HIGHER SECONDARY FIRST YEAR PUBLIC EXAMINATION
MARCH– 2025
MATHEMATICS – ANSWER KEY
PART-I

Note: i) Answer all the questions. $[20 \times 1 = 20]$
ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer

	TYPE-A		TYPE-B	
1.	(d)	$[2, \infty)$	(d)	$P(A \cap \bar{B}) + P(\bar{A} \cap B)$
2.	(b)	6	(c)	3
3.	(c)	$n(n+1)$	(d)	$[2, \infty)$
4.	(a)	$y = e^x$	(c)	$n(n+1)$
5.	(c)	3	(a)	512
6.	(a)	Right triangle	(c)	$\frac{10}{21}$
7.	(d)	5^5	(a)	Value does not exist
8.	(d)	$90°$	(b)	6
9.	(a)	6	(d)	7
10.	(a)	$-\frac{1}{\log 3}$	(a)	right triangle
11.	(d)	$\frac{1}{90}\cos x°$	(d)	5^5
12.	(c)	$\frac{2^{3x+5}}{3\log 2} + c$	(b)	$4x - 3y - 7 = 0$
13.	(a)	$\alpha + 3\beta = 11$	(c)	$\frac{2^{3x+5}}{3\log 2} + c$
14.	(d)	$P(A \cap \bar{B}) + P(\bar{A} \cap B)$	(a)	$y = e^x$
15.	(a)	512	(a)	$\alpha + 3\beta = 11$
16.	(a)	Value does not exist	(d)	$90°$
17.	(b)	$\frac{k^3}{\sqrt{2}}$	(a)	$-\frac{1}{\log 3}$
18.	(b)	$4x - 3y - 7 = 0$	(b)	$\frac{k^3}{\sqrt{2}}$
19.	(d)	7	(d)	$\frac{1}{90}\cos x°$
20.	(c)	$\frac{10}{21}$	(a)	6

PART-II

Note: $[7 \times 2 = 14]$

(i) Answer any **SEVEN** questions
(ii) Question number **30** is compulsory.

21. **If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, prove that the points P, Q, R are collinear.**
Solution:

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$-\overrightarrow{OP} + \overrightarrow{OQ} = -\overrightarrow{OQ} + \overrightarrow{OR}$$
$$\overrightarrow{OQ} - \overrightarrow{OP} = \overrightarrow{OR} - \overrightarrow{OQ}$$
$$\overrightarrow{PQ} = \overrightarrow{QR}$$
$\overrightarrow{PQ} \parallel \overrightarrow{QR}$ and Q is common

Hence P, Q, R are collinear

22. An experiment has the four possible mutually exclusive and exhaustive outcomes A, B, C, and D. Check whether the following assignments of probability are permissible. $P(A) = 0.15$, $P(B) = 0.30$, $P(C) = 0.43$, $P(D) = 0.12$.

Solution:

$$P(A) = 0.15 \geq 0;\ P(B) = 0.30 \geq 0;\ P(C) = 0.43 \geq 0;\ P(D) = 0.12 \geq 0$$
$$P(A) + P(B) + P(C) + P(D) = 0.15 + 0.30 + 0.43 + 0.12 = 1.00$$

∴ Given assignments are permissible.

23. Differentiate: $y = e^{\sin x}$.

Solution:

$$\text{Let } y = e^u \quad ; \quad u = \sin x$$
$$\frac{dy}{du} = e^u \quad ; \quad \frac{du}{dx} = \cos x$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$= e^u \times \cos x$$
$$= \cos x\, e^{\sin x}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

24. Find the positive integer 'n' so that $\lim\limits_{x\to 2} \frac{x^n - 2^n}{x-2} = 12$.

Solution:

$$\lim_{x\to 2} \frac{x^n - 2^n}{x-2} = 12 \qquad \because \lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$$
$$n(2)^{n-1} = 12$$
$$n(2)^{n-1} = 3(2)^2$$
$$n(2)^{n-1} = 3(2)^{3-1}$$
$$n = 3$$

25. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$.

Solution:

In general, a 2×3 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$$a_{11} = \frac{(1-1)^2}{2} = 0 \quad ; \quad a_{12} = \frac{(1-2(2))^2}{2} = \frac{9}{2} \quad ; \quad a_{13} = \frac{(1-2(3))^2}{2} = \frac{25}{2}$$
$$a_{21} = \frac{(2-2(1))^2}{2} = 0 \quad ; \quad a_{22} = \frac{(2-2(2))^2}{2} = 2 \quad ; \quad a_{23} = \frac{(2-2(3))^2}{2} = 8$$
$$A = \begin{bmatrix} 0 & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

26. Without expanding, evaluate the determinants: $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$.

Solution:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6 & 9 & 12 \end{vmatrix}$$

$$= (x)(3)\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}_{R_1 \equiv R_3}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$= 3x(0)$$
$$= 0$$

27. Solve $23x < 100$ when (i) x is a natural number, (ii) x is an integer.

Solution:

$$23x < 100$$
$$x < \frac{100}{23} \Longrightarrow x < 4.3$$

(i) x is a natural number

$$x \in \{1, 2, 3, 4\}$$

(ii) x is an integer

$$x \in \{\dots\dots\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

28. Write the first 6 terms of the sequences whose n^{th} term is $a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$.

Solution:

$$a_1 = 1 + 1 = 2 \;;\; a_2 = 2$$
$$a_3 = 3 + 1 = 4 \;;\; a_4 = 4$$
$$a_5 = 5 + 1 = 6 \;;\; a_6 = 6$$

First 6 terms are 2, 2, 4, 4, 6, 6

29. Find the locus of P, if for all values of α, the co-ordinates of a moving point P is $(9\cos\alpha, 9\sin\alpha)$.

Solution:

Let $P(h, k)$ be a point on the locus

Given $(9\cos\alpha, 9\sin\alpha)$ be the coordinate of moving point

$$h = 9\cos\alpha \quad \text{and} \quad k = 9\sin\alpha$$
$$\frac{h}{9} = \cos\alpha \;;\; \frac{k}{9} = \sin\alpha$$
$$\text{w.k.t.} \quad \cos^2\alpha + \sin^2\alpha = 1$$
$$\left(\frac{h}{9}\right)^2 + \left(\frac{k}{9}\right)^2 = 1$$
$$h^2 + k^2 = 9^2$$
$$\therefore \text{locus of the point } P(h,k) \text{ is } x^2 + y^2 = 81$$

30. What is the unit digit of the sum $3! + 4! + \dots + 20!$?

Solution:

From 5! onwards for the unit digit is zero and hence the contribution to the unit digit is through $3! + 4!$ only. which is $6 + 24 = 30$.

Therefore, the required unit digit is 2.

PART-III

Note: **[7 × 3 = 21]**

(i) Answer any **SEVEN** questions

(ii) Question number **40** is compulsory.

31. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

Solution:

$$-1 \le \cos x \le 1$$
$$3 \ge -3\cos x \ge -3$$
$$-3 \le -3\cos x \le 3$$
$$1-3 \le 1-3\cos x \le 1+3$$
$$-2 \le 1-3\cos x \le 4$$
$$-\frac{1}{2} \ge \frac{1}{1-3\cos x} \ge \frac{1}{4}$$
$$\frac{1}{1-3\cos x} \le -\frac{1}{2} \;;\; \frac{1}{1-3\cos x} \ge \frac{1}{4}$$
$$f(x) \le -\frac{1}{2} \;;\; f(x) \ge \frac{1}{4}$$
$$f(x) \in \left(-\infty, -\frac{1}{2}\right] \;;\; f(x) \in \left[\frac{1}{4}, \infty\right)$$

Hence the range of f is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{4}, \infty\right)$

32. Solve the equation $\sqrt{x^2 - x - 2} = x + 1$.

Solution:

$$\sqrt{x^2 - x - 2} = x + 1$$
$$\sqrt{x^2 - x - 2} \ge 0 \Rightarrow x + 1 \ge 0$$
$$\therefore x \ge -1$$
$$\sqrt{x^2 - x - 2} = x + 1$$
$$x^2 - x - 2 = (x+1)^2$$
$$x^2 - x - 2 = x^2 + 2x + 1$$
$$-3x = 3$$
$$\therefore x = -1$$

33. Prove that $\sin(45° + \theta) - \sin(45° - \theta) = \sqrt{2}\sin\theta$.

Solution:

$$\begin{aligned} \text{LHS} &= \sin(45° + \theta) - \sin(45° - \theta) \\ &= (\sin 45° \cos\theta + \cos 45° \sin\theta) - (\sin 45° \cos\theta - \cos 45° \sin\theta) \\ &= \sin 45° \cos\theta + \cos 45° \sin\theta - \sin 45° \cos\theta + \cos 45° \sin\theta \\ &= 2\cos 45° \sin\theta \\ &= 2\left(\frac{1}{\sqrt{2}}\right)\sin\theta \\ &= \sqrt{2}\sin\theta = \text{RHS} \end{aligned}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

34. If $^{(n+2)}P_4 = 42 \times {}^nP_2$, find n.

Solution:

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$^{(n+2)}P_4 = 42 \times {}^nP_2$$
$$\frac{(n+2)!}{(n+2-4)!} = 42 \times \frac{n!}{(n-2)!}$$
$$\frac{(n+2)(n+1)n!}{(n-2)!} = 42 \times \frac{n!}{(n-2)!}$$
$$(n+2)(n+1) = 7 \times 6$$
$$n+2 = 7$$
$$n = 5$$

35. Compute 102^4.

Solution:

$$(a+b)^n = {}^nC_0a^nb^0 + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-2}b^2 + \cdots + {}^nC_na^0b^n$$
$$102^4 = (100+2)^4$$
$$= {}^4C_0100^42^0 + {}^4C_1100^32^1 + {}^4C_2100^22^2 + {}^4C_3100^12^3 + {}^4C_4100^02^4$$
$$= 1(100000000)1 + 4(1000000)2 + 6(10000)(4) + 4(100)8 + (1)(16)$$
$$= 100000000 + 8000000 + 240000 + 3200 + 16$$
$$= 108243216$$

36. Show the points $\left(0, -\frac{3}{2}\right)$, $(1, -1)$, and $\left(2, -\frac{1}{2}\right)$ are collinear.

Solution:

Let A, B and C be $\left(0, -\frac{3}{2}\right)$, $(1, -1)$, and $\left(2, -\frac{1}{2}\right)$ respectively.

The slope of AB is $\frac{-1+\frac{3}{2}}{1-0} = \frac{1}{2}$

The slope of BC is $\frac{-\frac{1}{2}+1}{2-1} = \frac{1}{2}$

The slope of AB is equal to slope of BC.

Hence, A, B and C are lying on the same line.

37. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$.

Solution:

G. T. $\vec{a}, \vec{b}, \vec{c}$ be unit vectors $\Rightarrow |\vec{a}| = 1 ; |\vec{b}| = 1 ; |\vec{c}| = 1$

G. T. $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$

$$\left.\begin{array}{l}\vec{a}.\vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}\\ \vec{a}.\vec{c} = 0 \Rightarrow \vec{a} \perp \vec{c}\end{array}\right\} \Rightarrow \vec{a} \parallel (\vec{b} \times \vec{c})$$
$$\vec{a} = \lambda(\vec{b} \times \vec{c}) \rightarrow ①$$
$$|\vec{a}| = |\lambda(\vec{b} \times \vec{c})|$$
$$|\vec{a}| = |\lambda||\vec{b} \times \vec{c}| \quad \because \text{angle between } \vec{b} \text{ and } \vec{c} \text{ is } \frac{\pi}{3}$$
$$|\vec{a}| = |\lambda||\vec{b}||\vec{c}|\sin\frac{\pi}{3}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$1 = |\lambda|(1)(1)\frac{\sqrt{3}}{2}$$
$$|\lambda| = \frac{2}{\sqrt{3}}$$
$$\lambda = \pm\frac{2}{\sqrt{3}}$$
$$\text{sub } \lambda = \pm\frac{2}{\sqrt{3}} \text{ in } \textcircled{1} \text{ we get}$$
$$\vec{a} = \frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$$

38. Integrate with respect to x: cos 5x sin 3x

Solution:

$$\int \cos 5x \sin 3x\,dx = \int \frac{1}{2}[\sin(5x+3x) - \sin(5x-3x)]\,dx$$
$$= \frac{1}{2}\left[\int \sin 8x\,dx - \int \sin 2x\,dx\right]$$
$$= \frac{1}{2}\left[\frac{1}{8}(-\cos 8x) - \frac{1}{2}(-\cos 2x)\right] + c$$
$$= \frac{1}{2}\left[-\frac{\cos 8x}{8} + \frac{\cos 2x}{2}\right] + c$$

39. Eight coins are tossed once, find the probability of getting:
(i) exactly two tails (ii) at most two tails

Solution:

$$n(S) = 2^8 = 256$$

(Tossing 8 coins once and tossing one coin 8 times are same)

$$P(\text{ getting exactly two tails}) = \frac{{}^8C_2}{256}$$
$$= \frac{28}{256} = \frac{7}{64}$$
$$P\text{ (Atmost two tails)} = P(x \le 2)$$
$$= P(x=0) + P(x=1) + P(x=2)$$
$$= \frac{{}^8C_0}{256} + \frac{{}^8C_1}{256} + \frac{{}^8C_2}{256}$$
$$= \frac{1+8+28}{256}$$
$$= \frac{37}{256}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

40. Find $\frac{dy}{dx}$, if $y = \cos^{-1}(2\cos^2 x - 1)$.

Solution:

$$y = \cos^{-1}(2\cos^2 x - 1)$$
$$y = \cos^{-1}(\cos 2x)$$
$$y = 2x$$
$$\text{diff. w.r.to "x"}$$
$$\frac{dy}{dx} = 2$$

PART-IV

ANSWER ALL QUESTIONS. **[$7 \times 5 = 35$]**

41. (a) Resolve into partial fractions: $\frac{x^2+x+1}{x^2-5x+6}$.

Solution:

$$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{x^2-5x+6} \rightarrow ①$$

Consider, $$\frac{6x-5}{x^2-5x+6} = \frac{6x-5}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} \rightarrow ②$$

$$6x-5 = A(x-3) + B(x-2)$$

Put $x = 2$	Put $x = 3$
$6(2)-5 = A(2-3)+0$	$6(3)-5 = 0+B(3-2)$
$7 = A(-1)$	$13 = B$
$\boxed{A = -7}$	$\boxed{B = 13}$

Sub A, B values in ②

$$\frac{6x-5}{x^2-5x+6} = \frac{-7}{(x-2)} + \frac{13}{(x-3)}$$

From ①,

$$\therefore \frac{x^2+x+1}{x^2-5x+6} = 1 - \frac{7}{(x-2)} + \frac{13}{(x-3)}$$

(OR)

(b) Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form:
(i) Slope and Intercept form (ii) Intercept form (iii) Normal form

Solution:

(i) Slope and intercept form

$$\text{given that } \sqrt{3}x - y + 4 = 0$$
$$y = \sqrt{3}x + 4$$

Comparing the above equation with the equation $y = mx + b$, we have

$$\text{Slope } = \sqrt{3} \text{ and y - intercept } = 4$$

(ii) Intercept form

$$\sqrt{3}x - y + 4 = 0$$
$$\sqrt{3}x - y = -4$$
$$\frac{-\sqrt{3}}{4}x + \frac{y}{4} = 1$$
$$\frac{x}{\left(-\frac{4}{\sqrt{3}}\right)} + \frac{y}{4} = 1$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

Comparing the above equation with the equation $\frac{x}{a} + \frac{y}{b} = 1$

$$\text{x- intercept} = -\frac{4}{\sqrt{3}} \text{ and y- intercept} = 4$$

(iii) Normal form:

$$\text{The required form } x\cos\alpha + y\sin\alpha = p$$

$$\text{Given form } -\sqrt{3}x + y = 4(\because p \text{ is always positive})$$

$\div 2$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 2$$

$$-\cos\frac{\pi}{6}x + \sin\frac{\pi}{6}y = 2$$

$$\cos\left(\pi - \frac{\pi}{6}\right)x + \sin\left(\pi - \frac{\pi}{6}\right)y = 2$$

$$x\cos\frac{5\pi}{6} + y\sin\frac{5\pi}{6} = 2 \text{ This is a Normal form.}$$

$$\text{Here } \alpha = 150^\circ = \frac{5\pi}{6} \text{ and } p = 2$$

42. (a) **Prove that $\frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)} = \cos^2\theta\cot\theta$.**

Solution:

$$LHS = \frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)}$$

$$= \frac{(\cot\theta)(\cos\theta)(\cos\theta)}{(-\cos\theta)(-\tan\theta)(\operatorname{cosec}\theta)}$$

$$= \frac{\cot\theta\cos^2\theta}{(\cos\theta)\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)}$$

$$= \cos^2\theta\cot\theta = RHS$$

(OR)

(b) **If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$.**

Solution:

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$y^2 = \frac{(\sin^{-1}x)^2}{1-x^2}$$

$$(1-x^2)y^2 = (\sin^{-1}x)^2$$

$$(1-x^2)(2yy_1) + y^2(-2x) = 2\sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$(1-x^2)(2yy_1) + y^2(-2x) = 2y$$

$$(1-x^2)y_1 - xy = 1$$

$$(1-x^2)y_2 + y_1(-2x) - \left(xy_1 + y(1)\right) = 0$$

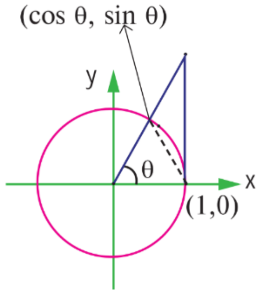
$$(1-x^2)y_2 - 2xy_1 - xy_1 - y = 0$$

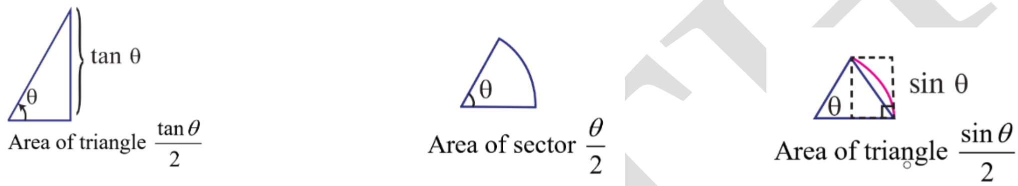
$$(1-x^2)y_2 - 3xy_1 - y = 0$$

43. (a) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$.

Solution:

Consider the circle with centre (0,0) and radius 1. Any point on this circle is R $(\cos\theta, \sin\theta)$.





By area property $\frac{\tan\theta}{2}\ge\frac{\theta}{2}\ge\frac{\sin\theta}{2}$

$\times$ by $\frac{2}{\sin\theta}$

$$\frac{1}{\cos\theta}\ge\frac{\theta}{\sin\theta}\ge 1$$

$$\cos\theta\le\frac{\sin\theta}{\theta}\le 1$$

Because $\cos(-\theta)=\cos\theta$ and $\frac{\sin(-\theta)}{-\theta}=\frac{\sin\theta}{\theta}$ one can conclude that this inequality is valid for all non-zero θ in the open interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

$$\lim_{\theta\to 0}\cos\theta=1\ ;\ \lim_{\theta\to 0}(1)=1$$

Sandwich theorem we get

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

(OR)

(b) A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

Solution:

Let M_1, M_2 be the production from Machine –I, Machine –II respectively

Let D be the Defective items produced from above Machines

Given

$$P(M_1)=\frac{60}{100}=\frac{6}{10}\ ;\ P(M_2)=\frac{40}{100}=\frac{4}{10}$$

$$P(D/M_1)=\frac{2}{100}\ ;\ P(D/M_2)=\frac{4}{100}$$

$$\begin{aligned}
P(\text{Selected item is defective}) &= P(D) \\
&= P(M_1)P(D/M_1) + P(M_2)P(D/M_2) \\
&= \left(\frac{6}{10}\right)\left(\frac{2}{100}\right) + \left(\frac{4}{10}\right)\left(\frac{4}{100}\right) \\
&= \frac{12}{1000} + \frac{16}{1000} \\
&= \frac{28}{1000} = 0.028
\end{aligned}$$

44. (a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

Solution:

$$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = (x^3+7)^{\frac{1}{3}} - (x^3+4)^{\frac{1}{3}}$$

$$x \text{ is sufficiently large} \Longrightarrow \frac{1}{x} \text{ is smaller}$$

$$\begin{aligned}
\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} &= (x^3)^{\frac{1}{3}}\left(1+\frac{7}{x^3}\right)^{\frac{1}{3}} - (x^3)^{\frac{1}{3}}\left(1+\frac{4}{x^3}\right)^{\frac{1}{3}} \\
&= x\left(1+\frac{7}{x^3}\right)^{\frac{1}{3}} - x\left(1+\frac{4}{x^3}\right)^{\frac{1}{3}} \\
&= x\left(1+\left(\frac{1}{3}\right)\left(\frac{7}{x^3}\right) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{7}{x^3}\right)^2 + \cdots\right) \\
&\quad - x\left(1+\left(\frac{1}{3}\right)\left(\frac{4}{x^3}\right) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{4}{x^3}\right)^2 + \cdots\right) \\
&= x\left(1+\frac{7}{3x^3} - \frac{1}{9}\left(\frac{7}{x^3}\right)^2 + \cdots\right) - x\left(1+\frac{4}{3x^3} - \frac{1}{9}\left(\frac{4}{x^3}\right)^2 + \cdots\right) \\
&= x + \frac{7}{3x^2} + \cdots - x - \frac{4}{3x^2} + \cdots \\
&\approx \frac{7-4}{3x^2} = \frac{1}{x^2}
\end{aligned}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

$$\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3} \approx \frac{1}{x^2}$$

(OR)

(b) If one root of $k(x-1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or -25.

Solution:

$$\begin{gathered}
k(x-1)^2 = 5x - 7 \\
k(x^2 - 2x + 1) = 5x - 7 \\
kx^2 - 2kx + k - 5x + 7 = 0 \\
kx^2 - (2k+5)x + k + 7 = 0 \\
\text{Let } \alpha \text{ be the root, then } 2\alpha \text{ is also a root} \\
\text{Sum} = \alpha + 2\alpha = -\frac{-(2k+5)}{k}
\end{gathered}$$

$$\Rightarrow 3\alpha = \frac{2k+5}{k} \Rightarrow \alpha = \frac{2k+5}{3k}$$

$$\text{Product} = (\alpha)(2\alpha) = \frac{k+7}{k}$$

$$\Rightarrow 2\alpha^2 = \frac{k+7}{k}$$

$$2\left(\frac{2k+5}{3k}\right)^2 = \frac{k+7}{k}$$

$$2\left(\frac{4k^2+20k+25}{9k^2}\right) = \frac{k+7}{k}$$

$$8k^2 + 40k + 50 = 9k^2 + 63k$$

$$k^2 + 23k - 50 = 0$$

$$(k-2)(k+25) = 0$$

$$k = 2 \text{ or } k = -25$$

45. (a) Show that $\tan 20° \tan 40° \tan 60° \tan 80° = 3$

Solution:

$$\begin{aligned} \text{LHS} &= \tan 20° \tan 40° \tan 60° \tan 80° \\ &= \tan 40° \tan 20° \tan 80° \tan 60° \\ &= \tan(60° - 20°) \tan 20° \tan(60° + 20°)(\sqrt{3}) \\ &= (\tan 3(20°))(\sqrt{3}) \qquad \because \tan(60° - A)\tan A \tan(60° + A) = \tan 3A \\ &= (\tan 60°)(\sqrt{3}) \\ &= (\sqrt{3})(\sqrt{3}) \\ &= 3 = \text{RHS} \end{aligned}$$

(OR)

(b) Evaluate: $\int \frac{1}{\sqrt{x^2+5x+4}} dx$

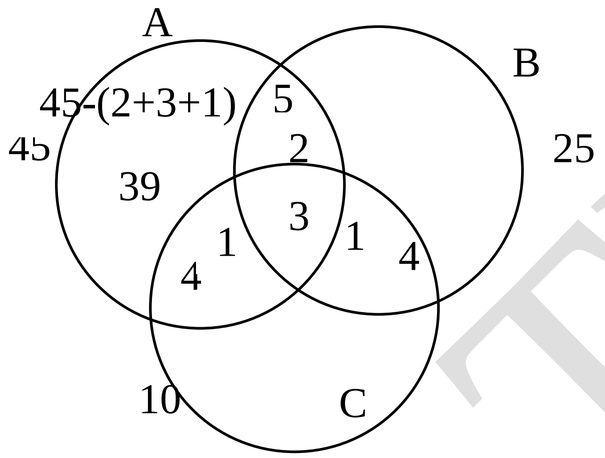
Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\sqrt{x^2+5x+4}} dx \\ &= \int \frac{1}{\sqrt{x^2+5x+\frac{25}{4}-\frac{25}{4}+4}} dx \\ &= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2 - \frac{9}{4}}} dx \\ &= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2}} dx \\ &= \log\left(\left(x+\frac{5}{2}\right) + \sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\right) + c \\ &= \log\left(\left(x+\frac{5}{2}\right) + \sqrt{x^2+5x+4}\right) + c \end{aligned}$$

46. (a) In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

Solution:

We draw the Venn diagram using percentage.



From diagram the percentage of persons who knows only Language A is 39.

The required number of persons $= 39\% \times 5000 = \frac{39}{100} \times 5000 = 1950.$

(OR)

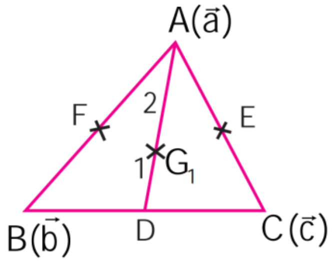
(b) Prove that the medians of a triangle are concurrent.

Solution:

Let ABC be a triangle and let D, E, F be the mid points of its sides BC, CA and AB respectively

We have to prove that the medians AD, BE, CF are concurrent.

Let O be the origin



$\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$

D, E, F be the mid points of its sides BC, CA and AB respectively

$\overrightarrow{OD} = \frac{\vec{b}+\vec{c}}{2}, \overrightarrow{OE} = \frac{\vec{c}+\vec{a}}{2}, \overrightarrow{OF} = \frac{\vec{a}+\vec{b}}{2}$

Let G_1 be the point on AD dividing it internally in the ratio $2:1$

$$\overrightarrow{OG_1} = \frac{1\overrightarrow{OA} + 2\overrightarrow{OD}}{1+2}$$

$$\overrightarrow{OG_1} = \frac{1\vec{a} + 2\left(\frac{\vec{b}+\vec{c}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

Let G_2 be the point on BE dividing it internally in the ratio $2:1$

$$\overrightarrow{OG_2} = \frac{1\overrightarrow{OB} + 2\overrightarrow{OE}}{1+2}$$

$$\overrightarrow{OG_2} = \frac{1\vec{b} + 2\left(\frac{\vec{c}+\vec{a}}{2}\right)}{3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS

Similarly, if G_3 divides CF in the ratio 2 : 1 then

$$\overrightarrow{OG_3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$$\overrightarrow{OG_1} = \overrightarrow{OG_2} = \overrightarrow{OG_3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

Hence, they are not different points. Let the common point be G.
Therefore, the three medians are concurrent and the point of concurrence is G.

47. (a) By the principle of mathematical induction, prove that, for $n \geq 1$
$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Solution:

$$P(n) \coloneqq 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Put $n = 1$

$$P(1) \coloneqq 1^3 = \left(\frac{1(1+1)}{2}\right)^2$$

$$1 = 1$$

P(1) is true

Let us assume that the statement is true for $n = k$

$$\text{i.e., } P(k) \coloneqq 1^3 + 2^3 + 3^3 + \cdots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$$

To prove that the statement is true for $n = k + 1$

$$\text{i.e., } P(k+1) = 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3$$

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2[k^2 + 4(k+1)]}{4}$$

$$= \frac{(k+1)^2[k^2 + 4k + 4]}{4}$$

$$P(k+1) = \frac{(k+1)^2(k+2)^2}{4}$$

$\Rightarrow$ P(k + 1) is true

$\therefore$ P(k + 1) is true whenever p(k) is true

Hence by the principle of mathematical induction for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(OR)

(b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

Solution:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix} \quad \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= a\begin{vmatrix} b & -c \\ 1 & 1+c \end{vmatrix} + b\begin{vmatrix} 0 & -c \\ 1 & 1+c \end{vmatrix} + 0\begin{vmatrix} 0 & b \\ 1 & 1 \end{vmatrix}$$

$$= a(b+bc+c) + b(0+c) + 0$$

$$= ab + abc + ac + bc$$

$$= abc + ab + bc + ac$$

$$= abc\left(1+\frac{ab}{abc}+\frac{bc}{abc}+\frac{ac}{abc}\right)$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Prepared by
M. SANKAR M.Sc., B.Ed.
POST GRADUATE TEACHER
BHARATHIDASAN MAT. HR. SEC. SCHOOL
Tiruvallur