S.K.A. MATRIC. HR. SEC. SCHOOL,

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UNIT – 1 (IMPORTANT 5 MARKS)

1. In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

2. Check the relation $R = \{(1,1),(2,2),(3,3),\dots\dots\dots(n,n)\}$ defined on the set $S = \{1,2,3,\dots\dots\dots n\}$ for the three basic relations.

3. In the set Z of integers, define mRn if $m-n$ is a multiple of 12. Prove that R is an equivalence relation.

4. Let $f,g: R \to R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find $f \circ g$

5. Let $f,g: R \to R$ be defined as $f(x) = |x| + x$ and $g(x) = |x| - x$. Find $f \circ g$ and $g \circ f$

6. Write the values of f at $-4,1,-2,7,0$ if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

7. Write the values of f at $-3,5,2,-1,0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0,2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

8. If $f: R \to R$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

9. If $f: R \to R$ is defined by $f(x) = 3x - 5$ prove that f is a bijection and find its inverse.

2 MARK AND 3 MARKS

10. Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \le n \le 5, n \in N\}$.

11. If $n(A) = 10$ and $n(A \cap B) = 3$, find $n\left((A \cap B)' \cap A\right)$.

12. If $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.

13. If $\wp(A)$ denotes the power set of A, then find $n\left(\wp(\wp(\wp(\phi)))\right)$.

14. If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(P(A))$.

15. Justify the trueness of the statement :
 "An element of a set can never be a subset of itself.

16. If $n(P(A)) = 1024, n(A \cup B) = 15$ and $n(P(B)) = 32$, then find $n(A \cap B)$.

17. If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n(P(A \Delta B))$.

18. If $A \times A$ has 16 elements, $S = \{(a,b) \in A \times A : a < b\}$; $(-1,2)$ and $(0,1)$ are two elements of S, then, find the remaining elements of S.

19. Let $X = \{a,b,c,d\}$ and $R = \{(a,a),(b,b),(a,c)\}$. Write down the minimum number of ordered pairs to be included to R to make it
 (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

20. Let $X = \{a,b,c,\}$ and $R = \{(a,a),(b,b),(a,c)\}$. Write down the minimum number of ordered pairs to be included to R to make it
 (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

21. In the set Z of integers, define mRn if $m - n$ is divisible by 7. Prove that R is an equivalence relation.

22. Find the domain of $f(x) = \frac{1}{1 - 2\cos x}$.

23. Find the range of the function $f(x) = \frac{1}{1 - 3\cos x}$.

24. Find the domain of $\frac{1}{1 - 2\sin x}$.

25. Find the range of the function $\frac{1}{2\cos x - 1}$.

26. Find the largest possible domain of the real valued function $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.

27. Find the largest possible domain for the real valued function f defined by $f(x) = \sqrt{x^2 - 5x + 6}$.

28. Let $f = \{(1,2),(3,4),(2,2)\}$ and $g = \{(2,1),(3,1),(4,2)\}$. Find $g \circ f$ and $f \circ g$.

29. Let $f = \{(1,4),(2,5),(3,5)\}$ and $g = \{(4,1),(5,2),(6,4)\}$. Find $g \circ f$. Can you find $f \circ g$?

30. Let f and g be the two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 4$ and $g(x) = x^2 + 3$. Find $g \circ f$ and $f \circ g$.

UNIT – 2 BASIC ALGEBRA

5 MARKS

1. Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$.
2. If one root of $k(x-1)^2 = 5x-7$ is double the other root, show that $k = 2$ or -25.
3. Solve the equation $\sqrt{6-4x-x^2} = x+4$.
4. If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.
5. Prove : $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$.
6. If $a^2+b^2 = 7ab$, show that $\log\frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$.
7. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that $xyz = 1$.
8. Resolve into partial fractions : $\frac{x^2+x+1}{x^2-5x+6}$
9. Resolve into partial fractions : $\frac{2x}{(x^2+1)(x-1)}$
10. Resolve into partial fractions : $\frac{x+1}{x^2(x-1)}$.

3 MARKS

11. Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$.
12. Solve : $2x^2+x-15 \le 0$.
13. Simplify : $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.
14. Prove $\log a + \log a^2 + \log a^3 + \cdots + \log a^n = \frac{n(n+1)}{2}\log a$.
15. If $\left(x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right)^2 = \frac{9}{2}$, then find the value of $\left(x^{\frac{1}{2}} - x^{\frac{-1}{2}}\right)$ for $x > 1$.
16. Solve : $\frac{x+1}{x+3} < 3$.
17. Find the number of solutions of $x^2 + |x-1| = 1$.
18. Solve : $|2x-3| = |x-5|$.

2 MARKS

19. Solve : $|x-9| < 2$ for x.
20. Solve : $3|x-2| + 7 = 19$ for x.
21. Solve : $23x < 100$ when (i) x is a natural number, (ii) x is an integer.
22. Construct a quadratic equation with roots 7 and -3.
23. Resolve into partial fractions $\frac{1}{x^2-a^2}$
24. Evaluate : $\left(\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right)^3$.
25. Find the radius of the spherical tank whose volume is $\frac{32\pi}{3}$ units.
26. Prove : $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = 0$.
27. Find the square root of $7-4\sqrt{3}$.
28. Find a positive number smaller than $\frac{1}{2^{1000}}$. Justify.

UNIT – 7 MATRICES AND DETERMINANTS

5 MARKS

1. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.
2. Express the following matrices as the sum of a symmetric and a skew-symmetric matrices:

   (i) $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$ and (ii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.
3. Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.
4. Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$.
5. Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a)$.
6. Using Factor Theorem , Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.
7. Prove that $\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr\,(p+q+r)^3$.
8. Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)\,(a-b)\,(b-c)\,(c-a)$.
9. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)\,(y-z)\,(z-x)\,(xy+yz+zx)$.

10. Solve $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0.$

11. Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$

12. In a triangle ABC, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A(1+\sin A) & \sin B(1+\sin B) & \sin C(1+\sin C) \end{vmatrix} = 0,$$

Prove that ΔABC is an isoceles triangle.

13. Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2.$

14. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}.$

15. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$

16. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$

2 MARKS

17. Construct a 2×3 matrix whose $(i, j)^{th}$ elements is given by

$$a_{ij} = \frac{\sqrt{3}}{2}|2i-3j| \quad (1 \le i \le 2, \quad 1 \le j \le 3).$$

18. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

(i) $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$ (ii) $a_{ij} = \frac{|3i-4j|}{4}$ with $m = 3, n = 4$

19. Find the sum $A + B + C$ if A, B, C are given by

$$A = \begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix}, B = \begin{bmatrix} \cos^2\theta & 0 \\ -\mathrm{cosec}^2\theta & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

20. Simplify :

$$\sec\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix} - \tan\theta \begin{bmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{bmatrix}.$$

21. Determine the value of $x + y$ if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}.$

22. Find $|A|$ if $A = \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}.$

23. Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}.$

24. Find the value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0.$

25. If A is square matrix and $|A| = 2$, find the value of $|AA^T|$.

26. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, find the value of $|3AB|$.

27. If the area of the triangle with vertices $(-3,0), (3,0)$ and $(0, k)$ is 9 square units, find the values of k.

28. Find the area of the triangle whose vertices are $(-2,-3), (3,2)$ and $(-1,-8)$.

29. Show that the points $(a, b+c), (b, c+a)$ and $(c, a+b)$ are collinear.

30. If $(k, 2), (2,4)$ and $(3,2)$ are vertices of the triangle of area 4 square units then determine the value of k.

31. Determine the values of a and b so that the following matrices are singular.

$$B = \begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$$

3 MARKS

32. Verify the property $A(B + C) = AB + AC$, when the matrices A, B, C are given by

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}.$$

33. Without expanding the determinants, show that $|B| = 2|A|$.

Where $B = \begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix}$ and $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}.$

34. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0.$

35. Show that $\begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix} = 0.$

36. Prove that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0.$

37. Without expanding, evaluate the following determinants :

(i) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ (ii) $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

38. Verify that $|AB| = |A|\,|B|$ if $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

39. Show that $\begin{vmatrix} 0 & c & b^2 \\ c & 0 & a \\ b & a & 0 \end{vmatrix} = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ab & bc & a^2+b^2 \end{vmatrix}$.

40. Find the value of the product $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$.

UNIT – 8 VECTOR ALGEBRA

5 MARKS

1. Prove that the medians of the triangle are concurrent.

2. If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$.

3. Prove that the points whose position vectors $2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}, 4\hat{\imath} + \hat{\jmath} + 9\hat{k}$ and $10\hat{\imath} - \hat{\jmath} + 6\hat{k}$ form a right angled triangle.

4. Show that the points whose position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}, -\hat{\jmath} - \hat{k}, 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ are coplanar.

5. If $|\vec{a}| = 5$, $|\vec{b}| = 6$, $|\vec{c}| = 7$ and if $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

6. If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ (iii) $\tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$

7. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

8. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3, |\vec{c}| = 4$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a}$

9. Find the cosine and sine angle between the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

10. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|$. Also reduce the condition for collinearity of the points A, B and C.

3 MARKS & 2 MARKS

11. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

12. Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

13. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.

14. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.

15. Find a unit vector parallel along the vector $5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

16. Find the value or values of m for which $m(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector.

17. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ prove that $\vec{a}$ and $\vec{b}$ are perpendicular.

18. Find the angle between the vectors $5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $6\hat{\imath} - 8\hat{\jmath} - \hat{k}$.

19. Find the area of the parallelogram whose two adjacent sides are determined by the vectors $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

20. For any vector $\vec{a}$ prove that $|\vec{a}\times\hat{\imath}|^2 + |\vec{a}\times\hat{\jmath}|^2 + |\vec{a}\times\hat{k}|^2 = 2|\vec{a}|^2$.

UNIT – 5 – BINOMIAL THEOREM, SEQUENCE AND SERIES

5 MARKS

1. Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

2. Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

3. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

4. Find $\sqrt[3]{65}$.

5. Find $\sqrt[3]{1001}$ approximately (two decimal places).

6. Find $\sqrt[3]{126}$ approximately(three decimal places)

7. Compute the sum of first n terms of the following series: $8 + 88 + 888 + 8888 + \cdots$

8. Compute the sum of first n terms of the following series: $6 + 66 + 666 + 6666 + \cdots$

9. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

10. The 2nd, 3rd and 4th terms in the binomial expansion of $(x+a)^n$ are 240, 720 and 1080 for a suitable value of x. Find x, a and n.

3 MARKS

11. Find the coefficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$.

12. Find the middle terms in the expansion of $(x+y)^7$.

13. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

14. If $a_1, a_2, a_3, \dots, a_n$ is a geometric progression, every term $a_k (k > 1)$ is the geometric mean of its immediate predecessor a_{k-1} and immediate successor a_{k+1}

15. Find the value of n, if the sum to n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\cdots$ is $435\sqrt{3}$.

16. Expand: $(x+2)^{-\frac{2}{3}}$ in powers of x

17. Expand $(1+x)^{\frac{2}{3}}$ upto four terms for $|x| < 1$.

2 MARKS

18. Find the middle term in the expansion of $(x+y)^6$.

19. Write the first 6 terms of the sequences whose n^{th} term a_n is given below:

$$a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

20. Write the first 6 terms of the sequences whose n^{th} term a_n is given below:

$$a_n = \begin{cases} n & \text{if } n \text{ is } 1,2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$$

21. Write the first 6 terms of the sequences whose n^{th} term a_n is given below:

$$a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

22. Find the coefficient of x^5 in $\left(x+\frac{1}{x^3}\right)^{17}$

23. Find the sum: $1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\cdots$.

24. Compute : 99^4

25. Find the expansion of $(2x+3)^5$..

UNIT – 4 COMBINATORICS AND MATHEMATICAL INDUCTION

1. Evaluate the following : (i) $10C_3$ (ii) $15C_{13}$ (iii) $100C_{99}$ (iv) $50C_{50}$.

   Evaluate : (i) $4P_4$ (ii) $5P_3$ (iii) $8P_4$ (iv) $6P_5$.

2. If $\frac{1}{7!}+\frac{1}{8!}=\frac{A}{9!}$ then find the value of A.

3. If $nC_4 = 495$, what is n?

4. Find the value of $\frac{8!}{5!\times 2!}$

5. Find the total number of outcomes when 5 coins are tossed once.

6. How many strings of length 5 can be formed out of the letters of the word PRIME taking all the letters at a time without repetition?

7. If $10P_r = {}^7P_{r+2}$ find r.

8. A polygon has 90 diagonals. Find the number of its sides?

9. Find the number of ways of arranging the letters of the word BANANA.

10. In how many ways 5 different balls be distributed among 3 boxes?

11. In how many ways 3 different balls be distributed among 5 boxes?

12. There are 10 bulbs in a room. Each one of them can be operated independently. Find the number of ways in which the room can be illuminated.

13. If $\frac{6!}{n!} = 6$, then find the value of n.

14. $\frac{1}{8!}+\frac{1}{9!}=\frac{n}{10!}$ find n?

15. If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in this list before the first word starting with B?

3 MARKS

16. Find the number of ways of arranging the letters of the word MATHEMATICS.

17. Find the number of ways of arranging the letters of the word ACCESSIBILITY

18. Find the distinct permutations of the letters of the word MISSISSIPPI?

19. Prove that $10C_2 + 2\times{}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$.

20 How many triangles can be formed by 15 points, in which 7 of them lie on one line and the remaining 8 on another parallel line?

21. If $nP_r = 720$, and $nC_r = 120$, find n, r.

22. Prove that $\frac{(2n)!}{n!} = 2^n(1.3.5\cdots(2n-1))$.

23. Count the number of positive integers greater than 7000 and less than 8000 which are divisible by 5, provided that no digits are repeated.

24. If $(n+1)C_8 : {}^{(n-3)}P_4 = 57:16$, find the value of n.

25. How many three-digit odd numbers can be formed by using the digits 0,1,2,3,4,5? If

(i) the repetition of digits is not allowed (ii)the repetition of digits is allowed

26. In how many ways 5 boys and 4 girls be seated in a row so that no two girls are together?

5 MARKS

27. By the principle of mathematical induction, prove that, for all integers $n \geq 1$.

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

28. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

29. Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

30. Prove that for any natural number $n, a^n - b^n$ is divisible by $a - b$, where $a > b$.

31. By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

32. By the principle of Mathematical induction, prove that, for $n \geq 1$.

$$1.2 + 2.3 + 3.4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

33. Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots + \frac{1}{n.(n+1).(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$

34. Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

35. Using the Mathematical induction, show that for any natural number n,

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}.$$

36. Using Mathematical induction, show that for any natural number $n \geq 2$,

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

37. In how many ways 4 mathematics books, 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together?

UNIT – 12 – INTRODUCTION TO PROBABILITY THEORY

5 MARKS

1. A die is rolled. If it shows an odd number, then find the probability of getting 5.

2. X speaks truth in70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

3. A main road in a City has 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the probability of 0.4 and 0.6 respectively. Determine the probability of

(i) a car crossing the first crossroad without stopping

(ii) a car crossing first two crossroads without stopping

(iii) a car crossing all the crossroads, stopping at third cross.

(iv) a car crossing all the crossroads, stopping at exactly one cross.

4. A factory has two machines I and II. Machine-I produces 40% of items of the Output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item.

5. A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work. Which engineer would you guess did the work?

6. The chances of X,Y and Z becoming managers of a certain company are 4:2:3.Theprobabilities that bonus scheme will be introduced if X,Y and Z become managers are 0.3,0.5and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

7. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions

(i) what is the probability that the firm will get a car in good condition?

(ii) if a car is in good condition, what is probability that it has come from agency N?

8. There are two identical Urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An Urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

9. A firm manufactures PVC pipes in three plants viz, *X*, *Y* and *Z*. The daily production volumes from the three firms *X*, *Y* and *Z* are respectively 2000 units, 3000 units and 5000 units respectively. It is known from the past experience that 3% of the output from plant *X*, 4% from plant *Y* and 2% from plant *Z* are defective. A pipe is selected at random from a day's total production,

(i) find the probability that the selected pipe is a defective one.

(ii) if the selected pipe is a defective, then what is the probability that it was produced by plant *Y*?

10. The chances of *A*,*B* and *C* becoming manager of a certain company are 5:3:2. The probabilities that the office canteen will be improved if *A*, *B* and *C* become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that *B* was appointed as the manager?

11. Suppose the chances of hitting a target by a person *X* is 3 times in 4 shots, by *Y* is 4 times in 5 shots, and by *Z* is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

2 MARKS

12. An integer is chosen at random from the first ten positive integers. Find the probability that it is (i) an even number (ii) multiple of three.

13. Three coins are tossed simultaneously, what is the probability of getting (i) exactly one head (ii) at least one head (iii) atmost one head.

14. (i) The odds that the event A occurs is 5 to 7. Find *P*(*A*)

(ii) Suppose $P(B) = \frac{2}{5}$. Express the odds that event *B* occurs.

15. A man has 2 ten rupee notes, 4 hundred rupee notes and 6 five hundred rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denomination and also its probability?

16. Given that *P*(*A*) = 0.52, *P*(*B*) = 0.43 and $P(A \cap B)$ = 0.24, find

(i) $P(A \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$

3 MARKS

17. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then the probability of

(i) $P(A \cup B)$ (ii) $P(A \cap \bar{B})$ (iii) $P(\bar{A} \cap B)$

18. If $P(A) = 0.5$, $P(B) = 0.8$ and $P(B/A) = 0.8$, Find $P(A/B)$ and $P(A \cup B)$

19. $P(A) = 0.52$, $P(B) = 0.43$, $P(A \cap B) = 0.24$

(i) $P(A \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$

UNIT – 6 TWO DIMENSIONAL ANALYTICAL GEOMETRY

5 MARKS

1. Write Five Different types of equation of straight lines.

2. If *Q* is a point on the locus of $x^2 + y^2 + 4x - 3y + 7 = 0$, then find the equation of locus of *P* which divides segment *OQ* externally in the ratio 3:4, where *O* is origin.

3. Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form :
(i) Slope and Intercept form (ii) Intercept form (iii) Normal form

4. Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form.

5. Find the points on the line $x + y = 5$, that lie at a distance 2 units from the line $4x + 3y - 12 = 0$.

6. If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines. $x \sec\theta + y \operatorname{cosec}\theta = 2a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$, then prove that $p_1^2 + p_2^2 = a^2$.

7. If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, find

(i) the value of λ and the separate equations of the lines

(ii) point of intersection of the lines (iii) angle between the lines.

8. If one of the straight lines of $ax^2 + 2hxy + by^2 = 0$ is perpendicular to $px + qy = 0$, then show that $ap^2 + 2hpq + bq^2 = 0$.

9. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $tan^{-1}(5)$.

10. Find the separate equation of the following pair of straight lines

(i) $3x^2 + 2xy - y^2 = 0$

(ii) $6(x-1)^2 + 5(x-1)(y-2) - 4(y-2)^2 = 0$

(iii) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$

11. (i) The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.
(ii) The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

12. Find the value of k, if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting,
$12x^2 + 7xy - 12y^2 - x + 7y + k = 0$.

13. For what value of k does the equation
$12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines.

14. Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

15. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a+b)^2 = 4h^2$.

3 MARKS

16. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a\cos^3\theta, y = a\sin^3\theta$.

17. Show the points $\left(0, -\frac{3}{2}\right)$, $(1,-1)$ and $\left(2, -\frac{1}{2}\right)$ are collinear.

18. Find the equations of the straight lines, making the y-intercept of 7 and angle between the line and the y-axis is 30°.

19. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of 45° with positive the x-axis.

20. Find the equation of the lines passing through the point (1,1)
(i) with y-intercept (-4)
(ii) with slope 3
(iii) and $(-2,3)$
(iv) and the perpendicular from the origin makes an angle 60° with x-axis

21. Show that the points $(1,3)$, $(2,1)$ and $\left(\frac{1}{2}, 4\right)$ are collinear, by using
(i) concept of slope(ii)using a straight line and (iii) any other method

22. Find the distance
(i) between two points $(5,4)$ and $(2,0)$
(ii) from a point $(1,2)$ to the line $5x + 12y - 3 = 0$

23. Find the distancebetween two parallel lines $3x + 4y = 12$ and $6x + 8y + 1 = 0$

24. Find the distance between the parallel lines
(i) $12x + 5y = 7$ and $12x + 5y + 7 = 0$
(ii)$3x - 4y + 5 = 0$ and $6x - 8y - 15 = 0$

25. If the line joining two points $A(2,0)$ and $B(3,1)$ is rotated about A in anticlockwise direction through an angle of 15°, then find the equation of the line in new position.

2 MARKS

26. Find the locus of P, if for all values of α, the co-ordinates of a moving point P is (i) $(9\cos\alpha, 9\sin\alpha)$ (ii) $(9\cos\alpha, 6\sin\alpha)$

27. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x-axis. Find the equation of the line.

28. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

29. Write the equation of the lines through the point $(1,-1)$
(i) parallel to $x + 3y - 4 = 0$ (ii) perpendicular to $3x + 4y = 6$

30. Separate the equations $5x^2 + 6xy + y^2 = 0$.

31. If exists, find the straight lines by separating the equations
$2x^2 + 2xy + y^2 = 0$.

CHAPTER 11 - INTEGRAL CALCULUS

2 MARKS & 3 MARKS

1. Integrate the following with respect to x : $\frac{1}{x^7}$

2. Evaluate: $\int \sqrt[3]{x^4}\, dx$

3. Integrate the following with respect to x: $(1+x^2)^{-1}$

4. Evaluate the following with respect to $\int (4x+5)^6\, dx$

5. Evaluate the following with respect to $\boldsymbol{x}$: $\int \operatorname{cosec}(ax+b)\cot(ax+b)dx$

6. Evaluate the following with respect to $\boldsymbol{x}$: $\int \sec(2-15x)\tan(2-15x)\, dx$

7. Evaluate the following integrals :
$\frac{12}{(4x-5)^3} + \frac{6}{3x+2} + 16e^{4x+3}$

8. If $f'(x) = 3x^2 - 4x + 5$ and $f(1) = 3$, then find $f(x)$.

9. Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x} dx$.

10. Integrate the following with respect to x.

$$\int \frac{2x+4}{x^2+4x+6}dx$$

11. Integrate the following with respect to x.

$$\int \frac{e^x}{e^x-1}dx$$

12. Integrate the following with respect to x.

$$\int \frac{1}{x\log x}dx$$

13. Integrate the following with respect to x.

$$\int \frac{\sin x+\cos x}{\sin x-\cos x}dx$$

14. Evaluate the following integrals : $\int xe^x dx$

5 MARKS

15. Evaluate: $\int x\cos x\,dx$
16. Evaluate: $\int x\log x\;dx$
17. Evaluate: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\;dx$
18. Evaluate the following integrals: $\int \frac{1}{x^2-2x+5}dx$
19. Evaluate the following integrals: $\int \frac{1}{\sqrt{x^2+12x+11}}dx$
20. Evaluate the following integrals: $\int \frac{1}{\sqrt{12+4x-x^2}}dx$
21. Evaluate the following integrals: $\int \frac{1}{\sqrt{x^2-4x+5}}dx$
22. Evaluate the following integrals :$\int \frac{3x+5}{x^2+4x+7}dx$
23. Evaluate the following integrals: $\int \frac{x+1}{x^2-3x+1}dx$
24. Evaluate the following integrals: $\int \frac{5x-2}{2+2x+x^2}dx$
25. Evaluate the following integrals: $\int \frac{2x+1}{\sqrt{9+4x-x^2}}\;dx$
26. Evaluate :$\int (x^2+x+1)dx$

CHAPTER 10 – DIFFERENTIAL CALCULUS – DIFFERENTIABILITY AND DIFFERENTIATION

5 MARK QUESTIONS

1. If $y = e^{\tan^{-1}x}$, show that $(1+x^2)y''+(2x-1)y'=0$
2. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2-3xy_1-y=0$.
3. If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ then prove that at $\theta=\frac{\pi}{2}, y''=\frac{1}{a}$.
4. If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$, $a\neq n\pi$.
5. If $y=(\cos^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$. Hence find y_2 when $x=0$
6. Find the derivative of x^x with respect to $x\log x$.
7. Differentiate : $y=\frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$.
8. Find y'' if $x^4+y^4=16$.
9. Find the second order derivative if x and y are given by $x=a\cos t, y=a\sin t$.
10. Find $\frac{d^2y}{dx^2}$ if $x^2+y^2=4$.

3 MARK QUESTIONS

11. Differentiate: $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
12. If $y=\tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'.
13. Find $\frac{dy}{dx}$ if $x^2+y^2=1$.
14. Differentiate: $y=x\sin x$
15. Differentiate $y=x\,e^x\log x$ with respect to x.
16. Differentiate: $y=\frac{\cos x}{x^3}$
17. Find $\frac{dy}{dx}$ if $x=a(t-\sin t), y=a(1-\cos t)$.
18. Differentiate $y=x^{\sqrt{x}}$.
19. Differentiate $y=x^{\cos x}$
20. If) $=x\cos x$, find $f''(x)$

2 MARK QUESTIONS:

21. Find $\frac{dy}{dx}$ if $x=at^2; y=2at, t\neq 0$.
22. Find y', y'' and y''' if $y=x^3-6x^2-5x+3$.
23. Differentiate: $y=x-3\sin x$

24. Differentiate: $y = \sqrt{x + \sqrt{x}}$
25. Differentiate: $y = e^x \sin x$
26. Find the slope of tangent line to the graph of $f(x) = -5x^2 + 7x$ at $(5, f(5))$.
27. Differentiate : $y = (x^3 - 1)^{100}$.
28. Differentiate 2^x.
29. Differentiate : $y = e^{\sin x}$.
30. Find $f'(x)$ if $f(x) = cos^{-1}(4x^3 - 3x)$.

CHAPTER – 9 : LIMITS AND CONTINUITY

5 MARK QUESTIONS

1. Prove that: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$
2. Calculate $\lim_{x\to 3} \frac{(x^2-6x+5)}{x^3-8x+7}$.
3. Calculate: $\lim_{x \to 0} \frac{\tan 2x}{\sin 5x}$
4. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} 4x+5, & \text{if } x \le 3 \\ 4x-5, & \text{if } x > 3 \end{cases}$$

5. Does the limits of following functions exist as $x \to 0$? State reasons for your answer. $\frac{\sin(x - \lfloor x \rfloor)}{x - \lfloor x \rfloor}$.
6. (i) Does the limits $\frac{\sin|x|}{x}$ exist as $x \to 0$?
   (ii) Evaluate: $\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{x}$
7. Show that $\lim_{x\to\infty^+} x\left[\left\lfloor\frac{1}{x}\right\rfloor + \left\lfloor\frac{2}{x}\right\rfloor + \cdots + \left\lfloor\frac{15}{x}\right\rfloor\right] = 120$.
8. Evaluate: $\lim_{x\to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$
9. Find the constant b that makes g continuous on $(-\infty, \infty)$. $g(x) = \begin{cases} x^2 - b^2 & \text{if } x < 4 \\ bx + 20 & \text{if } x \ge 4 \end{cases}$
10. A function f is defined as follows :

$$f(x) = \begin{cases} 0 & \text{for } x < 0; \\ x & \text{for } 0 \le x < 1; \\ -x^2 + 4x - 2 & \text{for } 1 \le x < 3; \\ 4 - x & \text{for } x \ge 3 \end{cases}$$

Is the function continuous?

11. For what value of α is this function $f(x) = \begin{cases} \frac{x^4-1}{x-1}, & \text{if } x \ne 1 \\ \alpha, & \text{if } x = 1 \end{cases}$ continuous at $x = 1$?
12. Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x^2}-1}{x^2}$
13. Evaluate: $\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$
14. Show that: $\lim_{n\to\infty} \frac{1+2+3+\ldots+n}{3n^2+7n+2} = \frac{1}{6}$

2 MARKS

15. Consider the function $f(x) = \sqrt{x}, x \ge 0$. Does $\lim_{x\to 0} f(x)$ exist?
16. Calculate: $\lim_{x\to 3} \frac{x^2-81}{\sqrt{x}-3}$
17. Examine the continuity of $f(x) = \frac{x^2-16}{x+4}$
18. Calculate $\lim_{x\to 0} |x|$.
19. Find the positive integer n so that $\lim_{x\to 3} \frac{x^n - 3^n}{x-3} = 27$.
20. Evaluate $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$.

CHAPTER 3 – TRIGONOMETRY

5 MARKS

1. State and Prove Napier's Formula.
2. Derive Projection formula from (i) Law of sines, (ii) Law of cosines.
3. Prove that $\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\operatorname{cosec}(360°+\theta)} = \cos^2\theta \cot\theta$.
4. Find the value of (i) $\cos 15°$ (ii) $\tan 165°$
5. If $\sin x = \frac{15}{17}$ and $\cos y = \frac{12}{13}, 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2}$, find the value of
   (i) $\sin(x + y)$ (ii) $\cos(x - y)$
6. Prove that
   (i) $\cos(30° + x) = \frac{\sqrt{3}\cos x - \sin x}{2}$ (ii) $\cos(\pi + \theta) = -\cos\theta$
   (iii) $\sin(\pi + \theta) = -\sin\theta$.
7. Prove that $\sin 105° + \cos 105° = \cos 45°$.
8. Show that
   (i) $\tan(45° + A) = \frac{1+\tan A}{1-\tan A}$ (ii) $\tan(45° - A) = \frac{1-\tan A}{1+\tan A}$.

9. If θ is an acute angle, then find

(i) $\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$, when $\sin\theta=\frac{1}{25}$ (ii) $\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$, when $\sin\theta=\frac{8}{9}$

10. If $A+B=45°$, show that $(1+\tan A)(1+\tan B)=2$.

11. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2\tan 2\theta$.

12. Show that $\cot\left(7\frac{1°}{2}\right)=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$.

13. If $A+B+C=180°$, prove that

(i) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

(ii) $\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(iii) $\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

(iv) $\sin^2 A+\sin^2 B-\sin^2 C=2\sin A\sin B\cos C$

(v) $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

(vi) $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(vii) $\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C)=4\sin A\sin B\sin C$.

2 MARK QUESTIONS

1. Express each of the following angles in radian measure :

(i) $30°$ (ii) $135°$ (iii) $-205°$ (iv) $150°$ (v) $330°$

2. Find the degree measure corresponding to the following radian measures

(i) $\frac{\pi}{3}$ (ii) $\frac{\pi}{9}$ (iii) $\frac{2\pi}{5}$ (iv) $\frac{7\pi}{3}$ (v) $\frac{10\pi}{9}$

3. Find the values of

(i) $\sin(480°)$ (ii) $\sin(-1110°)$ (iii) $\cos(300°)$ (iv) $\tan(1050°)$

(v) $\cot(660°)$ (vi) $\tan\left(\frac{19\pi}{3}\right)$ (vii) $\sin\left(-\frac{11\pi}{3}\right)$.

4. Express each of the following as a sum or difference:

(i) $\sin 35°\cos 28°$ (ii) $\sin 4x\cos 2x$ (iii) $2\sin 10\theta\cos 2\theta$

(iv) $\cos 5\theta\cos 2\theta$ (v) $\sin 5\theta\sin 4\theta$

5. Express each of the following as a product:

(i) $\sin 75°-\sin 35°$ (ii) $\cos 65°+\cos 15°$ (iii) $\sin 50°+\sin 40°$

(iv) $\cos 35°-\cos 75°$

6. Find the principal solution and general solutions of the following:

(i) $\sin\theta=-\frac{1}{\sqrt{2}}$ (ii) $\cot\theta=\sqrt{3}$ (iii) $\tan\theta=-\frac{1}{\sqrt{3}}$

3 MARKS

1. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$.

2. Prove that $(\sec A-\operatorname{cosec} A)(1+\tan A+\cot A)=\tan A\sec A-\cot A\operatorname{cosec} A$.

3. Find the values of other five trigonometric functions for the following:

(i) $\cos\theta=-\frac{1}{2}$, θ lies in the III quadrant

(ii) $\cos\theta=\frac{2}{3}$, θ lies in the I quadrant

(iii) $\sin\theta=-\frac{2}{3}$, θ lies in the IV quadrant

(iv) $\tan\theta=-2$, θ lies in the II quadrant

(v) $\sec\theta=\frac{13}{5}$, θ lies in the IV quadrant

4. If $\theta+\phi=\alpha$ and $\tan\theta=k\tan\phi$, then prove that $\sin(\theta-\phi)=\frac{k-1}{k+1}\sin\alpha$.

5. Prove that $\frac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x}=\tan 4x$.

6. Prove that $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x}=\tan 3x$.

7. Prove that $\frac{\sin(4A-2B)+\sin(4B-2A)}{\cos(4A-2B)+\cos(4B-2A)}=\tan(A+B)$.

8. If $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$, show that $\cos 3\theta=\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

9. A foot ball player can kick a football from ground level with an initial velocity of 80 ft/second. Find the maximum horizontal distance of the football travels and at what angle?

10. Show that $\cos 10°\cos 30°\cos 50°\cos 70°=\frac{3}{16}$.