# HIGHER SECONDARY SECOND YEAR PUBLIC EXAMINATION MARCH– 2025

## MATHEMATICS – ANSWER KEY

### PART-I

**Note: i) Answer all the questions.** **[$20 \times 1 = 20$]**

**ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer**

| | TYPE-A | | TYPE-B | |
|---|---|---|---|---|
| **1.** | **(a)** | N | **(b)** | $\frac{1}{\sqrt{2}}$ |
| **2.** | **(d)** | 2 | **(a)** | $\pi$ |
| **3.** | **(d)** | $\frac{1}{5}$ | **(d)** | 2xu |
| **4.** | **(d)** | 2.5 | **(a)** | N |
| **5.** | **(a)** | 1, 2 | **(d)** | $\frac{1}{5}$ |
| **6.** | **(a)** | 19 | **(a)** | $(1,0)$ |
| **7.** | **(b)** | $y = x^3 + 2$ | **(d)** | 2.5 |
| **8.** | **(b)** | [1,2] | **(b)** | $\frac{1}{f(x)} f'(x)\,dx$ |
| **9.** | **(d)** | 2xu | **(b)** | [1,2] |
| **10.** | **(b)** | 2 | **(d)** | 2 |
| **11.** | **(b)** | $\pm\frac{1}{\sqrt{2}}(1+i)$ | **(c)** | $\frac{\pi a^3}{6}$ |
| **12.** | **(b)** | $1 + i$ | **(a)** | 1, 2 |
| **13.** | **(d)** | 0.25 | **(b)** | $\frac{\pi}{6}$ |
| **14.** | **(c)** | $\frac{\pi}{2}$ | **(a)** | 19 |
| **15.** | **(a)** | $(1,0)$ | **(b)** | $1 + i$ |
| **16.** | **(b)** | $\frac{\pi}{6}$ | **(c)** | $\frac{\pi}{2}$ |
| **17.** | **(c)** | $\frac{\pi a^3}{6}$ | **(b)** | $\pm\frac{1}{\sqrt{2}}(1+i)$ |
| **18.** | **(b)** | $\frac{1}{\sqrt{2}}$ | **(b)** | $y = x^3 + 2$ |
| **19.** | **(a)** | $\pi$ | **(d)** | 0.25 |
| **20.** | **(b)** | $\frac{1}{f(x)} f'(x)\,dx$ | **(b)** | 2 |



### PART-II

**Note:** **[$7 \times 2 = 14$]**

(i) Answer any **SEVEN** questions

(ii) Question number **30** is compulsory.

**21.** If adj A $= \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $A^{-1}$.

Solution:

$$\text{W.K.T.} \quad A^{-1} = \pm \frac{1}{\sqrt{|\text{adj } A|}} \text{adj } A$$

$$|\text{adj } A| = -1(1-4) - 2(1-4) + 2(2-2) = 3 + 6 + 0 = 9 > 0$$

$$A^{-1} = \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}}$$

**22.** If $z = x + iy$, find $\text{Re}\left(\frac{1}{z}\right)$ in rectangular form.

Solution:

$$\text{Re}\left(\frac{1}{z}\right) = \text{Re}\left(\frac{1}{x+iy}\right) = \text{Re}\left(\frac{1}{x+iy} \times \frac{x-iy}{x-iy}\right) = \text{Re}\left(\frac{x - iy}{x^2+y^2}\right) = \frac{x}{x^2+y^2}$$

**23.** Find the value of $\tan^{-1}(-\sqrt{3})$.

Solution:

$$\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right) \qquad \because -\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$= -\frac{\pi}{3}$$



**24.** If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.

Solution:

$$\text{G.T.} \quad y = 4x + c \text{ and } x^2 + y^2 = 9$$

W.K.T. condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$

$$\text{Here } m = 4, \ a^2 = 9$$

$$c^2 = a^2(1+m^2)$$

$$c^2 = 9(1+4^2)$$

$$c^2 = 9(17)$$

$$c = \pm 3\sqrt{17}$$

**25.** Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2 - 6x + 7}{x + 5}$.

Solution:

Dividing $x^2 - 6x + 7$ by $x + 5$:

$$\begin{array}{r|l} & x - 11 \\ x+5 & x^2 - 6x + 7 \\ & x^2 + 5x \\ \hline & -11x + 7 \\ & -11x - 55 \\ \hline & 62 \end{array}$$

$\therefore y = x - 11$ is a slant asymptote

**26.** Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

Solution:

$$F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$$

$$F(\lambda x,\lambda y) = \frac{(\lambda x)^2+5(\lambda x)(\lambda y)-10(\lambda y)^2}{3(\lambda x)+7(\lambda y)} = \frac{\lambda^2x^2+\lambda^2 5xy-\lambda^2 10y^2}{\lambda(3x+7y)}$$

$$= \frac{\lambda^2(x^2+5xy-10y^2)}{\lambda(3x+7y)} = \lambda F(x,y)$$

$\therefore$ F(x, y) is homogeneous function of degree 1

**27.** Solve the differential equation $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Solution:

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}}dy = \frac{1}{\sqrt{1-x^2}}dx$$

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int \frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1} y = \sin^{-1} x + c$$



**28.** Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function of X.

Solution:

Given p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_1^4 Cx^2\,dx = 1$$

$$C\left[\frac{x^3}{3}\right]_1^4 = 1$$

$$C\left[\frac{64}{3} - \frac{1}{3}\right] = 1$$

$$C\left(\frac{63}{3}\right) = 1$$

$$C = \frac{1}{21}$$

**29.** Find a polynomial equation of minimum degree with rational coefficients having $i - 2$ as a root.

Solution:

$$\text{G.T.} -2 + i \text{ is a root} \implies -2 - i \text{ is also a root}$$

$$\text{Sum} = -2 + i - 2 - i = -4$$
$$\text{Product} = (-2+i)(-2-i) = (-2)^2 + (1)^2 = 4 + 1 = 5$$
$$\text{W.K.T.} \quad x^2 - (\text{sum})x + \text{product} = 0$$
i.e., $x^2 + 4x + 5 = 0$ is a minimum degree polynomial.

**30.** If $f(x) = \sin x$, then prove that $\int_0^{\pi} f(x)\,dx = 2\int_0^{\frac{\pi}{2}} f(x)\,dx$.

Solution:

$$\text{W.K.T} \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \qquad \text{If } f(2a-x) = f(x)$$

Take $2a = \pi$ and $f(x) = \sin x$

$$\therefore f(2a - x) = \sin(\pi - x) = \sin x = f(x)$$

$$\text{Hence} \quad \boxed{\int_0^{\pi} f(x)\,dx = 2\int_0^{\frac{\pi}{2}} f(x)\,dx, \quad \text{where } f(x) = \sin x}$$

## PART-III

**Note:** **[7 × 3 = 21]**

(i) Answer any **SEVEN** questions

(ii) Question number **40** is compulsory.

**31.** Solve the system of linear equations $2x + 5y = -2,\ x + 2y = -3$ by matrix inversion method.

Solution:

The matrix form of the system is $AX = B$



$$\text{Where } A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \end{bmatrix};\ B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$|A| = 4 - 5 = -1 \neq 0$$

$A^{-1}$ exists

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$$

The solution is,

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 15 \\ -2 + 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$$

$\therefore$ The solution is $x = -11,\ y = 4$

**32.** If $|z| = 2$, show that $8 \le |z + 6 + 8i| \le 12$.

Solution:

$$\text{W.K.T.} \ \big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2|$$

$$\big||z| - |6 + 8i|\big| \le |z + 6 + 8i| \le |z| + |6 + 8i|$$

$$\text{Since } |z| = 3$$

$$\left|2 - \sqrt{36 + 64}\right| \le |z + 6 + 8i| \le 2 + \sqrt{36 + 64}$$

$$|2-10| \le |z+6+8i| \le 2+10$$
$$|-8| \le |z+6+8i| \le 12$$
$$\therefore \boxed{8 \le |z+6+8i| \le 12}$$

**33.** Solve the equation $7x^3 - 43x^2 = 43x - 7$.

Solution:

$$\text{G.T.} \quad 7x^3 - 43x^2 = 43x - 7 \Rightarrow 7x^3 - 43x^2 - 43x + 7 = 0$$

This is the odd degree reciprocal equation of type I

$\therefore x = -1$ is a root

| | | | | |
|---|---|---|---|---|
| | 7 | −43 | −43 | 7 |
| −1 | 0 | −7 | 50 | −7 |
| | 7 | −50 | 7 | 0 |

$$\therefore 7x^2 - 50x + 7 = 0$$
$$\left(x - \frac{49}{7}\right)\left(x - \frac{1}{7}\right) = 0$$
$$(x-7)\left(x - \frac{1}{7}\right) = 0$$
$$x = 7\,;\, x = \frac{1}{7}$$

$$\text{Hence, the roots are } -1, 7 \text{ and } \frac{1}{7}$$

**34.** Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.



Solution:

$$\text{w.k.t} \quad \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\right)\left(\frac{7}{24}\right)}\right)$$
$$= \tan^{-1}\left(\frac{\frac{48+77}{264}}{\frac{264-14}{264}}\right)$$
$$= \tan^{-1}\left(\frac{125}{250}\right)$$
$$= \tan^{-1}\frac{1}{2}$$

**35.** If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

Solution:

$$[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$$
$$= \{1(1-0) - 0 + 1(1-1)\}[\vec{a}, \vec{b}, \vec{c}]$$
$$\therefore [\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$$

**36.** If $u(x,y) = x^2y + 3xy^4$, $x = e^t$ and $y = \sin t$, find $\frac{du}{dt}$.

Solution:

$$u(x,y) = x^2y + 3xy^4 \qquad x = e^t \qquad y = \sin t$$
$$\frac{\partial u}{\partial x} = 2xy + 3y^4 \qquad \frac{dx}{dt} = e^t \qquad \frac{dy}{dt} = \cos t$$
$$\frac{\partial u}{\partial y} = x^2 + 12xy^3$$
$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$
$$\frac{du}{dt} = (2xy + 3y^4)(e^t) + (x^2 + 12xy^3)(\cos t)$$
$$= (2(e^t)(\sin t) + 3(\sin t)^4)(e^t) + ((e^t)^2 + 12(e^t)(\sin t)^3)(\cos t)$$
$$= e^t(2e^t \sin t + 3\sin^4 t + e^t \cos t + 12\sin^3 t \cos t)$$

**37.** Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + 5\cos^2 x}$.

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 + 5\cos^2 x}$$

$\div$ by $\cos^2 x$ both Nr. and Dr.

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{\sec^2 x + 5} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{1 + \tan^2 x + 5} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x\, dx}{\tan^2 x + 6}$$



Put $t = \tan x$

$dt = \sec^2 x\, dx$

| x | 0 | $\pi/2$ |
|---|---|---|
| t | 0 | $\infty$ |

$$\therefore I = \int_0^{\infty} \frac{dt}{t^2 + 6} = \int_0^{\infty} \frac{dt}{(t)^2 + (\sqrt{6})^2} \qquad \because \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$
$$= \left[\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{t}{\sqrt{6}}\right)\right]_0^{\infty}$$
$$= \frac{1}{\sqrt{6}}[\tan^{-1}(\infty) - \tan^{-1}(0)]$$
$$= \frac{1}{\sqrt{6}}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2\sqrt{6}}$$
$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1 + 5\cos^2 x} = \frac{\pi}{2\sqrt{6}}$$

**38.** A lottery with 600 tickets gives one prize of ₹ 200, four prizes of ₹ 100, and six prizes of ₹ 50. If the ticket costs is ₹ 2, find the expected winning amount of a ticket.

Solution:

$$n(S) = 600$$

Let X be the random variable denotes the amount of winning.

i.e., $X = 200, 100, 50$ and $0$

$$f(200) = P(X = 200) = \frac{1}{600} \; ; \; f(100) = P(X = 100) = \frac{4}{600}$$
$$f(50) = P(X = 50) = \frac{6}{600} \; ; \; f(0) = P(X = 0) = \frac{589}{600}$$

probability mass function is

| X | 200 | 100 | 50 | 0 |
|---|---|---|---|---|
| f(x) | $\frac{1}{600}$ | $\frac{4}{600}$ | $\frac{6}{600}$ | $\frac{589}{600}$ |

$$E(x) = \sum_x x\,f(x) = 200\left(\frac{1}{600}\right) + 100\left(\frac{4}{600}\right) + 50\left(\frac{6}{600}\right) + 0\left(\frac{589}{600}\right)$$
$$= \frac{200 + 400 + 300 + 0}{600} = \frac{900}{600} = \frac{3}{2} = 1.50$$

Expected amount winning = ₹ 1.50

one ticket cost = ₹ 2.00

Profit = 1.50 − 2.00 = −0.50

∴ Loss = ₹ 0.50

**39.** Find the Taylor's series about x = 2 for $f(x) = x^3 + 2x + 1, (-\infty < x < \infty)$.

Solution:

| | |
|---|---|
| $f(x) = x^3 + 2x + 1$ | $f(2) = 2^3 + 2(2) + 1 = 13$ |
| $f'(x) = 3x^2 + 2$ | $f'(2) = 3(2)^2 + 2 = 14$ |
| $f''(x) = 6x$ | $f''(2) = 6(2) = 12$ |
| $f'''(x) = 6$ | $f'''(2) = 6$ |
| $f^{iv}(x) = 0$ | $f^{iv}(2) = 0$ |



$$f(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3 + \frac{f^{iv}(2)}{4!}(x-2)^4 + \cdots\infty$$

$$x^3 + 2x + 1 = 13 + \frac{14}{1!}(x-2) + \frac{12}{2!}(x-2)^2 + \frac{6}{3!}(x-2)^3 + 0$$
$$x^3 + 2x + 1 = 13 + 14(x-2) + 6(x-2)^2 + (x-2)^3$$

**40.** Let Q be the set all Rational numbers. If ∗ be binary operation defined on Q as $a * b = a + b - ab + 7$ and $\left(\frac{3}{2}\right) * m = \frac{87}{10}$, then find the value of m.

Solution:

$$\text{G.T.} \quad a * b = a + b - ab + 7$$
$$\left(\frac{3}{2}\right) * m = \frac{87}{10}$$
$$\frac{3}{2} + m - \left(\frac{3}{2}\right)m + 7 = \frac{87}{10}$$
$$m\left(1 - \frac{3}{2}\right) = \frac{87}{10} - \frac{3}{2} - 7$$
$$m\left(-\frac{1}{2}\right) = \frac{87 - 15 - 70}{10}$$
$$m\left(-\frac{1}{2}\right) = \frac{2}{10} \Rightarrow \boxed{m = -\frac{2}{5}}$$

## PART-IV

**ANSWER ALL QUESTIONS.** $[7 \times 5 = 35]$

**41. (a) Solve, by Cramer's rule, the system of equations $x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$.**

Solution:

$$x_1 - x_2 + 0x_3 = 3$$
$$2x_1 + 3x_2 + 4x_3 = 17$$
$$0x_1 + x_2 + 2x_3 = 7$$

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 1(6-4) + 1(4-0) + 0 = 2 + 4 = 6 \neq 0$$

$$\Delta_{x_1} = \begin{vmatrix} 3 & -1 & 0 \\ 17 & 3 & 4 \\ 7 & 1 & 2 \end{vmatrix} = 3(6-4) + 1(34-28) + 0 = 6 + 6 = 12$$

$$\Delta_{x_2} = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 17 & 4 \\ 0 & 7 & 2 \end{vmatrix} = 1(34-28) - 3(4-0) + 0 = 6 - 12 = -6$$

$$\Delta_{x_3} = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & 17 \\ 0 & 1 & 7 \end{vmatrix} = 1(21-17) + 1(14-0) + 3(2-0)$$
$$= 4 + 14 + 6 = 24$$

By Crammer's rule,



$$x_1 = \frac{\Delta_{x_1}}{\Delta} = \frac{12}{6} = 2$$
$$x_2 = \frac{\Delta_{x_2}}{\Delta} = \frac{-6}{6} = -1$$
$$x_3 = \frac{\Delta_{x_3}}{\Delta} = \frac{24}{6} = 4$$

Hence, The solution is $x_1 = 2,\ x_2 = -1,\ x_3 = 4$

**(OR)**

**41. (b) Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t$, $t \in \mathbb{R}$ at any point on the curve.**

Solution:

$$x = 7\cos t \quad ; \quad y = 2\sin t$$
$$\frac{dx}{dt} = 7(-\sin t) \quad ; \quad \frac{dy}{dt} = 2(\cos t)$$
$$\frac{dx}{dt} = -7\sin t \quad ; \quad \frac{dy}{dt} = 2\cos t$$
$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-7\sin t}$$
$$m = -\frac{2\cos t}{7\sin t}$$

∴ Equation of tangent at $(7\cos t, 2\sin t)$ is

$$y - 2\sin t = -\frac{2\cos t}{7\sin t}(x - 7\cos t)$$
$$(7\sin t)y - 14\sin^2 t = -(2\cos t)x + 14\cos^2 t$$
$$(2\cos t)x + (7\sin t)y = 14\cos^2 t + 14\sin^2 t$$
$$\boxed{(2\cos t)x + (7\sin t)y = 14}$$

∴ Equation of normal $(7\cos t, 2\sin t)$ is

$$y - 2\sin t = -\frac{1}{-\frac{2\cos t}{7\sin t}}(x - 7\cos t)$$
$$y - 2\sin t = \frac{7\sin t}{2\cos t}(x - 7\cos t)$$
$$(2\cos t)y - 4\sin t\cos t = (7\sin t)x - 49\sin t\cos t$$
$$49\sin t\cos t - 4\sin t\cos t = (7\sin t)x - (2\cos t)y$$
$$(7\sin t)x - (2\cos t)y = 45\sin t\cos t$$
$$\boxed{(7\sin t)x - (2\cos t)y = \frac{45}{2}(2\sin t\cos t) = \frac{45}{2}\sin 2t}$$

**42. (a) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.**

Solution:

$$\omega \text{ is a cube root of unity} \Longrightarrow \omega^3 = 1 \;\&\; 1 + \omega + \omega^2 = 0$$
$$(z-1)^3 + 8 = 0$$
$$(z-1)^3 = -8$$
$$(z-1) = (-8)^{\frac{1}{3}}(1)^{\frac{1}{3}}$$
$$(z-1) = -2(1)^{\frac{1}{3}}$$
$$(z-1) = -2(1, \omega, \omega^2)$$
$$(z-1) = -2\ ;\ (z-1) = -2\omega\ ;\ (z-1) = -2\omega^2$$
$$z = -2\ ;\ z = 1 - 2\omega\ ;\ z = 1 - 2\omega^2$$



**(OR)**

**42. (b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.**

Solution:

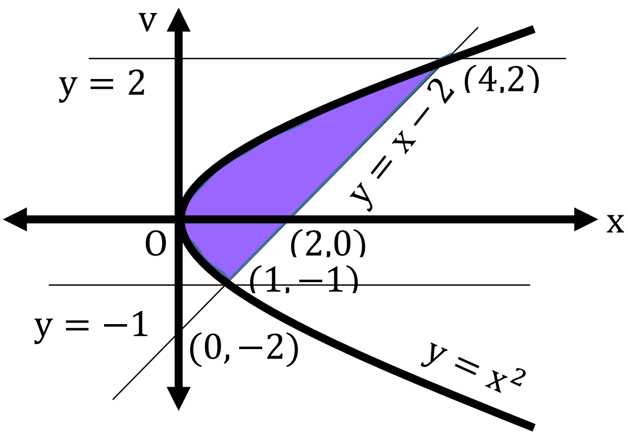
$$y^2 = x \rightarrow (1)$$
$$y = x - 2 \rightarrow (2)$$

Sub (1) and (2), we get

$$y = y^2 - 2$$
$$y^2 - y - 2 = 0$$
$$(y+1)(y-2) = 0$$
$$y = -1\ ;\ y = 2$$

when $y = -1$ ; $x = 1$

when $y = 2$ ; $x = 4$



$\therefore$ The point of intersection are $(1,-1)$ and $(4,2)$

The required area is

$$A = \int_c^d (x_R - x_L)\,dy$$

$$c = -1\ ;\ d = 2\ ;\ x_R = y + 2\ ;\ x_L = y^2$$

$$A = \int_{-1}^{2} (y + 2 - y^2)\,dx$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$$

$$= \left[\left(\frac{(2)^2}{2} + 2(2) - \frac{(2)^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right)\right]$$

$$= \left[\left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right]$$

$$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$$

$$= 8 - \frac{1}{2} - \frac{9}{3}$$

$$= 5 - \frac{1}{2} = \frac{9}{2}$$

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**43. (a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.**



Solution:

$$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0 \quad \rightarrow \quad (1)$$

is a even degree reciprocal equation of type I

Given that $\frac{1}{3}$ is a solution $\Longrightarrow$ 3 is also solution

$\therefore x = \frac{1}{3}$ and $x = 3$ are the roots

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{3}$ | 6 | −5 | −38 | −5 | 6 |
| | 0 | 2 | −1 | −13 | −6 |
| | 6 | −3 | −39 | −18 | 0 |
| 3 | 0 | 18 | 45 | 18 | |
| | 6 | 15 | 6 | 0 | |

$$\therefore 6x^2 + 15x + 6 = 0$$

$$\div 3 \Longrightarrow 2x^2 + 5y + 2 = 0$$

$$\left(x + \frac{4}{2}\right)\left(x + \frac{1}{2}\right) = 0$$

$$(x + 2)\left(x + \frac{1}{2}\right) = 0$$

$$x = -2,\ \ x = -\frac{1}{2}$$

Hence, the roots are $-2, -\frac{1}{2}, \frac{1}{3}$ and 3

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**(OR)**

**43. (b) Solve $(x^2 - 3y^2)dx + 2xydy = 0$.**

Solution:

$$(x^2 - 3y^2)dx + 2xydy = 0$$

$$\frac{dy}{dx} = \frac{3y}{2x} - \frac{x}{2y}$$

$$\text{Put } y = vx$$

$$v + x\frac{dv}{dx} = \frac{3v}{2} - \frac{1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\frac{2vdv}{v^2 - 1} = \frac{dx}{x}$$

$$\log|v^2 - 1| = \log|x| + \log|C|$$

Hence $|v^2 - 1| = |Cx|$, where C is an arbitrary constant

$$\left|\frac{y^2}{x^2} - 1\right| = |Cx|$$

$$|y^2 - x^2| = |Cx^3|$$

Hence, $y^2 - x^2 = \pm Cx^3$ (or) $y^2 - x^2 = kx^3$ gives the general solution.

**44. (a) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either side.**

Solution:



Let the vertex is A(0, 0) and the parabola is open down

then the equ. of parabola is $x^2 = -4ay \rightarrow (1)$

It passes through $(15, -10)$

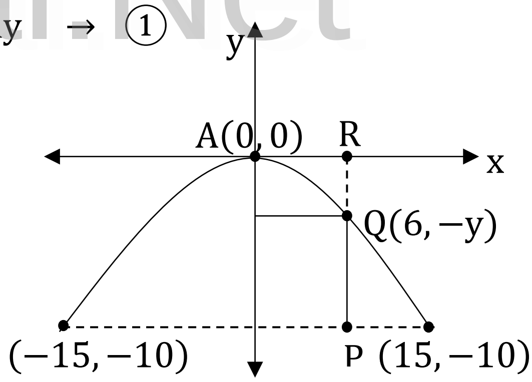
$$(15)^2 = -4a(-10)$$

$$15 \times 15 = 4a(10)$$

$$4a = \frac{45}{2} \Rightarrow a = \frac{45}{8}$$

Sub $4a = \frac{45}{2}$ in (1), we get

$$x^2 = -\frac{45}{2}y \rightarrow (2)$$




Since the point Q(6, −y) lies on parabola (2)

$$6^2 = -\frac{45}{2}(-y)$$

$$36 = \frac{45}{2}y \Rightarrow 12 = \frac{15}{2}y \Rightarrow y = \frac{24}{15} = 1.6$$

$$\therefore QR = 1.6 \text{ and } PR = 10$$

$$PR = PQ + QR$$

$$10 = PQ + 1.6$$

$$PQ = 10 - 1.6 = 8.4$$

$\therefore$ The req. hight of the arch at req. place is 8.4m

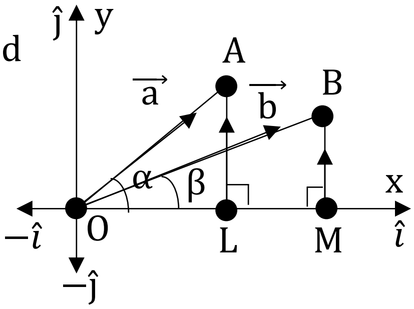
**(OR)**

**44. (b) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.**

Solution:

Let $\hat{a}=\overrightarrow{OA}$ and $\hat{b}=\overrightarrow{OB}$ be the unitvectors and which makes angle $\alpha$ and $\beta$ with positive $x-$axis respectively.

$\therefore$ The angle between $\hat{a}$ and $\hat{b}$ is $\alpha-\beta$




From the diagram

$$\cos\alpha=\frac{OL}{OA}=\frac{OL}{1}=OL \qquad \cos\beta=\frac{OM}{OB}=\frac{OM}{1}=OM$$

$$\sin\alpha=\frac{LA}{OA}=\frac{LA}{1}=LA \qquad \sin\beta=\frac{MB}{OB}=\frac{MB}{1}=MB$$

$$\hat{a}=\overrightarrow{OA}=\overrightarrow{OL}+\overrightarrow{LA} \qquad \hat{b}=\overrightarrow{OB}=\overrightarrow{OM}+\overrightarrow{MB}$$

$$=OL\hat{\imath}+LA\hat{\jmath} \qquad =OM\hat{\imath}+MB\hat{\jmath}$$

$$\hat{a}=\cos\alpha\,\hat{\imath}+\sin\alpha\,\hat{\jmath} \qquad \hat{b}=\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath}$$

$$\hat{a}\cdot\hat{b}=(\cos\alpha\,\hat{\imath}+\sin\alpha\,\hat{\jmath})\cdot(\cos\beta\,\hat{\imath}+\sin\beta\,\hat{\jmath})$$

$$\hat{a}\cdot\hat{b}=\cos\alpha\cos\beta+\sin\alpha\sin\beta \quad \rightarrow \quad (1)$$

$$\hat{a}\cdot\hat{b}=|\hat{a}||\hat{b}|\cos(\alpha-\beta)=(1)(1)\cos(\alpha-\beta)=\cos(\alpha-\beta)\rightarrow (2)$$

From (1) and (2), we get

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$



**45. (a) During war, 1 ship out of 9 was sunk on an average in making a certain voyage. What was the probability that:**

**(i) Exactly 3 out of a convoy of 6 ships would arrive safely?**

**(ii) No ships arrive safely from a convoy of 4 ships.**

Solution:

Let X denotes number of ships safely

$$\text{(i) } n=6\ ;\ q=\frac{1}{9}\ ;\ p=1-q=1-\frac{1}{9}=\frac{8}{9}$$

W. K. T. The binomial distribution is

$$f(x)={}^{n}C_x\,p^x q^{n-x}\ ;\ x=0,1,2,\dots n$$

$$f(x)={}^{6}C_x\left(\frac{8}{9}\right)^x\left(\frac{1}{9}\right)^{6-x}\ ;\ x=0,1,2,\dots 6$$

$$P(X=3)=f(3)={}^{6}C_3\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3=\frac{6\times5\times4}{3\times2\times1}\times\frac{8^3}{9^3}\times\frac{1}{9^3}=20\,\frac{8^3}{9^6}$$

$$\text{(ii) } n=4\ ;\ q=\frac{1}{9}\ ;\ p=1-q=1-\frac{1}{9}=\frac{8}{9}$$

$$f(x)={}^{4}C_x\left(\frac{8}{9}\right)^x\left(\frac{1}{9}\right)^{4-x}\ ;\ x=0,1,2,\dots 4$$

$$P(X=0)=f(0)={}^{4}C_0\left(\frac{8}{9}\right)^0\left(\frac{1}{9}\right)^4=\left(\frac{1}{9}\right)^4$$

**(OR)**

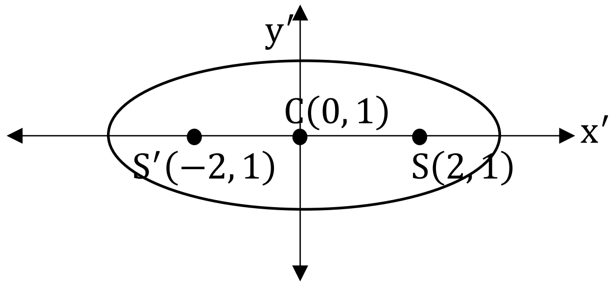
**45. (b) Find the equation of the ellipse whose Foci are $(2,1),(-2,1)$ and the length of the latus rectum is 6.**

Solution:

$$\text{G.T. foci} = (2,1)\ \&\ (-2,1)$$

$$\text{centre} = \text{mid point of } SS'$$

$$= \left(\frac{-2+2}{2}, \frac{1+1}{2}\right) = (0,1)$$




Major axis is along $x$ − axis and centre $C(h,k)$

then the equ. of ellipse is $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

$$C(h,k) = C(0,1)$$

$$\frac{x^2}{a^2} + \frac{(y-1)^2}{b^2} = 1 \rightarrow \text{①}$$

From dia.,

$$SS' = 2c = 4$$

$$c = 2 \Rightarrow c^2 = 4$$

Given L.L.R = 6

$$\frac{2b^2}{a} = 6$$

$$b^2 = 3a \rightarrow \text{②}$$

W.K.T. $c^2 = a^2 - b^2$

$$4 = a^2 - 3a$$

$$a^2 - 3a - 4 = 0$$

$$(a-4)(a+1) = 0$$

$$a - 4 = 0\ ;\ a + 1 = 0$$

$$a = 4\ ;\ a = -1 (\text{not possible})$$

$$\therefore a^2 = 16\ \&\ b^2 = 12$$

$$\therefore \frac{x^2}{16} + \frac{(y-1)^2}{12} = 1 \text{ is req. equation of ellipse}$$



**46. (a) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + s(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$.**

Solution:

The requied equation of plane passing through a point $(0,1,-5)$ and parallel to vectors $2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$.

$$\text{Let } \vec{a} = 0\hat{\imath} + \hat{\jmath} - 5\hat{k}\ ;\ \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\ ;\ \vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$$

Non – parametric vector equation:

$$(\vec{r}-\vec{a}).(\vec{b}\times\vec{c})=0$$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 6\\ 1 & 1 & -1\end{vmatrix}=\hat{\imath}(-3-6)-\hat{\jmath}(-2-6)+\hat{k}(2-3)$$

$$\vec{b}\times\vec{c}=-9\hat{\imath}+8\hat{\jmath}-\hat{k}$$

$$\therefore\left(\vec{r}-(0\hat{\imath}+\hat{\jmath}-5\hat{k})\right).\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)=0$$

$$\vec{r}.\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)-(0\hat{\imath}+\hat{\jmath}-5\hat{k}).\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)=0$$

$$\vec{r}.\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)-(0+8+5)=0$$

$$\boxed{\vec{r}.\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)=13}$$

Cartesian equation:

$$(x\hat{\imath}+y\hat{\jmath}+z\hat{k}).\left(-9\hat{\imath}+8\hat{\jmath}-\hat{k}\right)=13$$

$$-9x+8y-z=13$$

$$\boxed{9x-8y+z+13=0}$$

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**(OR)**

**46. (b) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?**

Solution:

Let x be the population at any time t.

$$\frac{dx}{dt}\propto x$$

$$\frac{dx}{dt}=kx$$

$$\frac{dx}{x}=kdt$$

$$\int\frac{dx}{x}=k\int dt$$

$$\log|x|=kt+\log|C|$$

$$\log|x|-\log|C|=kt$$

$$\log\left|\frac{x}{C}\right|=kt$$

$$\frac{x}{C}=e^{kt}$$

$$x=C\,e^{kt}\quad\rightarrow\quad(1)$$

$$\text{When } t=0,\ \ x=x_0(\text{initial})$$

$$x_0=C\,e^0=C(1)$$

$$\boxed{C=x_0}$$

$$\text{Sub } C=x_0 \text{ in } (1)$$

$$x=x_0\,e^{kt}\quad\rightarrow\quad(2)$$

$$\text{When } t=50,\ \ x=2x_0$$

$$2x_0=x_0\,e^{k(50)}$$

$$2=e^{50k}$$

$$2 = \left(e^k\right)^{50}$$

$$\boxed{e^k = 2^{\frac{1}{50}}}$$

$$\text{Sub } e^k = 2^{\frac{1}{50}} \text{ in } ②$$

$$x = x_0\left(e^k\right)^t$$

$$x = x_0\left(2^{\frac{1}{50}}\right)^t$$

$$x = x_0\, 2^{\frac{t}{50}} \quad \to \quad ③$$

$$\text{When } t = ?, \quad x = 3x_0$$

$$3x_0 = x_0\, 2^{\frac{t}{50}}$$

$$3 = 2^{\frac{t}{50}}$$

$$\log 3 = \log 2^{\frac{t}{50}}$$

$$\log 3 = \frac{t}{50}\log 2$$

$$\frac{\log 3}{\log 2} = \frac{t}{50}$$

$$50\left(\frac{\log 3}{\log 2}\right) = t$$

$\therefore$ The population is trippled in $50\left(\frac{\log 3}{\log 2}\right)$ years

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**47. (a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.**

Solution:

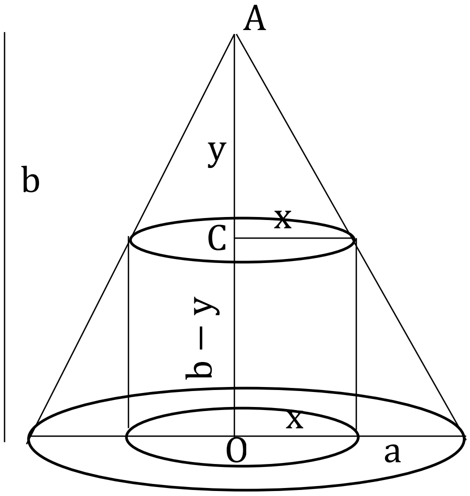
Let x be the radius of cylinder

height of cone = OA = b, Let AC = y

height of cylinder = OC = OA − AC

$$h = b - y$$

volume of the cylinder $= \pi x^2(b - y)$




From dia,

$$\frac{x}{a} = \frac{y}{b}$$

$$x = \frac{a}{b}y$$

$$V(y) = \pi\left(\frac{a^2}{b^2}y^2\right)(b - y)$$

$$V(y) = \frac{\pi a^2}{b^2}(by^2 - y^3)$$

$$V'(y) = \frac{\pi a^2}{b^2}(2by - 3y^2)$$

$$V''(y) = \frac{\pi a^2}{b^2}(2b - 6y)$$

$$V'(y) = 0$$
$$\frac{\pi a^2}{b^2}(2by - 3y^2) = 0$$
$$2by - 3y^2 = 0$$
$$y(2b - 3y) = 0$$
$$y = 0 \quad ; \quad 2b - 3y = 0$$
$$y = 0 \text{ (not possible)} \qquad -3y = -2b$$
$$y = \frac{2b}{3}$$
$$\text{Sub } y = \frac{2b}{3} \text{ in } V''(y)$$
$$V''\left(\frac{2b}{3}\right) = \frac{\pi a^2}{b^2}\left(2b - 6\left(\frac{2b}{3}\right)\right)$$
$$= \frac{\pi a^2}{b^2}(2b - 4b) = \frac{\pi a^2}{b^2}(-2b) < 0$$
$$\text{i.e., volume is maximum at } y = \frac{2b}{3}$$
$$\text{Maximum volume of cylinder} = V\left(\frac{2b}{3}\right) = \frac{\pi a^2}{b^2}\left(\frac{4b^2}{9}\right)\left(b - \frac{2b}{3}\right)$$
$$= \frac{\pi a^2}{b^2}\left(\frac{4b^2}{9}\right)\left(\frac{b}{3}\right)$$
$$= \frac{4}{9}\left(\frac{1}{3}\pi a^2 b\right)$$
$$\therefore \text{Maximum volume of cylinder} = \frac{4}{9}(\text{volume of cone})$$



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**(OR)**

**47. (b) Using truth table, prove that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.**

Solution:

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge (q \vee r)$ | $p \wedge q$ | $p \wedge r$ | $(p \wedge q) \vee (p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T |
| T | F | T | T | T | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F (1) | F | F | F (2) |

From (1) and (2), we get

$$\therefore p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

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