🎯 Straight Line - Full Formula Sheet (Hidden Formulas Included)

📌 1. Slope of a Line (முனை உருண்டை)

- **Slope (m) = $(y_2 - y_1) / (x_2 - x_1)$**
- **If θ is the angle with the positive x-axis:**
  m = tan θ

✅ **Hidden Formula:**

- **Angle between line and x-axis:**
  If the line equation is **Ax + By + C = 0**, slope **m = -A/B**, and **θ = $\tan^{-1}$(-A/B)**

📌 2. Straight Line Equations

(i) Point-Slope Form (புள்ளி-சரிவு வடிவம்)

- **$y - y_1 = m(x - x_1)$**

✅ **Hidden Insight:**

- If the line is vertical (parallel to y-axis): **x = constant**
- If the line is horizontal (parallel to x-axis): **y = constant**

(ii) Slope-Intercept Form (சரிவு-தடை வடிவம்)

- **y = mx + c**

✅ **Hidden Trick:**

- If the slope is undefined (vertical line), the equation has no **y** term:
  x = constant

(iii) Two-Point Form (இரு புள்ளி வடிவம்)

- $y - y_1 = ((y_2 - y_1) / (x_2 - x_1)) (x - x_1)$

✅ **Hidden Shortcut:**

- If two points (x_1, y_1) and (x_2, y_2) are symmetric about the y-axis:
  $x_1 + x_2 = 0$

(iv) Intercept Form (தடை வடிவம்)

- $(x/a) + (y/b) = 1$

✅ **Hidden Twist:**

- If the line is equally inclined to both axes (45° or 135°):
  $a = \pm b$

📌 3. Angle Between Two Lines (இரண்டு கோடுகளுக்கு இடையிலான கோணம்)

- $\tan \theta = |(m_2 - m_1) / (1 + m_1 m_2)|$

✅ **Hidden Formula:**

- If the lines are perpendicular, the product of slopes is **-1**:
  $m_1 \times m_2 = -1$
- If the lines are parallel:
  $m_1 = m_2$

📌 4. Distance Between Two Points (இரண்டு புள்ளிகளுக்கு இடையிலான தூரம்)

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

✅ **Hidden Shortcut:**

- If the points are symmetric about the origin:
  $x_1 = -x_2$ and $y_1 = -y_2$

📌 5. Distance from a Point to a Line

- From **(x_0, y_0)** to **$Ax + By + C = 0$**:
  $d = |Ax_0 + By_0 + C| / \sqrt{(A^2 + B^2)}$

✅ **Hidden Insight:**

- If the line passes through the origin **(0,0)**:
  C = 0, so the formula reduces to:
  $d = |Ax_0 + By_0| / \sqrt{(A^2 + B^2)}$

📌 6. Equation of a Line Perpendicular to Another Line

- **$Bx - Ay + k = 0$**

✅ **Hidden Trick:**

- If two lines are perpendicular, the perpendicular bisector passes through their midpoint:
  $M = ((x_1 + x_2)/2 , (y_1 + y_2)/2)$

📌 7. Area of a Triangle (முக்கோணத்தின் பரப்பளவு)

- **Area = $(1/2) |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$**

✅ **Hidden Formula:**

- If area = 0, the points are collinear (same straight line).
- For **isosceles triangles**, two sides must have equal distances.

📌 8. Normal Form (நேர்கோண வடிவம்)

- **$x \cos\theta + y \sin\theta = p$**

✅ **Hidden Insight:**

- If the perpendicular distance **$p = 0$**, the line passes through the origin.

📌 9. Pair of Straight Lines (நேர்கோடுகளின் ஜோடி)

- **$Ax^2 + 2Hxy + By^2 = 0$** (represents a pair of straight lines passing through the origin)

✅ **Hidden Formula:**

- Angle between the pair of lines:
  $\tan \theta = \pm\sqrt{(H^2 - AB)/(A + B)}$
- If **$H^2 = AB$**, the lines are perpendicular!

✨ **Bonus: Quick Revision Tips:**

1️⃣ **Vertical Line:** No y-term → **x = constant**
2️⃣ **Horizontal Line:** No x-term → **y = constant**
3️⃣ **Line passing through origin:** No constant → **Ax + By = 0**
4️⃣ **Collinear Points:** Area = 0
5️⃣ **Perpendicular Lines:** Product of slopes = -1
6️⃣ **Parallel Lines:** Slopes are equal

📌 9. Pair of Straight Lines (நேர்கோடுகளின் ஜோடி)

Formula:

$Ax^2 + 2Hxy + By^2 = 0$

(Represents a pair of straight lines passing through the origin)

✅ Hidden Formula:

- **Angle between the pair of lines:**
  $\tan \theta = \pm\sqrt{(H^2 - AB)/(A + B)}$
- **Special Case:**
  If $H^2 = AB$, the lines are **perpendicular**!

1. Limit of a function f(x) is said to exist as $x \to a$ when,

$$\underset{h\to 0^+}{\text{Limit}}\ f(a-h) = \underset{h\to 0^+}{\text{Limit}}\ f(a+h) = \text{some finite value M.}$$

(Left hand limit) (Right hand limit)

2. Indeterminant Forms:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, \infty^0, 0^0, \text{ and } 1^\infty.$$

3. Standard Limits:

$$\underset{x\to 0}{\text{Limit}}\ \frac{\sin x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\tan x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\tan^{-1}x}{x}$$

$$= \underset{x\to 0}{\text{Limit}}\ \frac{\sin^{-1}x}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{e^x-1}{x} = \underset{x\to 0}{\text{Limit}}\ \frac{\ell n(1+x)}{x} = 1$$

$$\underset{x\to 0}{\text{Limit}}\ (1+x)^{1/x} = \underset{x\to\infty}{\text{Limit}}\left(1+\frac{1}{x}\right)^x = e,\quad \underset{x\to 0}{\text{Limit}}\ \frac{a^x-1}{x} = \log_e a,\ a>0,$$

$$\underset{x\to a}{\text{Limit}}\ \frac{x^n-a^n}{x-a} = na^{n-1}.$$

4. Limits Using Expansion

(i) $a^x = 1 + \frac{x\ln a}{1!} + \frac{x^2\ln^2 a}{2!} + \frac{x^3\ln^3 a}{3!} + \ldots\ldots\ldots a>0$

(ii) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(iii) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots \text{for } -1 < x \le 1$

(iv) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$

(v) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$

(vi) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + ...$

(vii) for $|x| < 1$, $n \in R$ $(1 + x)^n$

$$= 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + ...\infty$$

5. Limits of form 1^{∞}, 0^0, ∞^0

Also for $(1)^{\infty}$ type of problems we can use following rules.

$\lim_{x \to 0} (1 + x)^{1/x} = e$, $\lim_{x \to a} [f(x)]^{g(x)}$,

where $f(x) \to 1$; $g(x) \to \infty$ as $x \to a$ $= e^{\lim_{x \to a} [f(x) - 1]\, g(x)}$

6. Sandwich Theorem or Squeeze Play Theorem:

If $f(x) \le g(x) \le h(x) \; \forall x$ & $\underset{x \to a}{\text{Limit}} f(x) = \ell = \underset{x \to a}{\text{Limit}} h(x)$ then $\underset{x \to a}{\text{Limit}} g(x) = \ell$.

METHOD OF DIFFERENTIATION

1. Differentiation of some elementary functions

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(a^x) = a^x \ell n\, a$
3. $\frac{d}{dx}(\ell n\,|x|) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x\, \ell n\, a}$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$
7. $\frac{d}{dx}(\sec x) = \sec x \tan x$
8. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
9. $\frac{d}{dx}(\tan x) = \sec^2 x$
10. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

2. Basic Theorems

1. $\frac{d}{dx}(f \pm g) = f'(x) \pm g'(x)$

2. $\frac{d}{dx}(k\,f(x)) = k\,\frac{d}{dx}\,f(x)$

3. $\frac{d}{dx}(f(x)\,.\,g(x)) = f(x)\,g'(x) + g(x)\,f'(x)$

4. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\,f'(x) - f(x)g'(x)}{g^2(x)}$

5. $\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$

Derivative Of Inverse Trigonometric Functions.

$$\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}},\quad \frac{d\cos^{-1}x}{dx} = -\frac{1}{\sqrt{1-x^2}},\ \text{for } -1 < x < 1.$$

$$\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2},\quad \frac{d\cot^{-1}x}{dx} = -\frac{1}{1+x^2}\ (x \in R)$$

$$\frac{d\sec^{-1}x}{dx} = \frac{1}{|x|\sqrt{x^2-1}},\quad \frac{d\operatorname{cosec}^{-1}x}{dx}$$

$$= -\frac{1}{|x|\sqrt{x^2-1}},\ \text{for } x \in (-\infty, -1) \cup (1, \infty)$$

3. Differentiation using substitution

Following substitutions are normally used to simplify these expression.

(i) $\sqrt{x^2 + a^2}$ by substituting $x = a\tan\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(ii) $\sqrt{a^2 - x^2}$ by substituting $x = a\sin\theta$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(iii) $\sqrt{x^2 - a^2}$ by substituting $x = a\sec\theta$, where $\theta \in [0, \pi]$, $\theta \ne \frac{\pi}{2}$

(iv) $\sqrt{\frac{x+a}{a-x}}$ by substituting $x = a\cos\theta$, where $\theta \in (0, \pi]$.

4. Parametric Differentiation

If $y = f(\theta)$ & $x = g(\theta)$ where θ is a parameter, then $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

5. Derivative of one function with respect to another

Let $y = f(x)$; $z = g(x)$ then $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$.

1. If f & g are functions of x such that $g'(x) = f(x)$ then,

$\int f(x)\,dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the **constant of integration**.

2. **Standard Formula:**

(i) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\,\ell n\,(ax+b) + c$

(iii) $\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

(iv) $\int a^{px+q}\,dx = \frac{1}{p}\,\frac{a^{px+q}}{\ell n\,a} + c;\ a > 0$

(v) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \tan(ax+b)\,dx = \frac{1}{a}\,\ell n\,\sec(ax+b) + c$

(viii) $\int \cot(ax+b)\,dx = \frac{1}{a}\,\ell n\,\sin(ax+b) + c$

(ix) $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$

(x) $\int \mathrm{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + c$

(xi) $\int \sec x\, dx = \ell n(\sec x + \tan x) + c$ **OR** $\ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(xii) $\int \operatorname{cosec} x\, dx = \ell n(\operatorname{cosec} x - \cot x) + c$

OR $\ell n \tan\frac{x}{2} + c$ **OR** $-\ell n(\operatorname{cosec} x + \cot x) + c$

(xiii) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xiv) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xv) $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ell n\left[x + \sqrt{x^2 + a^2}\right] + c$

(xvii) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n\left[x + \sqrt{x^2 - a^2}\right] + c$

(xviii) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ell n\left|\frac{a+x}{a-x}\right| + c$

(xix) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ell n\left|\frac{x-a}{x+a}\right| + c$

(xx) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxi) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ell n\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) + c$

(xxii) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ell n\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) + c$

3. Integration by Subsitutions

If we subsitute f(x) = t, then f '(x) dx = dt

4. Integration by Part :

$$\int (f(x)\, g(x))\, dx = f(x) \int (g(x))\, dx - \int \left(\frac{d}{dx}(f(x)) \int (g(x))\, dx \right) dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\, dx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type

$$\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx, \int (px+q)\sqrt{ax^2+bx+c}\, dx$$

Make the substitution x + $\frac{b}{2a}$ = t , then split the integral as some of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions

(i) $\int \frac{dx}{a+b\sin^2 x}$ **OR** $\int \frac{dx}{a+b\cos^2 x}$

OR $\int \frac{dx}{a\sin^2 x + b\sin x \cos x + c\cos^2 x}$ **put tan x = t.**

(ii) $\int \frac{dx}{a+b\sin x}$ **OR** $\int \frac{dx}{a+b\cos x}$

OR $\int \frac{dx}{a+b\sin x + c\cos x}$ **put tan** $\frac{x}{2}$ **= t**

(iii) $\int \frac{a.\cos x + b.\sin x + c}{\ell.\cos x + m.\sin x + n}$ **dx. Express Nr ≡ A(Dr) + B** $\frac{d}{dx}$ **(Dr) + c & proceed.**

8. **Integration of type**

$\int \frac{x^2+1}{x^4+Kx^2+1}dx$ where K is any constant.

Divide Nr & Dr by x² & put $x \mp \frac{1}{x} = t$.

9. **Integration of type**

$\int \frac{dx}{(ax+b)\sqrt{px+q}}$ **OR** $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$; put px + q = t^2.

10. **Integration of type**

$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put ax + b = $\frac{1}{t}$;

$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ **, put $x = \frac{1}{t}$**

Intervals :

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers a, b $\in$ R such that a < b, we can define four types of intervals as follows :

Symbols Used

(i) Open interval : $(a, b) = \{x : a < x < b\}$ i.e. end points are not included. ( ) or] [

(ii) Closed interval : $[a, b] = \{x : a \le x \le b\}$ i.e. end points are also included. [ ]
This is possible only when both a and b are finite.

(iii) Open-closed interval : $(a, b] = \{x : a < x \le b\}$
( ] or]]

(iv) Closed - open interval : $[a, b) = x : a \le x < b\}$
[ ) or [[

The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$
(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(\infty, b] = \{x : x \le b\}$
(v) $(-\infty, \infty) = \{x : x \in R\}$

Properties of Modulus :

For any a, b $\in$ R

$|a| \ge 0$, $|a| = |-a|$, $|a| \ge a$, $|a| \ge -a$, $|ab| = |a|\,|b|$,

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $|a + b| \le |a| + |b|$, $|a - b| \ge ||a| - |b||$

Trigonometric Functions of Sum or Difference of Two Angles:

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\therefore 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$ and
and $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\therefore 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ and $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B).\sin(A-B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B).\cos(A-B)$

(e) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(f) $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$.

Factorisation of the Sum or Difference of Two Sines or Cosines:

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

Multiple and Sub-multiple Angles :

(a) $\cos 2A = \cos^2A - \sin^2A = 2\cos^2A - 1 = 1 - 2\sin^2A;\ 2\cos^2\frac{\theta}{2}$

$= 1 + \cos\theta,\ 2\sin^2\frac{\theta}{2} = 1 - \cos\theta.$

(b) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(c) $\sin 3A = 3\sin A - 4\sin^3 A$

(d) $\cos 3A = 4\cos^3 A - 3\cos A$

(e) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Important Trigonometric Ratios:

(a) $\sin n\pi = 0$; $\cos n\pi = \pm 1$; $\tan n\pi = 0$, where $n \in I$

(b) $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$;

$\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$;

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$; $\tan 75°$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$

(c) $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$ & $\cos 36°$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

Range of Trigonometric Expression:

$$-\sqrt{a^2+b^2} \le a \sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$$

Sine and Cosine Series :

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\beta\right)$$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\beta\right)$$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

Trigonometric Equations

Principal Solutions: Solutions which lie in the interval $[0, 2\pi)$ are called **Principal solutions.**

General Solution :

(i) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

(ii) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

(iv) $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.

PERMUTATION & COMBINNATION

1. **Arrangement :** number of permutations of n different things taken r at a time = $^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$

2. **Circular Permutation :**
The number of circular permutations of n different things taken all at a time is; $(n-1)!$

3. **Selection :** Number of combinations of n different things taken r at a time = $^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{^nP_r}{r!}$

4. The number of permutations of 'n' things, taken all at a time, when 'p' of them are similar & of one type, q of them are similar & of another type, 'r' of them are similar & of a third type & the remaining $n - (p + q + r)$ are all different is $\frac{n!}{p!\,q!\,r!}$.

5. **Selection of one or more objects**
   (a) Number of ways in which atleast one object be selected out of 'n' distinct objects is
   $$^nC_1 + {^nC_2} + {^nC_3} + \ldots\ldots\ldots\ldots + {^nC_n} = 2^n - 1$$
   (b) Number of ways in which atleast one object may be selected out of 'p' alike objects of one type 'q' alike objects of second type and 'r' alike of third type is
   $$(p + 1)(q + 1)(r + 1) - 1$$
   (c) Number of ways in which atleast one object may be selected from 'n' objects where 'p' alike of one type 'q' alike of second type and 'r' alike of third type and rest
   $n - (p + q + r)$ are different, is
   $$(p + 1)(q + 1)(r + 1)\,2^{n-(p+q+r)} - 1$$

6. **Multinomial Theorem :**
Coefficient of x^r in expansion of $(1 - x)^{-n} = {^{n+r-1}C_r}$ $(n \in N)$

7. Let $N = p^a\, q^b\, r^c \ldots\ldots$ where p, q, r...... are distinct primes & a, b, c..... are natural numbers then :
   (a) The total numbers of divisors of N including 1 & N is
   $$= (a + 1)(b + 1)(c + 1)\ldots\ldots\ldots$$
   (b) The sum of these divisors is =
   $$(p^0 + p^1 + p^2 + \ldots + p^a)(q^0 + q^1 + q^2 + \ldots + q^b)(r^0 + r^1 + r^2 + \ldots + r^c)\ldots\ldots\ldots$$

(c) Number of ways in which N can be resolved as a product of two factors is

$$= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1).... & \text{if N is not a perfect square} \\ \frac{1}{2}[(a+1)(b+1)(c+1)....+1] & \text{if N is a perfect square} \end{cases}$$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

8. Dearrangement :

Number of ways in which 'n' letters can be put in 'n' corresponding envelopes such that no letter goes to correct envelope is n !

$$\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}..........+(-1)^n\frac{1}{n!}\right)$$

PROBABILITY

1. Classical (A priori) Definition of Probability :

If an experiment results in a total of (m + n) outcomes which are equally likely and mutually exclusive with one another and if 'm' outcomes are favorable to an event 'A' while 'n' are unfavorable, then the probability of occurrence of the event 'A' = $P(A) = \frac{m}{m+n} = \frac{n(A)}{n(S)}$.

We say that odds in favour of 'A' are m : n, while odds against 'A' are n : m.

$$P(\bar{A}) = \frac{n}{m+n} = 1 - P(A)$$

2. Addition theorem of probability : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

De Morgan's Laws :

(a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$

Distributive Laws :

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A, B, C occur) $= P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A, B, C occur) $= P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A, B, C occur) =

$P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

3. Conditional Probability : $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

given by $\sum p_i m_i$.

6. **Total Probability Theorem :** $P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)$

7. **Bayes' Theorem :**

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then $P(B_i/A) = \dfrac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$ $B_1, B_2, B_3, \ldots, B_n$

I. Position Vector Of A Point:

let O be a fixed origin, then the position vector of a point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then,

$\overrightarrow{AB} = \vec{b} - \vec{a}$ = pv of B – pv of A.

DISTANCE FORMULA : Distance between the two points A $(\vec{a})$ and B $(\vec{b})$ is AB = $\left|\vec{a} - \vec{b}\right|$

SECTION FORMULA : $\vec{r} = \dfrac{n\vec{a} + m\vec{b}}{m+n}$. Mid point of AB = $\dfrac{\vec{a} + \vec{b}}{2}$.

II. Scalar Product Of Two Vectors: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where $|\vec{a}|, |\vec{b}|$ are magnitude of $\vec{a}$ and $\vec{b}$ respectively and θ is angle between $\vec{a}$ and $\vec{b}$.

1. i.i = j.j = k.k = 1; i.j = j.k = k.i = 0 ☞ projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

2. If $\vec{a} = a_1 i + a_2 j + a_3 k$ & $\vec{b} = b_1 i + b_2 j + b_3 k$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

3. The angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \frac{\vec{a}.\vec{b}}{|\vec{a}|\ |\vec{b}|}$, $0 \le \phi \le \pi$

4. $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ $(\vec{a} \neq 0\ \vec{b} \neq 0)$

III. Vector Product Of Two Vectors:

1. If $\vec{a}$ & $\vec{b}$ are two vectors & θ is the angle between them then $\vec{a}\mathrm{x}\vec{b} = |\vec{a}||\vec{b}|\sin\theta\vec{n}$, where $\vec{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ such that $\vec{a}, \vec{b}$ & $\vec{n}$ forms a right handed screw system.

2. Geometrically $|\vec{a}\mathrm{x}\vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$.

3. $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$; $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

4. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

5. $\vec{a}\times\vec{b} = \vec{o} \Leftrightarrow \vec{a}$ and $\vec{b}$ are parallel (collinear)

$(\vec{a} \neq 0, \vec{b} \neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar.

SOLUTION OF TRIANGLE

1. **Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

2. **Cosine Formula:**

(i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (ii) $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

(iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

3. **Projection Formula:**
(i) a = b cosC + c cosB (ii) b = c cosA + a cosC (iii) c = a cosB + b cosA

4. **Napier's Analogy - tangent rule:**

(i) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ (ii) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

(iii) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

5. **Trigonometric Functions of Half Angles:**

(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$;

$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ where $s = \frac{a+b+c}{2}$ is semi perimetre of triangle.

(iv) $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} = \frac{2\Delta}{bc}$

6. Area of Triangle (Δ) :

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \sqrt{s(s-a)(s-b)(s-c)}$$

BINOMIAL THEOREM

1. Statement of Binomial theorem : If a, b $\in$ R and n $\in$ N, then

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \ldots + {}^nC_r\, a^{n-r} b^r + \ldots + {}^nC_n\, a^0 b^n$$

$$= \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

2. Properties of Binomial Theorem :

(i) **General term :** $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

(ii) **Middle term (s) :**

(a) If n is even, there is only one middle term,

which is $\left(\frac{n+2}{2}\right)$th term.

(b) If n is odd, there are two middle terms,

which are $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+1}{2}+1\right)$th terms.

5. Properties of Binomial Coefficients :

(i) $^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots.. + {}^nC_n = 2^n$

(ii) $^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots\ldots\ldots\ldots + (-1)^n\, {}^nC_n = 0$

(iii) $^nC_0 + {}^nC_2 + {}^nC_4 + \ldots. = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots. = 2^{n-1}$

(iv) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (v) $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$

6. Binomial Theorem For Negative Integer Or Fractional Indices

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots. +$$

$$\frac{n(n-1)(n-2)\ldots\ldots.(n-r+1)}{r!}x^r + \ldots.,|x| < 1.$$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)}{r!}x^r$$

📌 7th Chapter — Determinant Properties

1. **Interchanging two rows/columns:**

   ⇒ Changes the sign of the determinant.

2. **If two rows/columns are identical:**

   ⇒ Determinant = 0.

3. **Multiplying a row/column by a constant:**

   ⇒ Determinant gets multiplied by that constant.

4. **Adding/subtracting a multiple of one row/column to another:**

   ⇒ Determinant remains unchanged.

5. **Determinant of a triangular matrix:**

   ⇒ Product of diagonal elements.

🔥 Logarithm Properties

1. $\log_a(1) = 0$
2. $\log_a(a) = 1$
3. $\log_a(xy) = \log_a(x) + \log_a(y)$
4. $\log_a(x/y) = \log_a(x) - \log_a(y)$
5. $\log_a(x^n) = n \times \log_a(x)$
6. Change of base: $\log_a(b) = \log(b) / \log(a)$

🔥 Exponential Properties

1. $a^0 = 1$ $(a \neq 0)$
2. $a^1 = a$
3. $a^m \times a^n = a^{m+n}$
4. $a^m / a^n = a^{m-n}$
5. $(a^m)^n = a^{mn}$
6. $(ab)^n = a^n \times b^n$

🔥 Partial Fractions

For **proper fractions**:

- Linear factors: A/(x - a) + B/(x - b)
- Repeated linear factors: A/(x - a) + B/$(x - a)^2$ + ...
- Quadratic factors: (Ax + B)/(x^2 + px + q)

For **improper fractions**:

- Divide first, then apply partial fractions.

🔥 Binomial Expansion

For $(a + b)^n$:

$C_0a^n + C_1a^{n-1}b + C_2a^{n-2}b^2 + \ldots + C_nb^n$

Where $C_r = n! / (r!(n - r)!)$

🔥 Logarithmic Expansion (log(1 + x))

$\log(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots \ (|x| < 1)$

🔥 Exponential Expansion (e^x)

$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$

(v) If the determinant is in **cyclic symmetric form** and if **m** is the difference between the degree of the product of the factors (obtained by substitution) and the degree of the product of the leading diagonal elements, then:

1. If **m** is **0**, the required factor is a **constant k**.
2. If **m** is **1**, the required factor is **k(a + b + c)**.
3. If **m** is **2**, the required factor is **$k(a^2 + b^2 + c^2) + l(ab + bc + ca)$**.