TN - MATHS → MARCH - 25

ANSWER KEY.

I. ONE MARKS:

1. d) N
2. d). 2
3. d). 1/5
4. d). 2.5
5. a). 1, 2
6. a). 19
7. b). $y = x^3 + 2$
8. b). [1, 2]
9. d). $2xu$
10. b) 2
11. b) $\pm \frac{1}{2}(1+i)$
12. b) $1+i$
13. d) 0.25
14. c) $\pi/2$
15. a) (1, 0)
16. b) $\pi/6$
17. c) $\pi a^3/6$
18. b) $1/\sqrt{2}$
19. a) π
20. b) $\frac{1}{f(x)} f'(x)dx$.

PART - II

21. $A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$

$|adj A| = 9 \quad \sqrt{|adj A|} = \sqrt{9} = 3.$

$\therefore A^{-1} = \pm \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

22. $Re[1/z] = Re\left(\frac{1}{x+iy}\right) = Re\left(\frac{x-iy}{x^2+y^2}\right)$

$\therefore x/x^2+y^2.$

23. $\tan^{-1}(\sqrt{3}) = \tan^{-1}(\tan(-\pi/3))$

$\therefore = \frac{-\pi}{3} \in (-\pi/2, \pi/2).$

24. $y = mx + c, \quad x^2 + y^2 = a^2$

$c^2 = a^2(1+m^2) \Rightarrow c = \sqrt{9(17)}$

$\therefore c = 3\sqrt{17}$

25. $\frac{x^2 - 6x + 7}{x+5} \quad q = x - 11 \quad r = 62.$

$\therefore$ slant tangent is $y = x - 11$.

26. $f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + 5\lambda x \lambda y - 10\lambda^2 y^2}{3\lambda x + 7\lambda y}$

$= \frac{\lambda^2}{\lambda}\left(\frac{x^2 + 5xy - 10y^2}{7x + 7y}\right)$

$= \lambda f(x, y).$

$\therefore$ It is an equivalence Relation.

27. $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = \sin^{-1} x + c.$

28. $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_1^4 cx^2 dx = 1.$

$c\left[\frac{64-1}{3}\right] = 1 \quad \therefore c = \frac{1}{21}$

29. $S.R = -2+i-2-i = -4$

$P.R = (-2+i)(-2-i) = 5$

$\therefore$ equation $x^2 - (S.R)x + P.R = 0$

$x^2 + 4x + 5 = 0$

30. $\int_0^{\pi} f(x)dx = 2\int_0^{\pi/2} f(x)dx$

$f(2a - x) = f(x).$

$\therefore \int_0^{\pi} f(x)dx = 2\int_0^{\pi/2} \sin x\,dx.$

$\therefore 2\int_0^{\pi/2} f(x)dx.$

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PART-III

31. $x = A^{-1}B$, $A^{-1} = \frac{1}{|A|} adj$

$A^{-1} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$ $x = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$x = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$ $\therefore x = -11$ $y = 4$.

32. $|z+6+8i| \leq |z| + |6+8i|$

$\leq 2+10 \leq 12 \rightarrow (1)$

$|z+6+8i| \geq |z| - |6+8i|$

$\geq |2-10|$

$\geq 8 \rightarrow (2)$

$\therefore 8 \leq |z+6+8i| \leq 12$.

33. Solve using SD.

$(x+1)(x-7)(7x-1) = 0$

$\therefore x=-1$ $x=7$ $x = 1/7$.

34. $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$\tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right) = \tan^{-1}(1/2)$

35. $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}]$

$= \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a},\vec{b},\vec{c}] = 1[\vec{a},\vec{b},\vec{c}]$

$\therefore = [\vec{a},\vec{b},\vec{c}]$.

36. $u = x^2y + 3xy^4$, $x = e^t$, $y = \sin t$.

$\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt}$

$= (2xy+3y^4)e^t + (x^2+12xy^3)\cos t$

$= e^t(2e^t \sin t + 3\sin^4 t)$

$+ (e^{2t}\cos t + 12 e^t \sin^3 t \cos t)$

38. profit

$E(x) = 2\cancel{00} \times 1/6\cancel{00} + 1\cancel{00} \times \frac{4}{6\cancel{00}}$

$= + 5\cancel{0} \times \frac{6}{6\cancel{00}} + 0 \times \frac{589}{600}$

$1/2$

$= 3/2$

$= 1.5$

37. $I = \int_0^{\pi/2} \frac{dx}{1+5\cos^2 x} = \int_0^{\pi/2} \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}+5}$

$= \int_0^{\pi/2} \frac{\sec^2 x}{6+\tan^2 x} dx$

$t = \tan x$; $dt = \sec^2 x\, dx$

$I = \int_0^{\infty} \frac{dt}{6+t^2} dt = \frac{1}{\sqrt{6}}\left[\tan^{-1}\frac{t}{\sqrt{6}}\right]_0^{\infty}$

$I = \frac{\pi}{2\sqrt{6}}$.

39. $f(x) = x^3+2x+1$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$

$= 13 + 14(x-2) + 6(x-2)^2 + (x-2)^3 +$

40. $a * b = a+b-ab+7$

$\frac{3}{2} * m = \frac{87}{10}$

$\frac{3}{2} + m - \frac{3}{2}m + 7 = \frac{87}{10}$

$\frac{17-m}{2} = \frac{87}{10}$

$\boxed{m = -\frac{2}{5}}$

PART-III.

41.(a) $\Delta = 6$ $\Delta_1 = 12$ $\Delta_2 = -6$ $\Delta_3 = 24$

$x_1 = \frac{\Delta_1}{\Delta} = 2$. $x_2 = \frac{\Delta_2}{\Delta} = -1$

$x_3 = \frac{\Delta_3}{\Delta} = 4$.

$(x_1, x_2, x_3) = (2,-1,4)$.

(b) $\frac{dy}{dx} = \frac{-2\cos t}{7\sin t}$.

tangent $\Rightarrow y - 2\sin t = \frac{-2\cos}{7\sin t}(x - 7\cos t)$

$2x\cos t + 7y\sin t = 14$.

Perpendicular

$y - 2\sin t = \frac{7\sin t}{2\cos t}(x - 7\cos t)$

$7x\sin t - 2y\cos t = 45\sin t\cos t$.

42(a) $(z-1)^3 = -8$

$(1-z)^3 = 8$ $(1-z) = 8^{1/3} = 2.$ $= (1, \omega, \omega^2)$

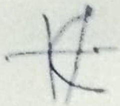
$\cancel{1,\omega,\omega^2}$. $z = 1-2 = -1$

$z = 1-2\omega$, $z = 1-2\omega^2$.

42.

b). $y = x$

$y = x-2$



$y=2,\ y=-1$

Area $= \int_{-1}^{2} (x_2 - x_1)$

$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$

$= \frac{9}{2}$.

43. (a) $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

root $1/3$.

other roots $3, -1/2, -2$.

(b) $\frac{dy}{dx} = \frac{3y}{2x} - \frac{x}{2y}$.

$y = vx \Rightarrow v + x\frac{dv}{dx} = \frac{3v}{2}$.

$\frac{2v\,dv}{v^2-1} = \frac{dx}{x}$

$\log|v^2-1| = \log|x| + \log|c|$

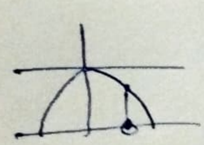
$v^2 - 1 = cx$

$\frac{y^2}{x^2} - 1 = cx$

$y^2 - x^2 = \pm cx^3$.

44.

(a) $x^2 - 4ay$



$15^2 = -4a(-10)$

$4a = \frac{225}{10}$

$x^2 = \frac{-225}{10} y$

$(6, -y_1)$ lies on parabola.

$36 = \frac{225}{10} y$.

$y = 1.6$.

Distance $= 10 - 1.6 = 8.4$ m.

(b). $\vec{a} = \cos\alpha \vec{i} + \sin\alpha \vec{j}$

$\vec{b} = \cos\beta \vec{i} + \sin\beta \vec{j}$

$\vec{a}\cdot\vec{b} = \cos(\alpha-\beta)$

$\vec{a}\cdot\vec{b} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

45(a) $p = \frac{8}{9},\ q = 1/9$.

$P(X=x) = nC_x p^x q^{n-x}$.

$= nC_x\left(\frac{8}{9}\right)^x (1/9)^{n-x}$

$n=6 \Rightarrow x = 3$.

$P(X=3) = 6C_3\left(\frac{8}{9}\right)^3 (1/9)^3$.

$= 20\left(\frac{8^3}{9^6}\right)$

$n=4 \quad x=0$.

$P(X=0) = 4C_0 (8/9)^0 (1/9)^4$

$= 1/9^4$.

b). $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

$(h,k) = (0,1)$

$b^2 = 3a$.

$c^2 = a^2 - b^2 \Rightarrow 4 = a^2 - 3a$.

$\therefore a^2 - 3a - 4 = 0$.

$a = -1 \quad a = 4 \qquad a^2 = 16$.

$b^2 = 3 \times 4 = 12$.

$\therefore \frac{x^2}{16} + \frac{(y-1)^2}{12} = 1$.

46 (a). $\vec{b} = 2\vec{i} + 3\vec{j} + 6\vec{k} \quad \vec{c} = \vec{i} + \vec{j} - \vec{k}$

$\vec{b}\times\vec{c} = -9\vec{i} + 8\vec{j} - \vec{k}$

$\vec{r} - (\vec{j} - 5\vec{k})\cdot(-9\vec{i} + 8\vec{j} - \vec{k}) = 0$.

$\vec{r}\cdot(-9\vec{i} + 8\vec{j} - \vec{k}) = 13$.

$\therefore$ Cartesian equation

$9x - 8y + z + 13 = 0$.

(b). $A = Ce^{kt}$.

$t = 0 \qquad C = A_0$ then

$A = A_0 e^{kt}$.

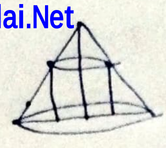
$t = 50 \qquad x = 2A_0$.

$\therefore k = \frac{1}{50}\log 2$

$t = t \qquad A = 3A_0$.

$t = 50\left(\frac{\log 3}{\log 2}\right)$.

47.(a) $h = \frac{b}{a}(a-r)$



$v = \pi r^2 h$.

$= \pi r^2 \left(\frac{b}{a}(a-r)\right)$

$v = \frac{\pi b}{a}(ar^2 - r^3)$

$v' = \frac{\pi b}{a}(2ar - 3r^2)$

$v'' = \frac{\pi b}{a}(2a - 6r)$

$v' = 0$ then $r = 0$.

$v = \frac{\pi b}{a}\left(\frac{4a^3}{9} - \frac{8a^3}{27}\right)$

$= \frac{\pi b}{a} \times \frac{4a^3}{27}$.

$= \frac{4}{9}\left(\frac{\pi a^2 b}{3}\right)$.

$\therefore \frac{4}{9}$ is volume of cone.

(b). $p \wedge (q \vee r) = TTTFFFFF$

$(p \wedge q) \vee (p \wedge r) = TTTFFFFF$.

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