

$b_{21} = \frac{a_{21}}{x_1} = \frac{5}{20} = \frac{1}{4}; \; b_{22} = \frac{a_{22}}{x_2} = \frac{4}{12} = \frac{1}{3}$

i) The technology matrix is $B = \begin{bmatrix} \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{2}{3} \end{bmatrix}$$

$$|I - B| = \begin{vmatrix} \frac{4}{5} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{2}{3} \end{vmatrix} = \left[\frac{4}{5}\right]\left[\frac{2}{3}\right] - \left[\frac{1}{4}\right]\left[\frac{1}{4}\right]$$

$$= \frac{8}{15} - \frac{1}{16} = \frac{128 - 15}{15 \times 16} = \frac{113}{240} > 0$$

(ii) Since the diagonals of (I – B) are positive and |I – B| is positive, the system has a solution.

$$\therefore (I - B)^{-1} = \frac{1}{|I - B|} \text{ adj } (I - B)$$

$$= \frac{1}{\frac{113}{240}} \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix} = \frac{240}{113} \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}$$

Also, $X = (I - B)^{-1} (D)$

$$\text{Where } D = \begin{bmatrix} 23 \\ 3 \end{bmatrix} = \frac{240}{113} \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \frac{1}{113} \begin{bmatrix} \frac{2}{3} \times 240 & \frac{1}{4} \times 240 \\ \frac{1}{4} \times 240 & \frac{4}{5} \times 240 \end{bmatrix} \begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \frac{1}{113} \begin{bmatrix} 160 & 60 \\ 60 & 192 \end{bmatrix} \begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \frac{1}{113} \begin{bmatrix} 160 \times 23 + 60 \times 3 \\ 60 \times 23 + 192 \times 3 \end{bmatrix} = \frac{1}{113} \begin{bmatrix} 3680 + 180 \\ 1380 + 576 \end{bmatrix}$$

$$= \frac{1}{113}\begin{bmatrix}3860\\1956\end{bmatrix} = \begin{bmatrix}34.16\\17.31\end{bmatrix}$$

The output for sector 1 is 34.16 units and 23 is 17.31 units.

(OR)

(b) Given $q = 13-2p_1-3p_2^2$

Differentiating partially with respect to 'p_1' we get,

$$\frac{\partial q}{\partial p_1} = 0-2-0=-2$$

$$\frac{Eq}{Ep_1} = \frac{-p_1}{q}\left(\frac{\partial q}{\partial p_1}\right) = \frac{-p_1}{13-2p_1-3p_2^2}(-2) = \frac{2p_1}{13-2p_1-3p_2^2}$$

$\frac{Eq}{Ep_1}$ when $p_1 = p_2 = 2$ is $\frac{2(2)}{13-2(2)-3(2^2)}$

$$\frac{Eq}{Ep_1} = \frac{4}{13-4-12} = \frac{4}{-3} = -\frac{4}{3}$$

Differentiating 'q' partially with respect to 'p_2' we get,

$$\frac{\partial q}{\partial p_2} = 0-0-6p_2$$

$$\frac{Eq}{Ep_2} = -\frac{p_2}{q}\left(\frac{\partial q}{\partial p_2}\right)$$

$$= \frac{-p_2}{13-2p_1-3p_2^2}(-6p_2) = \frac{6p_2^2}{13-2p_1-3p_2^2}$$

$\frac{Eq}{Ep_2}$ when $p_1 = p_2 = 2$ is

$$\frac{Eq}{Ep_2} = \frac{6(2)^2}{13-2(2)-3(2^2)}$$

$$= \frac{24}{13-4-12} = \frac{\overset{8}{\cancel{24}}}{-\cancel{3}} = -8$$

42. (a) Let B_1, B_2, B_3 and A be the events defined as follows:

B_1 = First box is chosen

B_2 = Second box is chosen

B_3 = Third box is chosen

A = Bulb drawn is defective

Since there are three urns and one of the three urns is chosen at random,

$$P(B_1) = \frac{1}{31},\ P(B_2) = \frac{1}{31},\ P(B_3) = \frac{1}{3}$$

Given $P(A/E_1) = \frac{1}{2}$

$P(A/E_2) = \frac{1}{8}$ and $P(A/E_3) = \frac{3}{4}$

By Bayes' theorem,

P(Defective bulb from box B_1)

$$P(B_1/A) = \frac{P(B_1)\cdot P\left(A/B_1\right)}{P(B_1)\cdot P\left(A/B_1\right)+P(B_2)\cdot P\left(A/B_2\right)+P(B_3)\cdot P\left(A/B_3\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{8}+\frac{1}{3}\times\frac{3}{4}} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{24}+\frac{3}{12}} = \frac{\frac{1}{6}}{\frac{4+1+6}{24}} = \frac{1/6}{11/24} = \frac{1}{6}\times\frac{24}{11}$$

$\therefore P(B_1/A) = \frac{4}{11}$

(OR)

(b) Let the given statement P(n) be defined as $1+2+3+\ldots+n = \frac{n(n+1)}{2}$ for $n \in \mathbb{N}$.

Step 1: Put $n = 1$

LHS P(1) $= 1$

RHS P(1) $= \frac{1(1+1)}{2} = 1$

LHS = RHS for $n = 1$

$\therefore$ P(1) is true

Step 2: Let us assume that the statement is true for $n = k$.

i.e., P(k) is true

$$1+2+3+\ldots+k = \frac{k(k+1)}{2} \text{ is true}$$

Step 3: To prove that P(k + 1) is true

$P(k+1) = 1+2+3+\ldots+k+(k+1)$

$$= P(k)+k+1 = \frac{k(k+1)}{2}+k+1 = \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$\therefore$ P(k + 1) is true

Thus if P (k) is true, then P (k + 1) is also true.

$\therefore$ P(n) is true for all $n \in \mathbb{N}$.

Hence $1+2+3+\ldots+n = \frac{n(n+1)}{2}$, $n \in \mathbb{N}$.

43. (a) The given equation is $4x^2 + 12xy + 9y^2 - 6x - 9y + 2 = 0$

Here $a = 4$, $2h = 12 \Rightarrow h = 6$ and $b = 9$

$h^2 - ab = 6^2 - 4 \times 9 = 36 - 36 = 0$

$\therefore$ The given equation represents a pair of parallel straight lines.

Consider $4x^2 + 12xy + 9y^2 = (2x)^2 + 12xy + (3y)^2$

$= 2x^2 + 2 \times (2x)(3y) + (3y)^2 = (2x + 3y)^2$

Here we have repeated factors.

Now consider, $4x^2 + 12xy + 9y^2 - 6x - 9y + 2 = 0$

$$(2x + 3y)^2 - 3(2x + 3y) + 2 = 0$$

$t^2 - 3t + 2 = 0$ where $t = 2x + 3y$

$$(t - 1)(t - 2) = 0$$

$$(2x + 3y - 1)(2x + 3y - 2) = 0$$

$\therefore$ Separate equations are $2x + 3y - 1 = 0$, $2x + 3y - 2 = 0$

(OR)

(b) Given regression lines are

$$4X - 5Y = -33 \quad \ldots (1)$$

$$20X - 9Y = 107 \quad \ldots (2)$$

$(1) \times 5 \Rightarrow 20X - 25Y = -165$

$\quad (-) \quad (+) \quad (-)$

$(2) \Rightarrow 20X - 9Y = 107$

Subtracting, $-16Y = 272$

$$Y = \frac{-272}{-16} = 17 \Rightarrow \bar{Y} = 17$$

Substituting, Y = 17 in (1) we get,

$$4X - 5(17) = -33$$

$$\Rightarrow 4X = -33 + 85 \Rightarrow 4X = 52$$

$$\Rightarrow X = \frac{52}{4} = 13$$

$$X = 13$$

$\therefore \bar{X} = 13$ and $\bar{Y} = 17$

Let the regression line of Y on X is

$$4X - 5Y = -33$$

$$4X + 33 = 5Y$$

$$\frac{4}{5}X + \frac{33}{5} = Y$$

$$Y = \frac{4}{5}X + \frac{33}{5}$$

$\therefore$ Regression Co-efficient of Y on X is

$$b_{yx} = \frac{4}{5}, \text{ which is less than one}$$

Let the regression line of X on Y be

$$20X - 9Y = 107$$

$$20X = 9Y + 107$$

$$X = \frac{9}{20}X + \frac{107}{20}$$

Regression Co-efficient of X on Y is

$$b_{xy} = \frac{9}{20} \text{ which is also less than one.}$$

Since the two regression co-efficients are positive, then the correlation co-efficient is also positive and it is given by

$$r = \sqrt{b_{yx} \cdot b_{xy}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = \sqrt{\frac{36}{100}} = \frac{6}{10} = 0.6$$

$$\therefore r = 0.6$$

44. (a) First we have to find the feasible region using the given conditions.

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant.

Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines $x_1 + 4x_2 = 24$; $3x_1 + x_2 = 21$; $x_1 + x_2 = 9$

$x_1 + 4x_2 = 24$ is a line passing through the points (0 , 6) and (24 , 0).

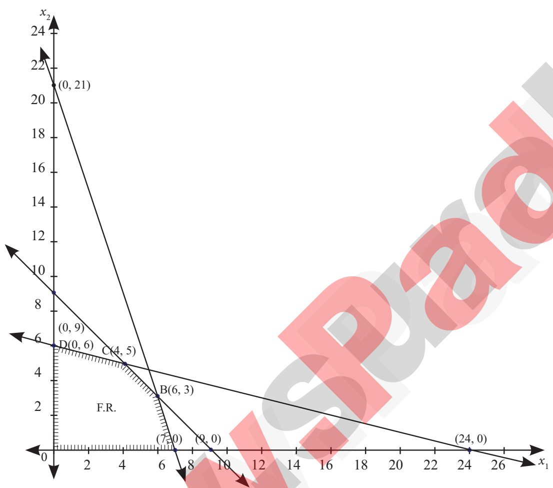
[(0,6) is obtained by taking $x_1 = 0$ in $x_1 + 4x_2 = 24$, (24, 0) is obtained by taking $x_2 = 0$ in $x_1 + 4x_2 = 24$].

Any point lying on or below the line $x_1 + 4x_2 = 24$ satisfies the constraint $x_1 + 4x_2 \leq 24$.

$3x_1 + x_2 = 21$ is a line passing through the points (0, 21) and (7, 0). Any point lying on or below the line $3x_1 + x_2 = 21$ satisfies the constraint $3x_1 + x_2 \leq 21$.

$x_1 + x_2 = 9$ is a line passing through the points (0 , 9) and (9 , 0) .Any point lying on or below the line $x_1 + x_2 = 9$ satisfies the constraint $x_1 + x_2 \leq 9$.

Now we draw the graph



The feasible region satisfying all the conditions is OABCD. The co-ordinates of the points are O(0,0) A(7,0);B(6,3) [the point B is the intersection of two lines $x_1 + x_2 = 9$ and $3x_1 + x_2 = 21$]; C(4,5) [the point C is the intersection of two lines $x_1 + x_2 = 9$ and $x_1 + 4x_2 = 24$] and D(0,6).

Corner points	$Z = 2x_1 + 5x_2$
O(0,0)	0
A(7,0)	14
B(6,3)	27
C(4,5)	33
D(0,6)	30

Maximum value of Z occurs at C. Therefore the solution *is* $x_1 = 4$, $x_2 = 5$, Z max = 33.

(OR)

(b) Given $\left(x+\frac{1}{x}\right)^{2n}$

$$\text{Here } n = 2n,\ x = x,\ a = \frac{1}{x}$$

The general term is

$$t_{r+1} = nC_r\, x^{n-r} a^r$$

$$\therefore t_{r+1} = 2nC_r x^{2n-r}\left(\frac{1}{x}\right)^r = 2nC_r x^{2n-r}\left(\frac{1}{x}\right)^r$$

$$= 2nC_r x^{2n-2r}$$

Since the term is independent of x, $2n - 2r = 0$

$$\Rightarrow \quad 2n = 2r \Rightarrow r = n$$

$$\therefore t_{n+1} = 2nC_n \cdot x^{2n-2n} = 2nC_n$$

$$= \frac{2n!}{n!n!} = \frac{(2n)(2n-1)(2n-2)(2n-3).....4.3.2.1}{n!\ n!}$$

Separate the even and odd terms in the numerator, we get

$$= \frac{[(2n)(2n-2)(2n-4)...4.2][(2n-1)(2n-3)...3.1]}{n!\,n!}$$

$$= \frac{2^n(n)(n-1)(n-2)...2.1.[(2n-1)(2n-3)...3.1]}{n!\,n!}$$

[Taking 2 common from each term]

$$= \frac{2^n.\ n!\ [(2n-1)(2n-3)...3.1]}{n!\ n!}$$

$$= \frac{1.3.5...(2n-3)(2n-1).2^n}{n!}$$

Hence proved.

45. (a)

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right) \quad ...(1)$$

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right) \quad ...(2)$$

Squaring and adding (1) and (2),

$(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right) + 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)\left[\cos^2\left(\frac{\alpha+\beta}{2}\right) + \sin^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

(OR)

(b) **(A)**

Requirement R $= 800$ unit per year

$C_1 = 10\%$ of 0.02

$= \frac{10}{100} \times 0.02 = \frac{.2}{100} = .002$

$C_3 = ₹\,5$

(i) $\text{EOQ} = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 5 \times 800}{.002}} = \sqrt{\frac{8000}{.002}}$

$q_0 = \sqrt{4{,}000{,}000} = 2000$ units

(ii) Minimum inventory cost $= \sqrt{2C_1C_3R} = \sqrt{2 \times .002 \times 5 \times 800} = \sqrt{8000 \times .002} = \sqrt{16} = ₹4$

(iii) EOQ in rupees $= 2000 \times 0.02$ [$\because$ unit price is 0.02] $= 2000 \times \frac{2}{100} = ₹\,40$

(iv) EOQ in years of supply $= \frac{q_0}{R} = \frac{2000}{800} = \frac{20}{8} = \frac{5}{2} = 2.5$

(v) Number of orders per year $= \frac{R}{q_0} = \frac{800}{2000} = \frac{8}{20} = \frac{2}{5} = 0.4$

46. (a) Given $x^m.y^n = (x+y)^{m+n}$ ... (1)

Taking logarithm on both sides we get,

$$m \log x + n \log y = [(m+n).\log(x+y)$$

Now, differentiating with respect to 'x' we get,

$$m\left(\frac{1}{x}\right) + n\left(\frac{1}{y}\right)\frac{dy}{dx} = (m+n)\left[\frac{1}{x+y}\right]\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y} + \frac{m+n}{x+y}\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y\,(x+y)}\right) = \frac{mx+nx-mx-my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{nx-my}{y\,(x+y)}\right) = \frac{nx-my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{nx-my}{x(x+y)} \times \frac{y\,(x+y)}{nx-my}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Hence proved.

(OR)

(b) Number of shares = $\frac{4500}{10} = 450$

Income from 12% stock = Number of shares × face value × Rate of dividend

$= 450 \times 10 \times \frac{12}{100} = ₹540$

Selling price of 450 shares = $450 \times 23 = ₹10,350$

Number of shares bought in 10% stock

$= \frac{\text{Selling price of 450 shares at ₹23}}{\text{Market value}}$

$= \frac{10,350}{18} = ₹575$

Income from 10% stock = No of shares × face value × Rate of dividend

$= 575 \times 25 \times \frac{10}{100} = ₹1437.5$

Change in his income = $₹1437.5 - ₹540 = ₹897.50$

47. (a)

$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$AB = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (1-2) & (3+4) \\ (-1-1) & (-3+2) \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ -2 & -1 \end{bmatrix}$

$\det(AB) = (-1)(-1) - (7)(-2)$

$= 1 + 14 = 15$

$(AB)^{-1} = \frac{1}{15}\begin{bmatrix} -1 & -7 \\ 2 & -1 \end{bmatrix}$

where $\det(A) = (1 \times 1) - (-1 \times 2) = 1 + 2 = 3$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

For Matrix B

$\det(B) = (1 \times 2) - (-1 \times 3) = 2 + 3 = 5$

$B^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \times \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} = \frac{1}{15}\begin{pmatrix} 2(1)+(-3)(1) & 2(-2)+(-3)(1) \\ 1(1)+1(1) & 1(-2)+1(1) \end{pmatrix}$

$= \frac{1}{15}\begin{pmatrix} -1 & -7 \\ 2 & -1 \end{pmatrix}$ Hence it is proved.

(OR)

(b)

Marks	Frequency	Cumulative frequency
X	*f*	*cf*
10	5	5
20	8	13
30	10	23
40	8	31
50	7	38
60	2	40

$N = \Sigma f = 40$

$Q_1 = \text{size of value} \left(\frac{N+1}{4}\right)^{th} = \text{size of} \left(\frac{40+1}{4}\right)^{th} \text{value}$

$= \text{size of } 11^{th} \text{ value} = 20$

$Q_3 = \left(\frac{3(N+1)}{4}\right)^{th} \text{value} = \text{size of } 30.75 \text{ value} = 40$

$QD = \frac{1}{2}(Q_3 - Q_1) = \frac{40-20}{2} = 10$

$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{40-20}{40+20} = \frac{20}{60} = 0.333$

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