11th STD

PUBLIC EXAMINATION - MARCH - 2025

Reg. No.

PART - III

Mathematics (with answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If A and B are any two events, then the probability that exactly one of them occurs is :

(a) $P(A) + P(B) - P(A \cap B)$

(b) $P(A \cup \overline{B}) + P(\overline{A} \cup B)$

(c) $P(A) + P(B) + 2P(A \cap B)$

(d) $P(A \cap \overline{B}) + P(\overline{A} \cap B)$

2. If the points $(x, -2)$, $(5, 2)$, $(8,8)$ are collinear, then x is equal to :

(a) 1 (b) -3 (c) 3 (d) $\frac{1}{3}$

3. The solution set of the following inequality $|x - 1| \geq |x - 3|$ is :

(a) (0, 2) (b) [0, 2] (c) $(-\infty, 2)$ (d) $[2, \infty)$

4. The value of $2 + 4 + 6 + ...+ 2n$ is :

(a) $\frac{2n(2n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $n(n + 1)$ (d) $\frac{n(n+1)}{2}$

5. The number of relations on a set containing 3 elements is :

(a) 512 (b) 9 (c) 1024 (d) 81

6. Four persons are selected at random from a group of 3 men, 2 women and 4 children. The probability that exactly two of them are children is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{10}{21}$ (d) $\frac{10}{23}$

7. $\lim_{x \to 3} \lfloor x \rfloor =$

(a) Value does not exist (b) 2

(c) 0 (d) 3

8. The derivative of $f(x) = x|x|$ at $x = -3$ is :

(a) does not exist (b) 6

(c) 0 (d) -6

9. If 3 is the logarithm of 343, then the base is :

(a) 6 (b) 5 (c) 9 (d) 7

10. In a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is :

(a) right triangle (b) equilateral triangle

(c) scalene triangle. (d) isosceles triangle

11. The number of 5 digit numbers, all digits of which are odd is :

(a) 5^6 (b) 25 (c) 625 (d) 5^5

12. The equation of the line through the point $(1, -1)$ and perpendicular to $3x + 4y = 6$ is :

(a) $4x + 3y + 7 = 0$ (b) $4x - 3y - 7 = 0$

(c) $3x + 4y + 7 = 0$ (d) $3x + 4y - 7 = 0$

13. $\int 2^{3x+5} dx$ is :

(a) $\frac{2^{3x+5}}{2\log 3} + c$ (b) $\frac{3(2^{3x+5})}{\log 2} + c$

(c) $\frac{2^{3x+5}}{3\log 2} + c$ (d) $\frac{2^{3x+5}}{2\log(3x+5)} + c$

14. The inverse function of $y = \log_e x$ is :

(a) $y + e^x$ (b) $y = \log_e x$

(c) $y = e^{-x}$ (d) $y = -\log_e x$

15. Straight line joining the points (2, 3) and (–1, 4) passes through the point (α, β) if :

(a) $\alpha + 3\beta = 11$ (b) $\alpha + 2\beta = 7$

(c) $3\alpha + \beta = 11$ (d) $3\alpha + \beta = 9$

16. If $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) 60° (b) 0 (c) 45° (d) 90°

17. If $\int \frac{3^{\frac{1}{x}}}{x^2} dx = k(3^{\frac{1}{x}}) + c$, then the value of k is :

(a) $-\frac{1}{\log 3}$ (b) log 3

(c) $\frac{1}{\log 3}$ (d) −log 3

18. If cos 28° + sin 28° = k^3, then cos 17° is equal to :

(a) $\pm\frac{k^3}{\sqrt{2}}$ (b) $\frac{k^3}{\sqrt{2}}$ (c) $-\frac{k^3}{\sqrt{3}}$ (d) $-\frac{k^3}{\sqrt{2}}$

19. $\frac{d}{dx}\left(\frac{2}{\pi}\sin x^{o}\right)$ is :

(a) $\frac{\pi}{90}\cos x^{o}$ (b) $\frac{\pi}{180}\cos x^{o}$

(c) $\frac{2}{\pi}\cos x^{o}$ (d) $\frac{1}{90}\cos x^{o}$

20. If $\vec{a}+2\vec{b}$ and $3\vec{a}+m\vec{b}$ are parallel, then the value of m is :

(a) 6 (b) 3 (c $\frac{1}{6}$ (d) $\frac{1}{3}$

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, prove that the points P, Q, R are collinear.

22. An experiment has the four possible mutually exclusive and exhaustive outcomes A, B, C, and D. Check if the following assignments of probability are permissible.

P(A) = 0.15, P(B) = 0.30, P(C) = 0.43, P(D)= 0.12

23. Differentiate : $y = e^{\sin x}$.

24. Find the positive integer 'n' so that $\lim_{x\to 2}\frac{x^n - 2^n}{x-2} = 12$.

25. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$.

26. Without expanding, evaluate $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$.

27. Solve $23x < 100$ when (i) x is a natural number, (ii) x is an integer.

28. Write the first 6 terms of the sequences whose n^{th} term, $a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$.

29. Find the equation of the locus of P, if for all values of α, the co-ordinates of a moving point P is (9 cos α, 9 sin α), where α is a parameter.

30. What is the unit digit of the sum 3! + 4! + ...+20!?

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory.** **7 × 3 = 21**

31. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

32. Solve : $\sqrt{x^2 - x - 2} = x + 1$

33. Prove that sin (45° + θ) – sin (45° – θ) = $\sqrt{2}$ sin θ

34. If $(n + 2)P_4 = 42 \times {}^nP_2$, find n.

35. Compute $(102)^4$

36. Show that the points $\left(0, -\frac{3}{2}\right)$, (1, –1) and $\left(2, -\frac{1}{2}\right)$ are collinear.

37. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$.

38. Integrate with respect to x

cos 5x sin 3x

39. Eight coins are tossed once, find the probability of getting :

(i) exactly two tails (ii) atmost two tails

40. Find $\frac{dy}{dx}$, if $y = \cos^{-1}(2\cos^2 x - 1)$.

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) Resolve into partial fractions : $\frac{x^2 + x + 1}{x^2 - 5x + 6}$

(OR)

(b) Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form : (i) Slope and intercept form (ii) Intercept form (iii) Normal form.

42. (a) Prove that $\dfrac{\cos(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\ \mathrm{cosec}\ (360°+\theta)} = \cos^2\theta\cot\theta$

(OR)

(b) If $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$

43. (a) Prove that $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1.$

(OR)

(b) A factory has two Machines - I and II. Machine - I produces 60% of items and Machine - II produces 40% of the items of the total output. Further 2% of the items produced by Machine - I are defective whereas 4% produced by Machine - II are defective. If an item is drawn at random what is the probability that it is defective?

44. (a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\dfrac{1}{x^2}$ when x is large.

(OR)

(b) If one root of $k(x-1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or -25.

45. (a) Show that $\tan 20° \tan 40° \tan 60° \tan 80° = 3$..

(OR)

(b) Integrate : $\dfrac{1}{\sqrt{x^2+5x+4}}$

46. (a) In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who know only Language A.

(OR)

(b) Prove that the medians of a triangle are concurrent.

47. (a) By the principle of mathematical induction, prove that, for $n > 1$.

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(OR)

(b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)$

❄❄❄❄❄

PART - I

1. (d) $P(A\cap\overline{B}) + P(\overline{A}\cap B)$
2. (c) 3
3. (d) $[2,\infty)$
4. (c) $n(n+1)$
5. (a) 512
6. (c) $\dfrac{10}{21}$
7. (a) Value does not exist
8. (b) 6
9. (d) 7
10. (a) right triangle
11. (d) 5^5
12. (b) $4x - 3y - 7 = 0$
13. (c) $\dfrac{2^{3x+5}}{3\log 2} + c$
14. (a) $y = e^x$
15. (a) $\alpha + 3\beta = 11$
16. (d) 90°
17. (a) $-\dfrac{1}{\log 3}$
18. (b) $\dfrac{k^3}{\sqrt{2}}$
19. (d) $\dfrac{1}{90}\cos x^{\circ}$
20. (a) 6

PART - II

21. Given $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$

$\Rightarrow \quad \overrightarrow{PQ} = \overrightarrow{QR}$ [By triangle law of addition]

$\Rightarrow \quad \overrightarrow{PQ} = \overrightarrow{QR}$ and Q is a common point.

Hence, the points P, Q, R are collinear.

22. $P(A) = 0.15, P(B) = 0.30, P(C) = 0.43, \ P(D)= 0.12$

Given $P(A) = 0.15, P(B) = 0.30, P(C) = 0.43,$

$P(D) = 0.12$

$P(A), P(B), P(C)$ and $P(D) \geq 0$

Also, $P(S) = P(A) + P(B) + P(C) + P(D)$

$= 0.15 + 0.30 + 0.43 + 0.12 = 1$

$\therefore$ The assignment of probability is permissible.

23. Take $u = \sin x$ so that

$y = e^u$

$$\frac{dy}{dx} = \frac{d(e^u)}{du} \times \frac{du}{dx} = e^u \times \cos x = \cos x \, e^{\sin x}.$$

24. $\lim\limits_{x \to 2} \dfrac{x^n - 2^n}{x - 2} = 12$

$$\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$n\,(2)^{n-1} = 12$

$n\,(2)^{n-1} = 3\,(2)^2$

$n\,(2)^{n-1} = 3\,(2)^{3-1}$

$n = 3$

25. Given $a_{ij} = \dfrac{(i - 2j)^2}{2}$ with $m = 2, n = 3$

we need to construct a 2×3 matrix.

$$\therefore a_{11} = \frac{(1 - 2(1))^2}{2} = \frac{(-1)^2}{2} = \frac{1}{2}$$

$$a_{12} = \frac{(1 - 2(2))^2}{2} = \frac{(-3)^2}{2} = \frac{9}{2}$$

$$a_{13} = \frac{(1 - 2(3))^2}{2} = \frac{(-5)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2 - 2(1))^2}{2} = \frac{0}{2} = 0$$

$$a_{22} = \frac{(2 - 2(2))^2}{2} = \frac{(-2)^2}{2} = \frac{4}{2}$$

$$a_{23} = \frac{(2-2(3))^2}{2} = \frac{(-4)^2}{2} = \frac{16}{2}$$

$$\therefore A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & \frac{4}{2} & \frac{16}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 9 & 25 \\ 0 & 4 & 16 \end{pmatrix}$$

26. Let $A = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Taking $(3x)$ common from R_3 we get,

$$A = 3x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 3x\,(0) = 0 \qquad [\because R_1 \equiv R_3]$$

27. Given $23x < 100$.

(i) when x is a natural number $23x < 100$

$\Rightarrow x < \frac{100}{23} \Rightarrow x < 4.348 \Rightarrow x = \{1, 2, 3, 4\}$

(ii) when x is an integer $x < 4.348$

$\Rightarrow x = \{\cdots\cdots -3, -2, -1, 0\ 1, 2, 3, 4\}$

Hence solution set is $\{\cdots\cdots -3, -2, -1, 0\ 1, 2, 3, 4\}$

28. $a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$

$a_1 = 1 + 1 = 2,\quad a_2 = 2,\quad a_3 = 3 + 1 = 4$

$a_4 = 4,\quad a_5 = 5 + 1 = 6,\quad a_6 = 6$

Hence the first 6 terms are 2, 2, 4, 4, 6, 6, ...

29. $(9 \cos \alpha, 9 \sin \alpha)$

Let P (h, k) be any point on the required path

From the given information, we have

$h = 9 \cos \alpha$ and $k = 9 \sin \alpha$

$$\Rightarrow \frac{h}{9} = \cos \alpha \text{ and } \frac{k}{9} = \sin \alpha$$

To eliminate the parameter α, squaring and adding we get

$$\left(\frac{h}{9}\right)^2 + \left(\frac{k}{9}\right)^2 = \cos^2 \alpha + \sin^2 \alpha$$

$$\Rightarrow \frac{h^2}{81} + \frac{k^2}{81} = 1 \qquad [\because \sin^2 \alpha + \cos^2 \alpha = 1]$$

$$\Rightarrow h^2 + k^2 = 81$$

$\therefore$ Locus of (h, k) is $x^2 + y^2 = 81$

30.
$$3! = 6 \to \text{unit digit is } 6$$
$$4! = 24 \to \text{unit digit is } 4$$
$$5! = 120 \to \text{unit digit is } 0$$
$$6! = 720 \to \text{unit digit is } 0$$
$$n \geq 5! = 0$$
$$3! + 4! = 6 + 4 = 10$$

The unit digit of the sum 3! + 4! + ... + 20! is **0**

PART - III

31.
$$-1 \leq \cos x \leq 1$$
$$\Rightarrow \quad 3 \geq -3\cos x \geq -3$$
$$\Rightarrow \quad -3 \leq -3\cos x \leq 3$$
$$\Rightarrow \quad 1-3 \leq 1-3\cos x \leq 1+3$$

Thus we get $-2 \leq 1 - 3\cos x$ and $1 - 3\cos x \leq 4$.

By taking reciprocals, we get $\frac{1}{1-3\cos x} \leq -\frac{1}{2}$ and $\frac{1}{1-3\cos x} \geq \frac{1}{4}$.

Hence the range of f is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{4}, \infty\right)$

32.
$$\sqrt{x^2-x-2} = x+1$$
$$\sqrt{x^2-x-2} \geq 0 \Rightarrow x+1 \geq 0$$
$$x \geq -1$$
$$\sqrt{x^2-x-2} = x+1$$
$$x^2-x-2 = (x+1)^2$$
$$x^2-x-2 = x^2+2x+1$$
$$-3x = 3$$
$$\therefore x = -1$$

33. $\sin(45° + \theta) - \sin(45° - \theta) = \sqrt{2}\sin\theta$

$$\sin(45°+\theta) - \sin(45°-\theta) = (\sin 45° \cos\theta + \cos 45° \sin\theta) - (\sin 45° \cos\theta - \cos 45° \sin\theta)$$
$$= \sin 45° \cos\theta + \cos 45° \sin\theta - \sin 45° \cos\theta + \cos 45° \sin\theta$$
$$= 2\cos 45° \sin\theta = 2 \times \frac{1}{\sqrt{2}} \sin\theta$$
$$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \sin\theta = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} \sin\theta = \sqrt{2}\sin\theta = \text{RHS}$$

Hence proved.

34. $^{(n+2)}P_4 = 42 \times {}^nP_2$

$\Rightarrow \dfrac{^{(n+2)}P_4}{^nP_2} = 12$

$\Rightarrow \dfrac{(n+2)(n+1)(n)(n-1)}{n(n-1)} = 42$

$\Rightarrow (n+2)(n+1) = 12 = 7 \times 6$

$\Rightarrow n+2 = 7 \Rightarrow n = 5.$

35. $102^4 = (100+2)^4$

$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_n a^0 b^n, n \in \mathbb{N}$

$= 100^4 + {}^4C_1(100)^3(2)^1 + {}^4C_2(100)^2(2^2) + {}^4C_3(100)^1(2^3) + 2^4$

$= 100000000 + 4(1000000)(2) + \dfrac{\overset{2}{\cancel{4}} \times (3)}{2(1)}(10000)(4) + 400(8) + 16$

$= 100000000 + 8000000 + 240000 + 3200 + 16 = 108,243,216$

36. Let A, B and C be $\left(0, -\frac{3}{2}\right)$, $(1, -1)$ and $\left(2, -\frac{1}{2}\right)$ respectively.

The slope of AB is $\dfrac{-1+\frac{3}{2}}{1-0} = \dfrac{1}{2}$

The slop of BC is $\dfrac{-\frac{1}{2}+1}{2-1} = \dfrac{1}{2}$

Thus, the slope of AB is equal to slope of BC.

Hence, A, B and C are lying on the same line.

37. Given $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors.

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1 \quad \vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$, and angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{3}$

$\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$

$\Rightarrow \vec{a}$ is $\perp^r$ to both $\vec{b}$ and $\vec{c}$. $\Rightarrow \vec{a}$ is parallel to $\vec{b} \times \vec{c}$

$\Rightarrow \vec{a} = \pm\lambda(\vec{b} \times \vec{c})$ (1) for some scalar λ.

$\Rightarrow |\vec{a}| = \lambda|\vec{b} \times \vec{c}|$

$\Rightarrow |\vec{a}| = \lambda|\vec{b}||\vec{c}|\sin\theta$

$\Rightarrow 1 = \lambda \sin\dfrac{\pi}{3} \Rightarrow 1 = \lambda\left(\dfrac{\sqrt{3}}{2}\right) \quad \Rightarrow \lambda = \dfrac{2}{\sqrt{3}}$

Sub $\lambda = \dfrac{2}{\sqrt{3}}$ in (1)

$\vec{a} = \pm\dfrac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$

38.
$$\int \cos 5x \sin 3x\, dx = \frac{1}{2}\int 2\cos 5x \sin 3x\, dx$$
$$= \frac{1}{2}\int (\sin 8x - \sin 2x)\, dx$$
$$\int \cos 5x \sin 3x\, dx = \frac{1}{2}\left(-\frac{\cos 8x}{8} + \frac{\cos 2x}{2}\right) + c$$

39. Since 8 coins are tossed,

(i) $n(S) = 2^8 = 256$

Let A be the event of getting exactly two tails

$$\Rightarrow \quad \therefore n(A) = {}^8C_2 = \frac{\overset{4}{\not 8} \times 7}{\not 2 \times 1} = 4 \times 7$$

$$\therefore P(S) = \frac{n(A)}{n(S)} = \frac{4\times 7}{2^8} = \frac{2^2 \times 7}{2^8} = \frac{7}{2^6} = \frac{7}{64}$$

(ii) Let C be the event of getting atmost two tails

$$\therefore n(C) = {}^8C_0 + {}^8C_1 + {}^8C_2$$
$$= 1 + 8 + \frac{\overset{4}{\not 8} \times 7}{\not 2 \times 1}$$
$$= 9 + 28 = 37$$
$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{37}{256}$$

40.
$$y = \cos^{-1}(2\cos^2 x - 1)$$
$$y = \cos^{-1}(\cos 2x)$$
$$y = 2x$$

differentiating with respect to x

$$\frac{dy}{dx} = 2$$

PART - IV

41. **(a)** Since the degree of the numerator is equal to the degree of the denominator, let us divide the numerator by the denominator.

$$\therefore \frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{x^2-5x+6} \quad \text{...(1)}$$

$$\begin{array}{r|l} & \quad 1 \\ x^2-5x+6 & x^2+x+1 \\ & \underset{(-)}{x^2} \underset{(+)}{-5x} \underset{(-)}{+6} \\ \hline & \quad 6x-5 \end{array}$$

Consider
$$\frac{6x-5}{x^2-5x+6} = \frac{6x-5}{(x-3)(x-2)}$$
$$= \frac{A}{(x-3)} + \frac{B}{(x-2)}$$

$$\Rightarrow \quad \frac{6x-5}{x^2-5x+6} = \frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$$

$$\Rightarrow \quad 6x-5 = A(x-2)+B(x-3) \quad ...(2)$$

Putting $x = 2$ in (2) we get,

$$7 = B(-1) \Rightarrow \boxed{B=-7}$$

Putting $x = 3$ in (2) we get,

$$13 = A(1) \Rightarrow \boxed{A=13}$$

$$\therefore \frac{6x-5}{x^2-5x+6} = \frac{13}{x-3} - \frac{7}{x-2} \quad ... (3)$$

Substituting (3) in (1) we get,

$$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{13}{x-3} - \frac{7}{x-2}$$

(OR)

(b) (i) Slope and intercept form

It is given that $\sqrt{3}x - y + 4 = 0$

$$\Rightarrow \quad y = \sqrt{3}x + 4 \quad ... (1)$$

Comparing the above equation with the equation $y = mx + b$, we have

$$\text{Slope} = \sqrt{3} \text{ and } y\text{ - intercept} = 4$$

(ii) Intercept form

$$\sqrt{3}x - y + 4 = 0 \Rightarrow \sqrt{3}x - y = -4$$

$$\frac{-\sqrt{3}}{4}x + \frac{y}{4} = 1$$

$$\text{That is } \frac{x}{\left(-\frac{4}{\sqrt{3}}\right)} + \frac{y}{4} = 1 \quad ... (2)$$

Comparing the equations (1) and (2) with the equation $\frac{x}{a} + \frac{y}{b} = 1$

We get, x-intercept $= -\frac{4}{\sqrt{3}}$ and y-intercept $= 4$

(iii) Normal form $-\sqrt{3}x - y + 4 = 0$

$$\left(-\sqrt{3}\right)x + y = 4 \quad ... (3)$$

Comparing the above equation with the equation $Ax + By + C = 0$.

Here $A = -\sqrt{3}$, and $B = 1$, $\sqrt{A^2+B^2} = 2$

Therefore, dividing the above equation by 2, we get

$$\frac{-\sqrt{3}x}{2} + \frac{y}{2} = 2 \quad ... (4)$$

Comparing the above equations (3) and (4) with the equation $x \cos\alpha + y \sin\alpha = p$

If we take

$$\cos\alpha = \frac{-\sqrt{3}}{2} \text{ and } \sin\alpha = \frac{1}{2} \text{ and } p = 2$$

$\Rightarrow \quad \alpha = 150° = \frac{5\pi}{6}$ and length of the normal $(p) = 2$.

The normal form is $x \cos \frac{5\pi}{6} + y \sin \frac{5\pi}{6} = 2$

42. **(a)** $\text{LHS} = \frac{\cot(180°+\theta)\ \sin(90°-\theta)\ \cos(-\theta)}{\sin(270°+\theta)\ \tan(-\theta)\ \text{cosec}(360°+\theta)}$

$$\cot(180° + \theta) = \cot\theta$$
$$\sin(270° + \theta) = -\cos\theta$$
$$\sin(90° - \theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\text{cosec}(360° + \theta) = \text{cosec}\,\theta.$$

$$\therefore \text{LHS} = \frac{\cot\theta\cdot\cos\theta\cdot\cos\theta}{(-\cos\theta)(-\tan\theta)(\text{cosec}\,\theta)}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}\cdot\cos\theta\cdot\cos\theta}{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}}$$

$$= \frac{\cos^3\theta}{\sin\theta} = \cos^2\theta\cdot\frac{\cos\theta}{\sin\theta}$$

$$= \cos^2\theta\cdot\cot\theta.$$

Hence proved.

(OR)

(b) Given $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$$\Rightarrow \quad y\sqrt{1-x^2} = \sin^{-1}x$$

Squaring, $y^2(1-x^2) = (\sin^{-1}x)^2$

Differentiating with respect to 'x', we get

$$y^2(-2x) + (1-x^2)\,2yy' = 2.\sin^{-1}x.\frac{d}{dx}(\sin^{-1}x)$$

$$\Rightarrow \quad -2xy^2 + (1-x^2)\,2yy' = 2.\sin^{-1}x.\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad -2xy^2 + (1-x^2)\,2yy' = 2y \qquad \left[\because y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}\right]$$

Dividing by $2y$ throughout we get,

$$-xy + (1-x^2)\,y' = 1$$

Differentiating again with respect to 'x', we get

$$\Rightarrow \quad -[xy' + y(1)] + (1-x^2)y'' + y'(-2x) = 0$$

$$\Rightarrow \quad -xy' - y + (1-x^2)\,y'' - 2xy' = 0$$

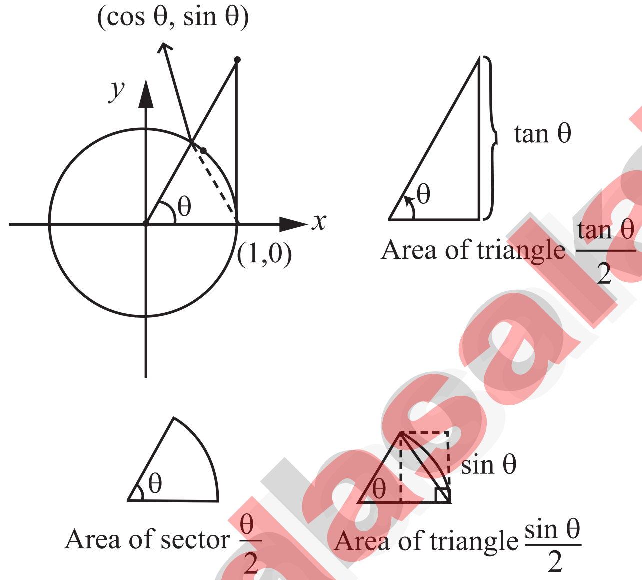
$$\Rightarrow \quad (1-x^2)y'' - 3xy' - y = 0$$

$$\text{Also, } (1-x^2)y_2 - 3xy_1 - y = 0$$

Hence proved.

43. **(a)** We use a circular sector to prove the result.

Consider the circle with centre (0,0) and radius 1. Any point on this circle is P (cos θ, sin θ).



By area property $\dfrac{\tan\theta}{2} \geq \dfrac{\theta}{2} \geq \dfrac{\sin\theta}{2}$.

Multiplying each expression by $\dfrac{2}{\sin\theta}$ produces $\dfrac{1}{\cos\theta} \geq \dfrac{\theta}{\sin\theta} \geq 1$ and taking reciprocals $\cos\theta \leq \dfrac{\sin\theta}{\theta} \leq 1$.

Because cos (–θ) = cos θ and $\dfrac{\sin(-\theta)}{-\theta} = \dfrac{\sin\theta}{\theta}$ one can conclude that this inequality is valid for all non-zero in the open interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

We know that $\lim\limits_{\theta\to 0} \cos\theta = 1$; $\lim\limits_{\theta\to 0} (1) = 1$ and applying Sandwich theorem we get $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$.

(OR)

(b) Let A_1 be the event that the items are produced by Machine - I,

A_2 be the event that the items are produced by Machine - II.

Let B be the event of drawing a defective item.

$$\text{Given } P(A_1) = \frac{60}{100}$$

$$P(B/A_1) = \frac{2}{100}$$

$$P(A_2) = \frac{40}{100}$$

$$P(B/A_2) = \frac{4}{100}$$

We have to find the total probability of event B. Since A_1 and A_2 are mutually exclusive and exhaustive events, we have

$$P(B) = P(A_1).P(B/A_1)+P(A_2).P(B/A_2)$$

$$P(B) = \frac{60}{100}\times\frac{2}{100}+\frac{40}{100}\times\frac{4}{100}$$

$$P(B) = \frac{120}{10000}+\frac{160}{10000}=\frac{280}{10000}$$

$$P(B) = 0.028$$

44. (a)

$$\sqrt[3]{x^3+7} = (x^3+7)^{\frac{1}{3}} = \left[x^3\left(1+\frac{7}{x^3}\right)\right]^{\frac{1}{3}} \quad \left(\left|\frac{7}{x^3}\right|<1 \text{ as } x \text{ is large}\right)$$

$$= x\left(1+\frac{7}{x^3}\right)^{\frac{1}{3}} = x\left(1+\frac{1}{3}\times\frac{7}{x^3}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{7}{x^3}\right)^2+\ldots\right)$$

$$= x\left(1+\frac{7}{3}\times\frac{1}{x^3}-\frac{49}{9}\times\frac{1}{x^6}+\ldots\right) = x+\frac{7}{3}\times\frac{1}{x^2}-\frac{49}{9}\times\frac{1}{x^5}+\ldots$$

$$\sqrt[3]{x^3+4} = (x^3+4)^{\frac{1}{3}} = \left[x^3\left(1+\frac{4}{x^3}\right)\right]^{\frac{1}{3}} = x\left(1+\frac{4}{x^3}\right)^{\frac{1}{3}} \quad \left(\left|\frac{4}{x^3}\right|<1\right)$$

$$= x\left(1+\frac{1}{3}\times\frac{4}{x^3}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{4}{x^3}\right)^2+\ldots\right)$$

$$= x+\frac{4}{3}\times\frac{1}{x^2}-\frac{16}{9}\times\frac{1}{x^6}+\ldots$$

Since x is large, $\frac{1}{x}$ is very small and hence higher powers of $\frac{1}{x}$ are negligible.

Thus $\sqrt[3]{x^3+7} = x+\frac{7}{3}\times\frac{1}{x^2}$ and $\sqrt[3]{x^3+4} = x+\frac{4}{3}\times\frac{1}{x^2}$.

Therefore $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4} = \left(x+\frac{7}{3}\times\frac{1}{x^2}\right)-\left(x+\frac{4}{3}\times\frac{1}{x^2}\right) = \frac{1}{x^2}$

(OR)

(b) Given equation is $k(x-1)^2 = 5x - 7$

$\Rightarrow \quad k(x^2 - 2x + 1) = 5x - 7$

$\Rightarrow \quad kx^2 - 2kx + k - 5x + 7 = 0$

$\Rightarrow \quad kx^2 + x(-2k-5) + (k+7) = 0$

Let the roots be α and 2α.

$$\therefore \alpha + 2\alpha = \frac{+2k+5}{k}$$

$$\Rightarrow \quad 3\alpha = \frac{+2k+5}{k}$$

$$\Rightarrow \quad \alpha = \frac{+2k+5}{3k} \quad \dots (1)$$

$$\left[\alpha(2\alpha) = \frac{k+7}{k}\right]$$

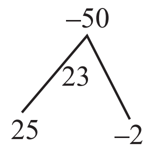
$$\Rightarrow \quad 2\alpha^2 = \frac{k+7}{k}$$

$$\Rightarrow \quad \alpha^2 = \frac{k+7}{2k} \quad \dots (2)$$

Substituting (1) in (2) we get,

$$\left(\frac{2k+5}{3k}\right)^2 = \frac{k+7}{2k} \Rightarrow \frac{4k^2+25+20k}{9k^2} = \frac{k+7}{2k}$$

$$\Rightarrow \quad \frac{4k^2+25+20k}{9k} = \frac{k+7}{2}$$



$\Rightarrow \quad 8k^2 + 50 + 40k = 9k^2 + 63k$

$\Rightarrow \quad k^2 + 23k - 50 = 0$

$\Rightarrow \quad (k-2)(k+25) = 0 \Rightarrow k = 2$ or -25.

Hence proved.

45. **(a)** LHS $= \tan 20° \tan 40° \tan 60° \tan 80°$

$= \tan 40° \tan 20° \tan 80° \tan 60°$

$= \tan(60° - 20°) \tan 20° \tan(60° + 20°)(\sqrt{3})$

$= \tan 3(20°)(\sqrt{3})$ $\quad [\because \tan(60° - A)\tan A \tan(60° + A) = \tan 3A]$

$= \tan 60° (\sqrt{3})$

$= (\sqrt{3})(\sqrt{3})$

$= 3 =$ RHS

Hence proved.

(OR)

(b) Let I $= \int \frac{1}{\sqrt{x^2+5x+4}}\,dx$

$$= \int \frac{1}{x^2+5x+\frac{25}{4}-\frac{25}{4}+4}\,dx = \int \frac{1}{\left(x+\frac{5}{2}\right)^2-\frac{9}{4}}\,dx = \int \frac{1}{\left(x+\frac{5}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

$$= \log\left(x+\frac{5}{2}\right)+\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{3}{2}\right)^2}+c$$

$$= \log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+4}\right|+c$$

46. (a) This problem can be solved either by property of cardinality or by venn diagram.

Cardinality:

Given that $n(A) = 45\%$ of $5000 = 2250$

Similarly, $n(B) = 1250$, $n(C) = 500$.

$n(A\cap B) = 250$, $n(B\cap C) = 200$,

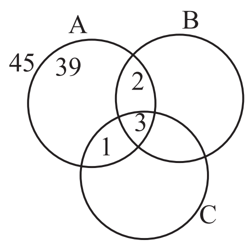
$n(C\cap A) = 200$ and $n(A\cap B\cap C) = 150$.

The number of persons who knows only language A is

$$n(A\cap B'\cap C') = n\{A\cap(B\cup C)'\}$$
$$= n(A) - n\{A\cap(B\cup C)\}$$
$$= n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$$
$$= 2250 - 250 - 200 + 150 = 1950$$

Thus the required number of persons is 1950.

Venn diagram:



We draw the venn diagram using percentage.

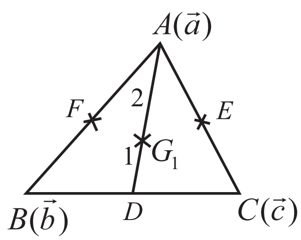
The percentage of persons who knows only language A is 39. Therefore, the required number of persons is $\overset{50}{\cancel{5000}} \times \frac{39}{\cancel{100}} = 1950$.

(OR)

(b) Let ABC be a triangle and let D, E, F be the mid points of its sides BC, CA and AB respectively. We have to prove that the medians AD, BE, CF are concurrent.

Let O be the origin and $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B, and C respectively.

The position vectors of D, E, and F are respectively $\frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{a}}{2}, \frac{\vec{a}+\vec{b}}{2}$.



Let G_1 be the point on AD dividing it internally in the ratio 2 : 1

Therefore, position vector of $G_1 = \dfrac{1\vec{OA} + 2\vec{OD}}{1+2}$

$$\vec{OG_1} = \frac{1\vec{a} + 2\left(\frac{\vec{b}+\vec{c}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad ...(1)$$

Let G_2 be the point on BE dividing it internally in the ratio 2 : 1

$$\text{Therefore, } \vec{OG_2} = \frac{1\vec{OB} + 2\vec{OE}}{1+2}$$

$$\vec{OG_2} = \frac{1\vec{b} + 2\left(\frac{\vec{c}+\vec{a}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad ...(2)$$

Similarly if G_3 divides CF in the ratio 2 : 1 then

$$\vec{OG_3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad ...(3)$$

From (1), (2), and (3) we find that the position vectors of the three points G_1, G_2, G_3 are one and the same. Hence they are not different points. Let the common point be G.

Therefore the three medians are concurrent and the point of concurrence is G.

47. (a) Let $p(n)$ be the statement

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Step 1: Put $n = 1$

$$\Rightarrow \quad 1^3 = \left[\frac{1(1+1)}{2}\right]^2 = \left[\frac{1(2)}{2}\right]^2$$

$$\Rightarrow \quad 1 = 1$$

$\therefore$ $p(n)$ is true.

Step 2: Let us assume that $p(K)$ is true.

$$\because 1^3 + 2^3 + 3^3 + ... + K^3 = \left[\frac{K(K+1)}{2}\right]^2 \quad ...(1)$$

Step 3: To prove that $p(K + 1)$ is true.

i.e to P. T. $1^3 + 2^3 + ... + K^3 + (K+1)^3$

$$= \left[\frac{(K+1)(K+2)}{2}\right]^2$$

$$\text{LHS} = 1^3 + 2^3 + \ldots + K^3 + (K+1)^3$$

$$= \left[K\frac{(K+1)}{2}\right]^2 + (K+1)^3 \quad \text{[Using (1)]}$$

$$= (K+1)^2\left[\frac{K^2}{4} + K + 1\right] = (K+1)^2\left[\frac{K^2+4K+4}{4}\right]$$

$$= \frac{(K+1)^2(K+2)^2}{4} = \left[\frac{(K+1)(K+2)}{2}\right]^2 = \text{RHS}$$

$\because p(K + 1)$ is true.

$\because$ By the principle of mathematical induction, $p(n)$ is true for all values of n.

(OR)

(b) LHS $= \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking out a, b, c common from R_1, R_2 and R_3 respectively.

$$\text{LHS} = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ we get, $= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from R_1 $= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$ we get,

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & \frac{1}{b} \\ 0 & -1 & \frac{1}{c}+1 \end{vmatrix}$$

Expanding along R_1 we get,

$$= abc\,(1 + 1/a + 1/b + 1/c)\left[0+0+1\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}\right]$$

$$= abc\,(1 + 1/a + 1/b + 1/c)\,[1]$$

$$= abc\,(1 + 1/a + 1/b + 1/c) = \text{RHS}$$

Hence Proved.

❁ ❁ ❁ ❁ ❁