12th Maths – Public Exam Creative Question Answers

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Government Model Question – 2019

21. Solve the following system of linear equations by Cramer's rule $2x-y=3$, $x+2y=-1$.

$2x-y=3, \quad x+2y=-1$

$$\Delta = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4-(-1) = 4+1 = 5$$

$$\Delta_x = \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = 6-1 = 5$$

$$\Delta_y = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2-3 = -5$$

$$x = \frac{\Delta_x}{\Delta} = \frac{5}{5} = 1 \Rightarrow \boxed{x=1}$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-5}{5} = -1 \Rightarrow \boxed{y=-1}$$

23. Find the value of $\sin\left(\frac{\pi}{3}+\cos^{-1}\left(\frac{-1}{2}\right)\right)$

$$\sin\left(\frac{\pi}{3}+\cos^{-1}\left(\frac{-1}{2}\right)\right) = \sin\left(\frac{\pi}{3}+\frac{2\pi}{3}\right)$$

$$= \sin\left(\frac{\pi+2\pi}{3}\right)$$

$$= \sin\left(\frac{3\pi}{3}\right)$$

$$= \sin\pi$$

$$\sin\left(\frac{\pi}{3}+\cos^{-1}\left(\frac{-1}{2}\right)\right) = 1$$

30. Let $*$ be a binary operation on set Q of rational numbers defined as $a*b = \frac{ab}{8}$. Write the identity for $*$, if any.

$a,b \in Q \Rightarrow a*b \in Q$

$$a*b = \frac{ab}{8}$$

$$a*e = e*a = a$$

$$a*e = a$$

$$\frac{ae}{8} = a$$

$$e = a \times \frac{8}{a}$$

$$\boxed{e=8}$$

34. Find the centre, foci and eccentricity of the hyperbola $12x^2-4y^2-24x+32y-127=0$

$$12x^2-4y^2-24x+32y-127=0$$

$$12x^2-24x-4y^2+32y-127=0$$

$$12(x^2-2x)-4(y^2-8y)-127=0$$

$$12\left[(x-1)^2-1\right]-4\left[(y-4)^2-16\right]-127=0$$

$$12(x-1)^2-12-4(y-4)^2+64-127=0$$

$$12(x-1)^2-4(y-4)^2-75=0$$

$$12(x-1)^2-4(y-4)^2=75$$

$$\frac{12(x-1)^2}{75}-\frac{4(y-4)^2}{75}=1$$

$$\frac{(x-1)^2}{75/12}-\frac{(y-4)^2}{75/4}=1$$

$$\frac{(x-1)^2}{25/4}-\frac{(y-4)^2}{75/4}=1$$

Centre $(1,4)$ h k

$$a^2=\frac{25}{4}, \quad b^2=\frac{75}{4}$$

$$c^2 = a^2+b^2$$

$$= \frac{25}{4}+\frac{75}{4}$$

$$= \frac{100}{4}$$

$$c^2 = 25$$

$$\boxed{c = \pm 5}$$

$$e = \frac{c}{a} = \frac{5}{5/2} = 5\times\frac{2}{5} = 2$$

The coordinates of the foci are $(h+c,k)$ and $(h-c,k)$

$$= (1+5,4) \text{ and } (1-5,4)$$

$$= (6,4) \text{ and } (-4,4)$$

38. By using the properties of definite integrals, evaluate $\int_0^3 |x-1|\,dx$

$$x-1=0$$

$$\boxed{x=1}$$

$$|x-1| = \begin{cases} -(x-1) & x<1 \\ (x-1) & x\geq 1 \end{cases}$$

$$\int_0^3 |x-1|\,dx = \int_0^1 -(x-1)\,dx + \int_1^3 (x-1)\,dx$$

$$= -\left[\frac{x^2}{2}-x\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^3$$

$$= -\left[\frac{1^2}{2}-1-(0)\right] + \left[\frac{3^2}{2}-3-\left(\frac{1^2}{2}-1\right)\right]$$

$$= -\left[\frac{1}{2}-1\right] + \left[\frac{9}{2}-3-\frac{1}{2}+1\right]$$

$$= -\left(\frac{-1}{2}\right) + \left(\frac{8}{2}-2\right)$$

$$= \frac{1}{2} + (4-2)$$

$$= \frac{1}{2}+2$$

$$= \frac{5}{2}$$

$$\therefore \int_0^3 |x-1|\,dx = \frac{5}{2}$$

40. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, find the probability of getting exactly two heads. JEE Main [26th Feb, 1st Shift 2021]

Let the coin be tossed n-times

$$P(H) = P(T) = \frac{1}{2}$$

$$P(7\text{ heads}) = {}^nC_7 \left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7 = {}^nC_7\left(\frac{1}{2}\right)^n$$

$$P(9\text{ heads}) = {}^nC_9 \left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^9 = {}^nC_9\left(\frac{1}{2}\right)^n$$

$$P(7\text{ heads}) = P(9\text{ heads})$$

$${}^nC_7\left(\frac{1}{2}\right)^n = {}^nC_9\left(\frac{1}{2}\right)^n$$

$${}^nC_7 = {}^nC_9$$

$$\Rightarrow n = 16$$

$$P(2\text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^{16-2}\left(\frac{1}{2}\right)^2$$

$$= {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^2$$

$$= \frac{16\times 15}{2\times 1}\left(\frac{1}{2}\right)^{14+2}$$

$$= 8\times 15\times\left(\frac{1}{2}\right)^{16}$$

$$= 2^3\times 15\times\frac{1}{2^{16}}$$

$$= 15/2^{13}$$

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42 (a) Solve the equation: $3x^4-16x^3+26x^2-16x+3=0$

$$3x^4-16x^3+26x^2-16x+3=0$$

$$x^2\left[3x^2-16x+26-\frac{16}{x}+\frac{3}{x^2}\right]=0$$

$$x^2\left(3\left(x^2+\frac{1}{x^2}\right)-16\left[x+\frac{1}{x}\right]+26\right)=0$$

$$3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0$$

Let $y = x+\frac{1}{x}$

$$3(y^2-2)-16y+26=0$$

$$3y^2-6-16y+26=0$$

$$3y^2-16y+20=0$$

$$3y^2-6y-10y+20=0$$

$$3y(y-2)-10(y-2)=0$$

$$(y-2)(3y-10)=0$$

$$y-2=0,\quad 3y-10=0$$

$$\boxed{y=2},\quad 3y=10$$

$$\boxed{y=10/3}$$

$$y=2 \Rightarrow x+\frac{1}{x}=2$$

$$\frac{x^2+1}{x}=2$$

$$x^2+1=2x$$

$$x^2-2x+1=0$$

$$x^2-x-x+1=0$$

$$x(x-1)-1(x-1)=0$$

$$(x-1)(x-1)=0$$

$$x-1=0,\quad x-1=0$$

$$\boxed{x=1,1}$$

$$y=\frac{10}{3} \Rightarrow x+\frac{1}{x}=\frac{10}{3}$$

$$\frac{x^2+1}{x}=\frac{10}{3}$$

$$3(x^2+1)=10x$$

$$3x^2+3-10x=0$$

$$3x^2-9x-x+3=0$$

$$3x(x-3)-1(x-3)=0$$

$$(x-3)(3x-1)=0$$

$$x-3=0\,,\quad 3x-1=0$$

$$\boxed{x=3}\,,\quad 3x=1$$

$$\boxed{x=\tfrac{1}{3}}$$

$\therefore$ The roots are $1, 1, 3, \frac{1}{3}$.

43(b) Find the non-parametric and Cartesian equations of the plane passing through the points $(4,2,4)$ and is perpendicular to the planes $2x+5y+4z+1=0$ and $4x+7y+6z+2=0$.

$$\vec{a} = 4\hat{i}+2\hat{j}+4\hat{k}$$

$$\vec{b} = 2\hat{i}+5\hat{j}+4\hat{k}$$

$$\vec{c} = 4\hat{i}+7\hat{j}+6\hat{k}$$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & 4 \\ 4 & 7 & 6 \end{vmatrix}$$

$$= \hat{i}(30-28)-\hat{j}(12-16)+\hat{k}(14-20)$$

$$= 2\hat{i}+4\hat{j}-6\hat{k}$$

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$$

$$(\vec{r}-4\hat{i}+2\hat{j}+4\hat{k})\cdot(2\hat{i}+4\hat{j}-6\hat{k})=0$$

$$\vec{r}\cdot(2\hat{i}+4\hat{j}-6\hat{k})-(8+8-24)=0$$

$$\vec{r}\cdot(2\hat{i}+4\hat{j}-6\hat{k})-(-8)=0$$

$$\vec{r}\cdot(2\hat{i}+4\hat{j}-6\hat{k})+8=0$$

Let $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+4\hat{j}-6\hat{k})+8=0$$

$$2x+4y-6z+8=0$$

Which is the required equation.

44.(b) Let $z(x,y)=xe^{y}+ye^{-x}$, $x=e^{-t}$, $y=st^2$, $s,t\in R$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

Given that,

$$Z = xe^{y}+ye^{-x} \qquad x=e^{-t} \qquad y=st^2$$

$$\frac{\partial z}{\partial x} = e^{y}(1)+y(e^{-x})(-1) \qquad \frac{\partial x}{\partial t}=-e^{-t} \qquad \frac{\partial y}{\partial t}=2st$$

$$= e^{y}-ye^{-x} \qquad \frac{\partial x}{\partial s}=0 \qquad \frac{\partial y}{\partial s}=t^2$$

$$\frac{\partial z}{\partial y} = x.e^{y}+e^{-x}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= (e^{y}-ye^{-x})(0)+(x.e^{y}+e^{-x})(t^2)$$

$$= 0+t^2(xe^{y}+e^{-x})$$

$$= t^2(e^{-t}e^{st^2}+e^{-e^{-t}})$$

$$\frac{\partial z}{\partial s} = t^2\left(e^{-t+st^2}+e^{-e^{-t}}\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= (e^{y}-ye^{-x})(-e^{-t})+(x.e^{y}+e^{-x})(2st)$$

$$\frac{\partial z}{\partial t} = -e^{-t}\left(e^{st^2}-st^2.e^{-e^{-t}}\right)+\left(e^{-t}.e^{st^2}+e^{-e^{-t}}\right)(2st)$$

47.(b) Find the equations of tangent and normal to the curve $y^2-4x-2y+5=0$ at the point where it cuts the x-axis.

$$y^2-4x-2y+5=0 \longrightarrow (1)$$

Curve cuts x axis, when $y=0$.

$$y=0 \Rightarrow -4x+5=0$$

$$-4x=-5$$

$$x=\frac{-5}{-4} \Rightarrow \boxed{x=\frac{5}{4}}$$

$\therefore$ Curve cuts x axis at $(5/4, 0)$

On differentiating (1) with respect to x

$$2y\frac{dy}{dx}-4-2\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(2y-2)=4$$

$$\frac{dy}{dx}=\frac{4}{(2y-2)}$$

$$\frac{dy}{dx} = \frac{4}{2(y-1)}$$

$$\frac{dy}{dx} = \frac{2}{y-1}$$

$$\left.\frac{dy}{dx}\right|_{(5/4,0)} = \frac{2}{0-1} = \frac{2}{-1} = -2$$

Equation of the tangent :

Slope $m_1 = -2$

$P(x_1, y_1) = P(5/4, 0)$

$$y - y_1 = m_1(x - x_1)$$

$$y - 0 = -2\left(x - \frac{5}{4}\right)$$

$$y = -2\left(\frac{4x-5}{4}\right)$$

$$y = -\left(\frac{4x-5}{2}\right)$$

$$2y = -4x + 5$$

$$4x + 2y - 5 = 0$$

Equation of the normal :

Slope $m_2 = \frac{-1}{m_1} = \frac{-1}{-2} = \frac{1}{2}$

$$y - y_1 = m_2(x - x_1)$$

$$y - 0 = \frac{1}{2}\left(x - \frac{5}{4}\right)$$

$$y = \frac{1}{2}\left(\frac{4x-5}{4}\right)$$

$$y = \frac{4x-5}{8}$$

$$8y = 4x - 5$$

$$0 = 4x - 8y - 5$$

$$\therefore 4x - 8y - 5 = 0$$

"SUCCESS IS NO ACCIDENT. IT IS HARD WORK, PERSEVERANCE, LEARNING, STUDYING, SACRIFICE AND **MOST** OF ALL, LOVE OF WHAT YOU ARE DOING OR LEARNING TO DO."

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④

MARCH 2020

27. Prove that $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx = \frac{\pi}{4}$

Class 12 (RD Sharma) [Chapter 20 – MCQ.32] Pg 119

Let $I = \int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx \longrightarrow (1)$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{f\left(\sin\left(\frac{\pi}{2}-x\right)\right)}{f\left(\sin\left(\frac{\pi}{2}-x\right)\right)+f\left(\cos\left(\frac{\pi}{2}-x\right)\right)}\,dx$$

$$I = \int_0^{\pi/2} \frac{f(\cos x)}{f(\cos x)+f(\sin x)}\,dx \longrightarrow (2)$$

$(1)+(2) \Rightarrow 2I = \int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx + \int_0^{\pi/2} \frac{f(\cos x)}{f(\cos x)+f(\sin x)}\,dx$

$$2I = \int_0^{\pi/2} \frac{f(\sin x)+f(\cos x)}{f(\sin x)+f(\cos x)}\,dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi}{4}}$$

30. Find the equation of the parabola if the curve is open leftward, vertex is (2,1) and passing through the point (1,3).

Equation of the parabola which open leftward is

$$(y-k)^2 = -4a(x-h)$$

Vertex $(h,k) = (2,1)$

$$(y-1)^2 = -4a(x-2) \longrightarrow (1)$$

Passing through the point $(x,y) = (1,3)$

$$(3-1)^2 = -4a(1-2)$$

$$2^2 = -4a(-1)$$

$$4 = 4a$$

$$\boxed{a = 1}$$

$(1) \Rightarrow (y-1)^2 = -4(x-2)$ is the equation of the parabola.

35. Find the Critical numbers (only x values) of the function $f(x) = x^{\frac{4}{5}}(x-4)^2$

$$f(x) = x^{4/5}(x-4)^2$$

$$f'(x) = x^{4/5} \cdot 2(x-4) + (x-4)^2 \cdot \frac{4}{5}x^{\frac{4}{5}-1}$$

$$= 2x^{4/5}(x-4) + \frac{4}{5}x^{-1/5}(x-4)^2$$

$$= 2x^{4/5}(x-4) + \frac{4(x-4)^2}{5x^{1/5}}$$

$$= \frac{10x^{4/5} \cdot x^{1/5}(x-4) + 4(x-4)^2}{5x^{1/5}}$$

$$= \frac{10x^{5/5}(x-4) + 4(x-4)^2}{5x^{1/5}}$$

$$= \frac{10x(x-4) + 4(x-4)^2}{5x^{1/5}}$$

$$= \frac{2(x-4)[5x + 2(x-4)]}{5x^{1/5}}$$

$$= \frac{2(x-4)(5x+2x-8)}{5x^{1/5}}$$

$$f'(x) = \frac{2(x-4)(7x-8)}{5x^{1/5}}$$

$$f'(x) = 0$$

$$\frac{2(x-4)(7x-8)}{5x^{1/5}} = 0$$

$x-4=0$ $7x-8=0$ $x=0$

$x=4$ $7x=8$

$x = \frac{8}{7}$

$\therefore$ Critical values are $0, \frac{8}{7}, 4$.

38. Let x be a continuous random variable and $f(x)$ is defined as:

$$f(x) = \begin{cases} kx(1-x)^{10}, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$ Find the value of k.

Example 10.4 [old text book] Volume II - page 174

Since the given function is a probability density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$k\int_0^1 x(1-x)^{10}\,dx = 1$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$k\int_0^1 (1-x)(1-(1-x))^{10}\,dx = 1$$

$$k\int_0^1 (1-x)x^{10}\,dx = 1$$

$$k\int_0^1 (x^{10} - x^{11})\,dx = 1$$

$$k\left[\frac{x^{11}}{11} - \frac{x^{12}}{12}\right]_0^1$$

$$k\left[\frac{1}{11} - \frac{1}{12}\right] = 1$$

$$k\left[\frac{12-11}{132}\right] = 1$$

$$k\left[\frac{1}{132}\right] = 1$$

$$\boxed{k = 132}$$

40. If the lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ lie on the same plane, then write the number of ways to find the cartesian equation of the above plane and explain in detail.

The required number of ways are 2.

One is, Equation of plane passing through a point and parallel to two vectors.

Another one is, equation of plane passing through the two points and parallel to a vector.

(i) Vector equation:

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$$

$$\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{u} \quad \text{(or)}$$

(ii) Cartesian equation:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

(or)

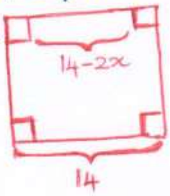
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$$

46. (a) A square shaped thin material with area 196 sq. units to make into an open box by cutting small equal squares from the four corners and folding the sides upward. Prove that the length of the side of a removed square is $\frac{7}{3}$ when the volume of the box is maximum.

Area = 196 sq. units

side = $\sqrt{196}$ = 14 sq. units

Let x = side of removed square



for open box

$l = 14-2x$

$b = 14-2x$

$h = x$

Volume = lbh cu. units

$= (14-2x)(14-2x)(x)$

$= (14-2x)^2(x)$

$= (14^2 - 2(14)(2x) + (2x)^2)\,x$

$= (196 - 56x + 4x^2)\,x$

$V = 196x - 56x^2 + 4x^3$

Differentiate with respect to x

$$\frac{dV}{dx} = 196 - 56(2x) + 4(3x^2)$$

$$= 196 - 112x + 12x^2$$

$$\frac{d^2V}{dx^2} = -112 + 24x$$

for V is maximum

(i) $\frac{dV}{dx} = 0$ (ii) $\frac{d^2V}{dx^2} < 0$

$$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 112x + 196 = 0$$

$$4(3x^2 - 28x + 49) = 0$$

$$3x^2 - 28x + 49 = 0$$

$$3x^2 - 21x - 7x + 49 = 0$$

$$3x(x-7) - 7(x-7) = 0$$

$$(x-7)(3x-7) = 0$$

$$x-7 = 0,\ 3x-7 = 0$$

$$x = 7,\quad 3x = 7$$

When $x = 7$, box cannot be formed.

$$\boxed{x = 7/3}$$

$$x = \frac{7}{3} \Rightarrow \frac{d^2V}{dx^2} = -112 + 24(7/3)$$

$$= -112 + 8\times 7$$

$$= -112 + 56$$

$$= -56 < 0$$

$\therefore$ V is maximum when $x = 7/3$

The volume of the open box is maximum, the side of removed square is $7/3$ units.

September 2020

21. Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$. Old text book Exercise 3.1 (3) Volume I - page 108

$$\left(\frac{1+i}{1-i}\right)^n = 1$$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$$

$$\left[\frac{(1+i)^2}{1^2 - i^2}\right]^n = 1$$

$$\left[\frac{1+i^2+2i}{1+1}\right]^n = 1$$

$$\left[\frac{1-1+2i}{2}\right]^n = 1$$

$$\left[\frac{2i}{2}\right]^n = 1$$

$$i^n = 1 \Rightarrow i^4 = 1$$

$$\boxed{n = 4}$$

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28. Find the differential equation of the family of $y = ax^2 + bx + c$ where a, b are parameters & c is a constant.

RD Sharma book - Exercise 22.02 (3.4)

$$y = ax^2 + bx + c \longrightarrow (1)$$

$$\frac{dy}{dx} = 2ax + b \longrightarrow (2)$$

$$\frac{d^2y}{dx^2} = 2a \Rightarrow a = \frac{1}{2}\frac{d^2y}{dx^2}$$

$$(2) \Rightarrow \frac{dy}{dx} = 2\left(\frac{1}{2}\frac{d^2y}{dx^2}\right)x + b$$

$$\frac{dy}{dx} = x \cdot \frac{d^2y}{dx^2} + b \Rightarrow b = \frac{dy}{dx} - x\frac{d^2y}{dx^2}$$

$$(1) \Rightarrow y = \frac{1}{2}\frac{d^2y}{dx^2}x^2 + x\left[\frac{dy}{dx} - x\frac{d^2y}{dx^2}\right] + c$$

$$2y = \frac{d^2y}{dx^2}x^2 + 2x\frac{dy}{dx} - 2x^2\frac{d^2y}{dx^2} + 2c$$

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y - 2c = 0$$

(or)

$$x^2 y_2 - 2xy_1 + 2y - 2c = 0$$

29. Examine the binary operation of the operation $a*b=\left(\frac{a-1}{b-1}\right), \forall a,b \in Q$

$$a*b=\frac{a-1}{b-1} \quad \forall a,b \in Q$$

If $a=1, b=1$

$$a*b=\frac{0}{0}=\text{undefined}$$

$$b-1 \neq 0$$

$$a*b=\frac{a-1}{b-1} \quad \forall a,b \in Q-\{1\}$$

$*$ is a binary operation on $Q-\{1\}$

30. Show that, if $x=r\cos\theta, y=r\sin\theta$, then $\frac{\partial r}{\partial x}$ is equal to $\cos\theta$.

$$x=r\cos\theta \quad ; \quad y=r\sin\theta$$

$$x^2=r^2\cos^2\theta \quad ; \quad y^2=r^2\sin^2\theta$$

$$x^2+y^2=r^2\cos^2\theta+r^2\sin^2\theta$$

$$=r^2(\cos^2\theta+\sin^2\theta)$$

$$x^2+y^2=r^2$$

$$2x+0=2r\frac{\partial r}{\partial x}$$

$$\frac{\partial r}{\partial x}=\frac{2x}{2r}=\frac{2r\cos\theta}{2r}=\cos\theta$$

$$\boxed{\frac{\partial r}{\partial x}=\cos\theta}$$

37. Suppose that $z=ye^{x^2}$ where $x=2t$ and $y=1-t$ then find $\frac{dz}{dt}$.

$$z=ye^{x^2} \qquad x=2t \qquad y=1-t$$

$$\frac{dz}{dx}=ye^{x^2}.2x \qquad \frac{dx}{dt}=2 \qquad \frac{dy}{dt}=-1$$

$$\frac{dz}{dy}=e^{x^2}.(1)$$

$$\frac{dz}{dt}=\frac{dz}{dx}\cdot\frac{dx}{dt}+\frac{dz}{dy}\cdot\frac{dy}{dt}$$

$$=2xy.e^{x^2}.(2)+e^{x^2}.(-1)$$

$$=4xy\,e^{x^2}-e^{x^2}$$

$$=e^{x^2}[4xy-1]$$

$$=e^{(2t)^2}[4(2t)(1-t)-1]$$

$$=e^{4t^2}(8t(1-t)-1)$$

$$=e^{4t^2}(8t-8t^2-1)$$

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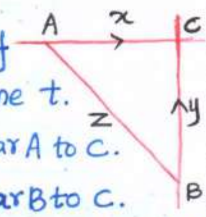
40. Show that $[(\neg q)\wedge p]\wedge q$ is a contradiction

Example 9.10 (ii) old text book volume II page 144

p	q	$\neg q$	$(\neg q)\wedge p$	$[(\neg q)\wedge p]\wedge q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

$\therefore [(\neg q)\wedge p]\wedge q$ is a contradiction

47(a) A car A is travelling from west at 50 km/hr and car B is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 kilometers and car B is 0.4 kilometers from the intersection?



Let C is the intersection of the two roads, at a given time t.
Let x be the distance from car A to C.
Let y be the distance from car B to C.
Let z be the distance between cars A and B.

Given that,

$$\frac{dx}{dt}=-50 \text{ and } \frac{dy}{dt}=-60$$

$$x=0.3 \text{ and } y=0.4.$$

By Pythagoras theorem,

$$x^2+y^2=z^2 \longrightarrow (1)$$

$$z^2=(0.3)^2+(0.4)^2$$

$$=0.09+0.16$$

$$=0.25$$

$$\boxed{z=0.5}$$

differentiate equation (1) with respect to t

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$z\frac{dz}{dt}=x\frac{dx}{dt}+y\frac{dy}{dt}$$

$$(0.5)\frac{dz}{dt}=(0.3)(-50)+(0.4)(-60)$$

$$=-15-24$$

$$(0.5)\frac{dz}{dt}=-39$$

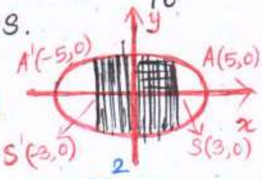
$$\frac{dz}{dt}=\frac{-39}{0.5}=\frac{-39}{1/2}=-39\times 2$$

$$dz/dt=-78$$

The cars are approaching each other at a rate of 78 km/hr.

47(b) Find the area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its latus rectums.

NCERT EXEMPLAR PROBLEMS (31) page - 178



$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

$$\frac{y^2}{16}=1-\frac{x^2}{25}$$

$$\frac{y^2}{16}=\frac{25}{25}-\frac{x^2}{25}$$

$$\frac{y^2}{16}=\frac{1}{25}(25-x^2)$$

$$y^2=\frac{16}{25}(25-x^2)$$

$$\boxed{y=\frac{4}{5}\sqrt{25-x^2}}$$

$a^2=25;\ b^2=16$

$$c^2=a^2-b^2$$
$$=25-16$$
$$c^2=9$$
$$\boxed{c=3}$$

The equation of latus rectum is $x=\pm 3$

Area required $=4\times$ Area of 1^{st} quadrant

$$=4\int_0^3 y\,dx$$

$$=4\int_0^3 \frac{4}{5}\sqrt{25-x^2}\,dx$$

$$=\frac{16}{5}\int_0^3\sqrt{25-x^2}\,dx$$

$$=\frac{16}{5}\int_0^3\sqrt{5^2-x^2}\,dx$$

$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$$=\frac{16}{5}\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^3$$

$$=\frac{16}{5}\left[\frac{3}{2}\sqrt{25-3^2}+\frac{25}{2}\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$=\frac{16}{5}\times\frac{1}{2}\left[3\sqrt{25-9}+25\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$=\frac{8}{5}\left[3\times\sqrt{16}+25\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$=\frac{8}{5}\left[12+25\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$=\frac{96}{5}+\frac{8\times 25}{5}\sin^{-1}\left(\frac{3}{4}\right)$$

$$=\frac{96}{5}+40\sin^{-1}\left(\frac{3}{4}\right)$$

September 2021

22. If α and β are the roots of $x^2-5x+6=0$ then prove that $\alpha^2-\beta^2=\pm 5$

$$x^2-5x+6=0$$

$$\alpha+\beta=\frac{-b}{a}=-\frac{(-5)}{1}=5$$

$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6$$

$$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$$
$$=\left[(\alpha+\beta)^2-2\alpha\beta\right]-2\alpha\beta$$
$$=(\alpha+\beta)^2-4\alpha\beta$$
$$=5^2-4(6)$$
$$=25-24$$
$$(\alpha-\beta)^2=1\Rightarrow(\alpha-\beta)=\pm 1$$
$$\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)$$
$$=(5)(\pm 1)$$
$$\alpha^2-\beta^2=\pm 5$$

30. Show that the differential equation Corresponding to $y=A\sin x$, where A is an arbitrary constant is $y=y'\tan x$

$$y=A\sin x$$
$$y'=A\cdot\cos x.$$

$(\times)\tan x$ on both sides.

$$\Rightarrow y'\tan x=A\cos x\cdot\tan x$$
$$=A\cos x\times\frac{\sin x}{\cos x}$$
$$=A\sin x$$
$$y'\tan x=y$$
$$\therefore y=y'\tan x.$$

36. A force $13\hat{i}+10\hat{j}-3\hat{k}$ acts on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$ show that the work done by the force is 69 units.

Text book sum partially asked
Exercise 6.1 – (12)
Volume I (231)

$$\vec{F}=13\hat{i}+10\hat{j}-3\hat{k}$$

$$\vec{d}=(6\hat{i}+\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}-2\hat{k})=2\hat{i}+4\hat{j}-\hat{k}$$

$$W=\vec{F}\cdot\vec{d}$$
$$=(13\hat{i}+10\hat{j}-3\hat{k})\cdot(2\hat{i}+4\hat{j}-\hat{k})$$
$$=26+40+3$$
$$W=69\text{ units}$$

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39. An egg of a particular bird is spherical in shape. If radius to the inside of the shell is 4mm and radius to the outside of the shell is 4.2mm, prove that the approximate volume of the shell is $12.8\pi\ mm^3$.

(r) Radius of the inside shell = 4mm

Radius of the outside shell = 4.2mm

$$dr = 4.2 - 4$$
$$dr = 0.2\ mm$$

Volume (v) $= \frac{4}{3}\pi r^3$ Cu.units

$$\frac{dv}{dr} = \frac{4}{3}\pi \times 3r^2$$
$$dv = 4\pi \times r^2 \times dr$$
$$= 4\pi \times (4)^2 \times (0.2)$$
$$= 0.8\pi \times 16$$
$$dv = 12.8\pi\ mm^3$$

40. $\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\,dx = \frac{1}{2}$

$$I = \int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\,dx \longrightarrow (1)$$
$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$
$$I = \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-(1-x)}+\sqrt{1-x}}\,dx$$
$$= \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-1+x}+\sqrt{1-x}}\,dx$$
$$I = \int_0^1 \frac{\sqrt{1-x}}{\sqrt{x}+\sqrt{1-x}}\,dx \longrightarrow (2)$$

$(1)+(2) \Rightarrow$ $2I = \int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\,dx + \int_0^1 \frac{\sqrt{1-x}}{\sqrt{x}+\sqrt{1-x}}\,dx$

$$2I = \int_0^1 \frac{\sqrt{x}+\sqrt{1-x}}{\sqrt{x}+\sqrt{1-x}}\,dx$$
$$2I = \int_0^1 [\,]\,dx$$
$$2I = [x]_0^1$$
$$2I = 1-0$$
$$2I = 1$$
$$I = \frac{1}{2}$$

41 (a) Solve the system of equations $x-y+2z=2$, $2x+y+4z=7$, $4x-y+z=4$ by Cramer's rule.

Exercise 1.6(1)(i) – Method changed

$$x-y+2z=2$$
$$2x+y+4z=7$$
$$4x-y+z=4$$

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix}$$
$$= 1(1+4)+1(2-16)+2(-2-4)$$
$$= 5-14-12$$
$$= 5-26$$
$$\boxed{\Delta = -21}$$

$$\Delta_x = \begin{vmatrix} 2 & -1 & 2 \\ 7 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix}$$
$$= 2(1+4)+1(7-16)+2(-7-4)$$
$$= 10-9-22$$
$$= 10-31$$
$$\boxed{\Delta_x = -21}$$

$$\Delta_y = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 7 & 4 \\ 4 & 4 & 1 \end{vmatrix}$$
$$= 1(7-16)-2(2-16)+2(8-28)$$
$$= -9+28-40$$
$$= -49+28$$
$$\boxed{\Delta_y = -21}$$

$$\Delta_z = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 7 \\ 4 & -1 & 4 \end{vmatrix}$$
$$= 1(4+7)+1(8-28)+2(-2-4)$$
$$= 11-20-12$$
$$= 11-32$$
$$\boxed{\Delta_z = -21}$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-21}{-21} = 1$$
$$y = \frac{\Delta_y}{\Delta} = \frac{-21}{-21} = 1$$
$$z = \frac{\Delta_z}{\Delta} = \frac{-21}{-21} = 1$$

$\therefore x=1, y=1, z=1$

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43 (b) Find the eccentricity, centre, vertices & foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ & also draw the rough diagram.

NCERT (11th text book) Exercise 11.3(3) page - 255 Volume II

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$a^2=16$; $b^2=9$

$a=4$; $b=3$

$c^2=a^2-b^2$

$=16-9$

$c^2=7$

$\therefore c=\pm\sqrt{7}$

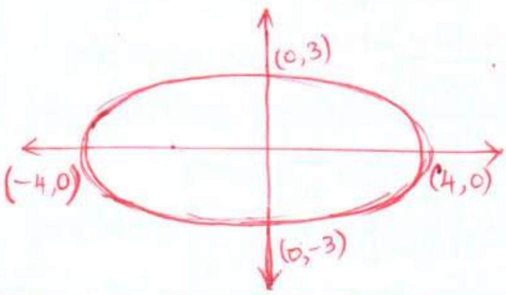
eccentricity $(e)=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Centre $=(0,0)$

Vertices $=(\pm a,0)$

$=(\pm 4,0)$

foci $=(\pm c,0)=(\pm\sqrt{7},0)$



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47 (b) Show that the Cartesian equation of the plane passing through the points (1,2,3) and (2,3,1) and also perpendicular to the plane $3x-2y+4z+5=0$ is $2y+z-7=0$

Exercise 2.8(11) - Old text book Volume 1 - 93

The normal vector to the plane $3x-2y+4z-5=0$ is $3\vec{i}-2\vec{j}+4\vec{k}$

This is parallel to the required plane.

The required plane passes through the point (1,2,3) and (2,3,1) and parallel to the vector $3\vec{i}-2\vec{j}+4\vec{k}$

$(x_1,y_1,z_1)=(1,2,3)$

$(x_2,y_2,z_2)=(2,3,1)$

$(l_1,m_1,n_1)=(3,-2,4)$

(10)

The cartesian equation of the plane

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 3 & -2 & 4 \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 3 & -2 & 4 \end{vmatrix}=0$$

$(x-1)(4-4)-(y-2)(4+6)+(z-3)(-2-3)=0$

$(x-1)(0)-(y-2)(10)+(z-3)(-5)=0$

$0-10y+20-5z+15=0$

$-10y-5z+35=0$

$10y+5z-35=0$

$\div 5$ $\boxed{2y+z-7=0}$

May 2022

40. Prove that the general equation of the circle whose diameter is the line segment joining the points (-4,-2) and (-1,-1), is $x^2+y^2+5x+3y+6=0$

Example 5.4 (values changed)

Equation of the circle with end points of the diameter as (x_1,y_1) and (x_2,y_2) given in theorem 5.2 is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow (x+4)(x+1)+(y+2)(y+1)=0$

$x^2+x+4x+4+y^2+y+2y+2=0$

$x^2+y^2+5x+3y+6=0$

45. (b) Show that the angle between the curves $y=x^2$ and $x=y^2$ at (1,1) is $\tan^{-1}\left(\frac{3}{4}\right)$

RD Sharma Mathematics Chapter 16 - Tangents and Normals Exercise 16.5(19) page-42

Given that,

$y=x^2 \longrightarrow (1)$

$x=y^2 \longrightarrow (2)$

Point $=(1,1)$

differentiating ① with respect to x

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}_{(1,1)} \Rightarrow m_1 = 2(1)$$

$$\boxed{m_1 = 2}$$

differentiate ② with respect to x

$$2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$\frac{dy}{dx}_{(1,1)} \Rightarrow m_2 = \frac{1}{2(1)}$$

$$\boxed{m_2 = \frac{1}{2}}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{2-\frac{1}{2}}{1+2\times\frac{1}{2}}\right|$$

$$= \left|\frac{1\frac{1}{2}}{1+1}\right|$$

$$= \left|\frac{3/2}{2}\right|$$

$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

46(a) The distribution function of a Continuous random variable X is:

$$F(x) = \begin{cases} 0, & x<1 \\ \frac{x-1}{4}, & 1\le x\le 5 \\ 1, & x>5 \end{cases}$$ find (i) $P(X<3)$ (ii) $P(2<X<4)$ (iii) $P(3\le X)$

$$F(x) = \begin{cases} 0, & x<1 \\ \frac{x-1}{4}, & 1\le x\le 5 \\ 1, & x>5 \end{cases}$$

(i) $P(X<3) = F(3) = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

(ii) $P(2<X<4) = F(4) - F(2)$

$$= \frac{4-1}{4} - \frac{2-1}{4}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4}$$

$$\boxed{P(2<X<4) = \frac{1}{2}}$$

(iii) $P(3\le X) = P(X\ge 3)$

$$= 1 - P(X<3)$$

$$= 1 - F(3)$$

$$= 1 - \frac{1}{2}$$

$$\boxed{P(3\le X) = \frac{1}{2}}$$

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July 2022

22. If α and β are the roots of $x^2+5x+6=0$, then show that $\alpha^2+\beta^2=13$

$$x^2+5x+6=0$$

$$\alpha+\beta = \frac{-b}{a} = \frac{-5}{1} = -5$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (-5)^2 - 2(6)$$

$$= 25-12$$

$$\boxed{\alpha^2+\beta^2 = 13}$$

23. Find the value of $\sin^{-1}(1)+\cos^{-1}(1)$

$$\sin^{-1}(1)+\cos^{-1}(1) = \frac{\pi}{2}+0 = \frac{\pi}{2}$$

30. Form the differential equation of the curve $y=ax^2+bx+c$, where a, b & c are arbitrary constants.

$$y = ax^2+bx+c \longrightarrow ①$$

$$y_1 = 2ax+b \longrightarrow ②$$

$$y_2 = 2a \Rightarrow \boxed{a = \frac{1}{2}y_2}$$

$$② \Rightarrow y_1 = 2\left(\frac{1}{2}y_2\right)x+b$$

$$y_1 = xy_2+b$$

$$y_1 - xy_2 = b$$

$$\boxed{b = y_1 - xy_2}$$

$$① \Rightarrow y = \frac{1}{2}y_2x^2+(y_1-xy_2)x+c$$

$$y = \frac{1}{2}y_2x^2+xy_1-x^2y_2+c$$

$$2y = y_2x^2+2xy_1-2x^2y_2+2c$$

$$2y = x^2y_2+2xy_1-2x^2y_2+2c$$

$$2y = 2xy_1-x^2y_2+2c$$

$$x^2y_2-2xy_1+2y-2c=0$$

34. Prove that the roots of the equation $x^4-3x^2-4=0$ are $\pm 2, \pm i$.

$$x^4-3x^2-4=0$$

Let $y=x^2$

$$y^2-3y-4=0$$

$$-4 ; -4, +1 ; -3$$

$$y^2-4y+y-4=0$$

$$y(y-4)+1(y-4)=0$$

$$(y-4)(y+1)=0$$

$$y-4=0 \; ; \; y+1=0$$

$$\boxed{y=4} \; ; \; \boxed{y=-1}$$

$$y=4 \Rightarrow x^2=4$$

$$x=\pm 2$$

$$y=-1 \Rightarrow x^2=-1$$

$$x=\pm i$$

$\therefore$ Roots are ± 2 & $\pm i$.

40. Prove that $\int_0^1 xe^x dx = 1$.

NCERT Volume II MISCELLANEOUS EXERCISES (7) - (35) page-353

We know that,

$$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int\left[f'(x)\int g(x)\,dx\right]dx$$

$$f(x)=x \text{ and } g(x)=e^x$$

$$\int_0^1 xe^x dx = \left[x\int e^x dx - \int\left(\frac{dx}{dx}\int e^x dx\right)dx\right]_0^1$$

$$=\left[xe^x - \int 1\cdot e^x dx\right]_0^1$$

$$\int_0^1 xe^x dx = \left[xe^x - e^x\right]_0^1$$

$$=\left[1\cdot e^1 - e^1\right]-\left[0\cdot e^0 - e^0\right]$$

$$=[e-e]-[0-e^0]$$

$$=0-(e^0)$$

$$=0-(-1)$$

$$\int_0^1 xe^x dx = 1$$

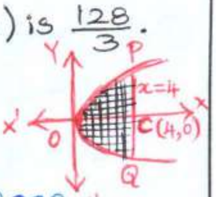
47.(a) Show that the area between the parabola $y^2=16x$ & its latus rectum (using integration) is $\frac{128}{3}$.

Maharashtra Text book Mathematical science PART II

Volume II Page-184

$$y^2=16x$$

$$y=4\sqrt{x}$$



A = Area POCP + Area QOCQ

$= 2$ (Area POCP)

$$=2\int_0^4 y\,dx$$

$$=2\int_0^4 4\sqrt{x}\,dx$$

$$=8\int_0^4 \sqrt{x}\,dx$$

$$=8\int_0^4 (x^{1/2})\,dx$$

$$=8\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$=8\times\frac{2}{3}\left[x^{3/2}\right]_0^4$$

$$=\frac{16}{3}(4)^{3/2}$$

$$=\frac{16}{3}\times(2^2)^{3/2}$$

$$=\frac{16}{3}\times 2^3$$

$$=\frac{16}{3}\times 8$$

$$\boxed{A=\frac{128}{3} \text{ sq.units}}$$

March 2023

30. Express $e^{\cos\theta + i\sin\theta}$ in $a+ib$ form

$$e^{\cos\theta+i\sin\theta} = e^{\cos\theta}\times e^{i\sin\theta} \rightarrow (1)$$

We know that

$$e^{i\theta}=\cos\theta+i\sin\theta$$

$$(1)\Rightarrow e^{\cos\theta}\left[\cos(\sin\theta)+i\sin(\sin\theta)\right]$$

$$=e^{\cos\theta}\cdot\cos(\sin\theta)+ie^{\cos\theta}\cdot\sin(\sin\theta).$$

40. If $a+b+c=0$ and a,b,c are rational numbers then, prove that the roots of the equation $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$ are rational numbers.

We need to show that discriminant is a perfect square.

$$A=(b+c-a),\; B=(c+a-b),\; C=(a+b-c)$$

$$\Delta = B^2-4AC$$

$$=(c+a-b)^2-4(b+c-a)\cdot(a+b-c)$$

$$=(-b-b)^2-4(-a-a)(-c-c)$$

$$=(-2b)^2-4(-2a)(-2c)$$

$[\because a+b+c=0$; $a+c=-b$; $b+c=-a$; $a+b=-c]$

$$=4b^2-16ac$$

$$=4(b^2-4ac)$$

$$=4\left((a+c)^2-4ac\right)$$

$$=4\left[(a-c)^2\right]$$

$$=\left[2(a-c)\right]^2$$

$\therefore$ The roots are rational

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47 (a) Find the maximum value of $\frac{\log x}{x}$.

Given that, RD sharma Exercise 18.6(10) page-80

$$f(x) = \frac{\log x}{x}$$

For critical points $f'(x) = 0$

$$f'(x) = \frac{x \times \frac{1}{x} - \log x (1)}{x^2}$$

$$f'(x) = \frac{1 - \log x}{x^2}$$

$$\frac{1-\log x}{x^2} = 0$$

$$1 - \log x = 0 \times x^2$$

$$1 - \log x = 0$$

$$1 = \log x$$

$$\log e = \log x$$

$$\Rightarrow e = x$$

$$\boxed{x = e}$$

$$f''(x) = \frac{-3x + 2x \log x}{x^4}$$

$$f''(e) = \frac{-3e + 2e \log e}{e^4}$$

$$= \frac{-1}{e^3} < 0$$

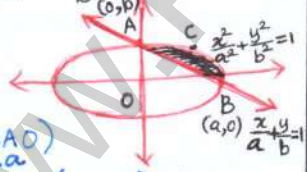
$$f''(e) < 0$$

Thus maximum value of $f(x)$ is $f(e)$

$$f(e) = \frac{\log e}{e} = \frac{1}{e}$$

$\therefore$ maximum value $= \frac{1}{e}$.

47 (b) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.



Area of BCAB

$= \text{Area}(OBCAO) - \text{Area}(OBAO)$

$$= \int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\, dx - \int_0^a b\left(1 - \frac{x}{a}\right) dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx - \frac{b}{a}\int_0^a (a - x)\, dx$$

$$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax - \frac{x^2}{2}\right\}_0^a\right]$$

$$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2 - \frac{a^2}{2}\right\}\right]$$

$$= \frac{b}{a}\left[\frac{a^2\pi}{4} - \frac{a^2}{2}\right]$$

$$= \frac{ba^2}{2a}\left[\frac{\pi}{2} - 1\right]$$

$$= \frac{ab}{2}\left[\frac{\pi}{2} - 1\right]$$

$$= \frac{ab}{4}(\pi - 2)$$

NCERT MISCELLANEOUS EXERCISE 8 9th sum - volume II page-375

MARCH 2024.

30. Show that the vectors $2\hat{i} - \hat{j} + 3\hat{k}$, $\hat{i} - \hat{j}$ and $3\hat{i} - \hat{j} + 6\hat{k}$ are coplanar.

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j}$$

$$\vec{c} = 3\hat{i} - \hat{j} + 6\hat{k}$$

The 3 vectors are said to be co-planar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 0 \\ 3 & -1 & 6 \end{vmatrix}$$

$$= 2(-6+0) + 1(6-0) + 3(-1+3)$$

$$= 2(-6) + 1(6) + 3(2)$$

$$= -12 + 6 + 6$$

$$= -12 + 12$$

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0$$

Hence the given vectors are coplanar

40. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$, then find $|\text{adj}(\text{adj} A)|$.

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix}$$

$$= 2(9-2) + 1(-15+3) + 3(-10+9)$$

$$= 2(7) + 1(-12) + 3(-1)$$

$$= 14 - 12 - 3$$

$$= 14 - 15$$

$$\boxed{|A| = -1}$$

$$|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$$

$$= (-1)^{(3-1)^2}$$

$$= (-1)^{2^2}$$

$$= (-1)^4$$

$$= 1$$

$$\therefore |\text{adj}(\text{adj} A)| = 1$$

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45 (b) Find the vector and cartesian equations of the plane containing $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$. page I 92

Old text book Exercise 2.8(7)

Since the required plane is containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ it contains the point (2,2,1) and parallel to the vector $2\vec{i}+3\vec{j}+3\vec{k}$.

The plane parallel to the line

$$\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$$

It is parallel to $3\vec{i}+2\vec{j}+\vec{k}$.

The equation of a plane passing through a point whose position vector is $\vec{a}$ and parallel to the vectors $\vec{u}$ and $\vec{v}$ is

$$\vec{r}=\vec{a}+s\vec{u}+t\vec{v}$$

$$\vec{a}=2\vec{i}+2\vec{j}+\vec{k}$$

$$\vec{u}=2\vec{i}+3\vec{j}+3\vec{k}$$

$$\vec{v}=3\vec{i}+2\vec{j}+\vec{k}$$

$$\therefore \vec{r}=(2\vec{i}+2\vec{j}+\vec{k})+s(2\vec{i}+3\vec{j}+3\vec{k})+t(3\vec{i}+2\vec{j}+\vec{k})$$

Cartesian form:

$(x_1,y_1,z_1)=(2,2,1)$

$(l_1,m_1,n_1)=(2,3,3)$

$(l_2,m_2,n_2)=(3,2,1)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0$$

$$(x-2)(3-6)-(y-2)(2-9)+(z-1)(4-9)=0$$

$$(x-2)(-3)-(y-2)(-7)+(z-1)(-5)=0$$

$$-3x+6+7y-14-5z+5=0$$

$$-3x+7y-5z-3=0$$

$$3x-7y+5z+3=0$$

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47 (a) Show that $p\leftrightarrow q \equiv ((\sim p)\vee q)\wedge((\sim q)\vee p)$ Volume II

Old text book Exercise 9.3(4). page 145

$p\leftrightarrow q$

p	q	$p\leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$((\sim p)\vee q)\wedge((\sim q)\vee p)$

p	q	$\sim p$	$\sim q$	$(\sim p)\vee q$ (m)	$(\sim q)\vee p$ (n)	$m\wedge n$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$$\therefore p\leftrightarrow q \equiv ((\sim p)\vee q)\wedge((\sim q)\vee p)$$