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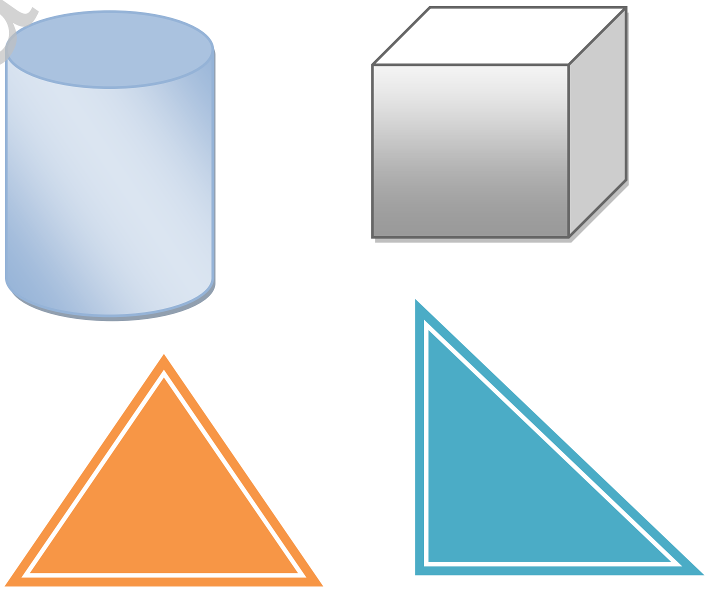
10th MATHEMATICS

COMPLETE QUESTION BANK CHAPTER WISE-2025-2026

NEW SYLLABUS 2024-2025



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10 TH MATHEMATICS COMPLETE QUESTION BANK 2024-2025

CHAPTER - 1. RELATIONS AND FUNCTIONS

Example 1.1 If $A = \{1,3,5\}$ and $B = \{2,3\}$ then **(i)** find $A \times B$ and $B \times A$.
(ii) Is $A \times B = B \times A$? If not why? **(iii)** Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$

Example 1.2 If $A \times B = \{(3,2),(3,4),(5,2),(5,4)\}$ then find A and B.

Example 1.3 Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}, B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$.
Then verify that **(i)** $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Exercise 1.1

1. Find $A \times B, A \times A$ and $B \times A$
(i) $A = \{2,-2,3\}$ and $B = \{1,-4\}$ (ii) $A = B = \{p,q\}$ (iii) $A = \{m,n\}; B = \phi$
2. Let $A = \{1,2,3\}$ and $B = \{x \mid x$ is a prime number less than $10\}$. Find $A \times B$ and $B \times A$.
3. If $B \times A = \{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$ find A and B.
4. If $A = \{5,6\}, B = \{4,5,6\}, C = \{5,6,7\}$, Show that $A \times A = (B \times B) \cap (C \times C)$.
5. Given $A = \{1,2,3\}, B = \{2,3,5\}, C = \{3,4\}$ and $D = \{1,3,5\}$, check if
$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?
6. Let $A = \{x \in \mathbb{W} \mid x < 2\}, B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$ and $C = \{3,5\}$. Verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
7. Let $A =$ The set of all natural numbers less than 8, $B =$ The set of all prime numbers less than 8, $C =$ The set of even prime number. Verify that **(i)** $(A \cap B) \times C = (A \times C) \cap (B \times C)$
(ii) $A \times (B - C) = (A \times B) - (A \times C)$.

Example 1.4 Let $A = \{3,4,7,8\}$ and $B = \{1,7,10\}$. Which of the following sets are relations from A to B?
(i) $R_1 = \{(3,7),(4,7),(7,10),(8,1)\}$
(ii) $R_2 = \{(3,1),(4,12)\}$
(iii) $R_3 = \{(3,7),(4,10),(7,7),(7,8),(8,11),(8,7),(8,10)\}$

Example 1.5 The arrow diagram shows (Fig.1.10) a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.

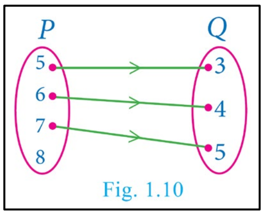


Fig. 1.10

Exercise 1.2

1. Let $A = \{1,2,3,7\}$ and $B = \{3,0,-1,7\}$, which of the following are relation from A to B ?
(i) $R_1 = \{(2,1),(7,1)\}$ (ii) $R_2 = \{(-1,1)\}$
(iii) $R_3 = \{(2,-1),(7,7),(1,3)\}$ (iv) $R_4 = \{(7,-1),(0,3),(3,3),(0,7)\}$
2. Let $A = \{1,2,3,4,\ldots,45\}$ and R be the relation defined as "square is of a number" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R.
3. A Relation R is given by the set $\{(x,y)/y = x+3, x \in \{0,1,2,3,4,5\}\}$. Determine its domain and range.
4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.
(i) $\{(x,y) \mid x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$
(ii) $\{(x,y) \mid y = x+3, x, y$ are natural numbers $< 10\}$

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹ 25,000 , ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Example 1.6 Let $X = \{1,2,3,4\}$ and $Y = \{2,4,6,8,10\}$ and $R = \{(1,2), (2,4), (3,6), (4,8)\}$. Show that R is a function and find its domain, co-domain and range?

Example 1.7 A relation $f: X \to Y$ is defined by $f(x) = x^2 - 2$ where, $X = \{-2, -1, 0, 3\}$ and $Y = \mathbb{R}$.
(i) List the elements of f (ii) Is f a function?

Example 1.8 If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$, then which of the following relations are functions from X to Y ?
(i) $R_1 = \{(-5, a), (1, a), (3, b)\}$
(ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$
(iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Example 1.9 Given $f(x) = 2x - x^2$, find (i) $f(1)$ (ii) $f(x+1)$ (iii) $f(x) + f(1)$

Exercise 1.3

1. Let $f = \{(x, y) \mid x, y \in N \text{ and } y = 2x\}$ be a relation on N . Find the domain, co-domain and range. Is this relation a function?
2. Let $X = \{3,4,6,8\}$. Determine whether the relation $R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$ is a function from X to N ?
3. Given the function $f: x \to x^2 - 5x + 6$, evaluate
(i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x-1)$
4. A graph representing the function $f(x)$ is given in Figure. It is clear that $f(9) = 2$.
(i) Find the following values of the function
(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
(ii) For what value of x is $f(x) = 1$?
(iii) Describe the following (i) Domain (ii) Range.
(iv) What is the image of 6 under f ?

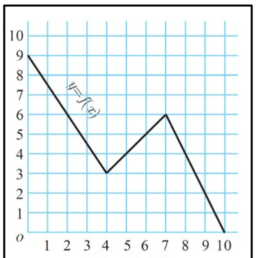


Fig. 1.16

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$.
6. A function f is defined by $f(x) = 2x - 3$
(i) find $\frac{f(0)+f(1)}{2}$.
(ii) find x such that $f(x) = 0$.
(iii) find x such that $f(x) = x$.
(iv) find x such that $f(x) = f(1 - x)$.
7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown (Fig.1.17). Express the volume V of the box as a function of x.

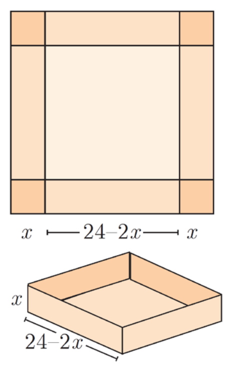


Fig. 1.17

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.
9. A plane is flying at a speed of 500 km per hour. Express the distance ' d ' travelled by the plane as function of time t in hours.
10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants.

(i) Check if this relation is a function.
(ii) Find a and b.
(iii) Find the height of a person whose forehand length is 40 cm
(iv) Find the length of forehand of a person if the height is 53.3 inches.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

Example 1.10 Using vertical line test, determine which of the following curves (Fig.1.18(a), 1.18(b), 1.18(c), 1.18(d)) represent a function?

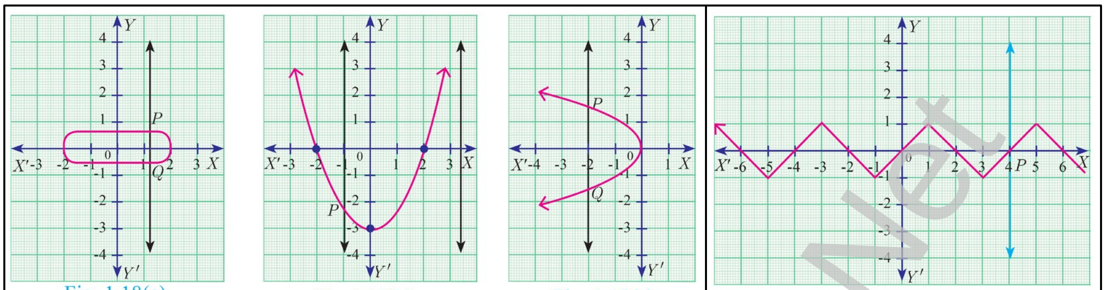


Fig. 1.18(a) Fig. 1.18(b) Fig. 1.18(c) Fig. 1.18(d)

Example 1.11 Let $A = \{1,2,3,4\}$ and $B = \{2,5,8,11,14\}$ be two sets. Let $f: A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function
(i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs
(iv) in a graphical form

Example 1.12 Using horizontal line test (Fig.1.35 (a), 1.35 (b), 1.35 (c)), determine which of the following functions are one - one.

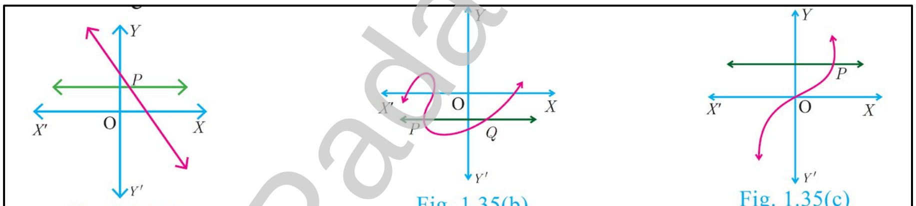


Fig. 1.35(a) Fig. 1.35(b) Fig. 1.35(c)

Example 1.13 Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that f is one - one but not onto function.

Example 1.14 If $A = \{-2,-1,0,1,2\}$ and $f: A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Example 1.15 Let f be a function $f: \mathbb{N} \to \mathbb{N}$ be defined by $f(x) = 3x + 2, x \in \mathbb{N}$
(i) Find the images of 1,2,3 (ii) Find the pre-images of 29,53
(ii) Identify the type of function

Example 1.16 Forensic scientists can determine the height (in cm) of a person based on the length of the thigh bone. They usually do so using the function $h(b) = 2.47b + 54.10$ where b is the length of the thigh bone.
(i) Verify the function h is one - one or not.
(ii) Also find the height of a person if the length of his thigh bone is 50 cm .
(iii) Find the length of the thigh bone if the height of a person is 147.96 cm .

Example 1.17 Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

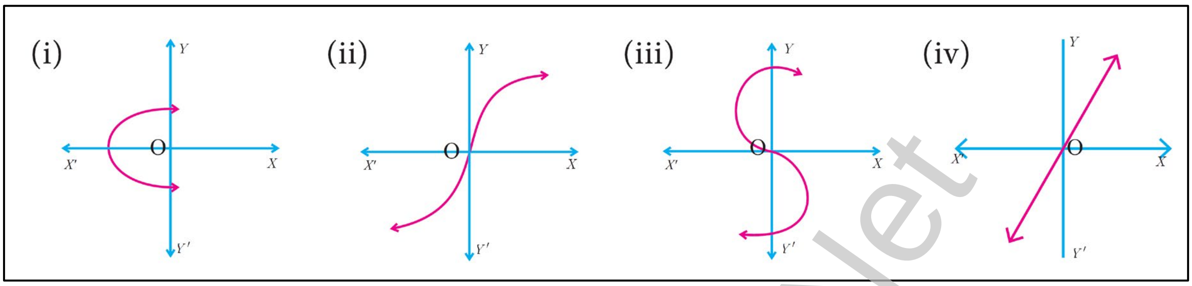
Example 1.18 If the function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=\begin{cases}2x+7; & x<-2\\ x^2-2; & -2\le x<3,\\ 3x-2; & x\ge 3\end{cases}$

then find the values of

(i) $f(4)$ (ii) $f(-2)$ (iii) $f(4)+2f(1)$ (iv) $\frac{f(1)-3f(4)}{f(-3)}$

Exercise 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



2. Let $f:A\to B$ be a function defined by $f(x)=\frac{x}{2}-1$, where $A=\{2,4,6,10,12\}$, $B=\{0,1,2,4,5,9\}$. Represent f by
(i) set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a graph
3. Represent the function $f=\{(1,2),(2,2),(3,2),(4,3),(5,4)\}$ through
(i) an arrow diagram (ii) a table form (iii) a graph
4. Show that the function $f:\mathbb{N}\to\mathbb{N}$ defined by $f(x)=2x-1$ is one-one but not onto.
5. Show that the function $f:\mathbb{N}\to\mathbb{N}$ defined by $f(m)=m^2+m+3$ is one-one function.
6. Let $A=\{1,2,3,4\}$ and $B=\mathbb{N}$. Let $f:A\to B$ be defined by $f(x)=x^3$ then,
(i) find the range of f (ii) identify the type of function
7. In each of the following cases state whether the function is bijective or not. Justify your answer.
(i) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+1$ (ii) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=3-4x^2$
8. Let $A=\{-1,1\}$ and $B=\{0,2\}$. If the function $f:A\to B$ defined by $f(x)=ax+b$ is an onto function? Find a and b.
9. If the function f is defined by $f(x)=\begin{cases}x+2; & x>1\\ 2; & -1\le x\le 1\\ x-1; & -3<x<-1\end{cases}$ find the values of
(i) $f(3)$ (ii) $f(0)$ (iii) $f(-1.5)$ (iv) $f(2)+f(-2)$
10. A function $f:[-5,9]\to\mathbb{R}$ is defined as follows:
$$f(x)=\begin{cases}6x+1; & -5\le x<2\\ 5x^2-1; & 2\le x<6\\ 3x-4; & 6\le x\le 9\end{cases}$$
Find (i) $f(-3)+f(2)$ (ii) $f(7)-f(1)$ (iii) $2f(4)+f(8)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$
11. The distance S an object travels under the influence of gravity in time t seconds is given by $S(t)=\frac{1}{2}gt^2+at+b$ where, (g is the acceleration due to gravity), a,b are constants. Verify wheather the function $S(t)$ is one-one or not.
12. The function ' t ' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C)=F$ where $F=\frac{9}{5}C+32$. Find,
(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C)=212$
(v) the temperature when the Celsius value is equal to the Faren-heit value.

Example 1.19 Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$

Example 1.20 Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Example 1.21 If $f(x) = 3x - 2, g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

Example 1.22 Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Example 1.23 If $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$

Example 1.24 Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$

Exercise 1.5

1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.
   (i) $f(x) = x - 6, g(x) = x^2$ (ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$
   (iii) $f(x) = \frac{x+6}{3}, g(x) = 3 - x$ (iv) $f(x) = 3 + x, g(x) = x - 4$
   (v) $f(x) = 4x^2 - 1, g(x) = 1 + x$
2. Find the value of k, such that $f \circ g = g \circ f$
   (i) $f(x) = 3x + 2, g(x) = 6x - k$ (ii) $f(x) = 2x - k, g(x) = 4x + 5$
3. If $f(x) = 2x - 1, g(x) = \frac{x+1}{2}$, show that $f \circ g = g \circ f = x$
4. If $f(x) = x^2 - 1, g(x) = x - 2$ find a, if $g \circ f(a) = 1$.
5. Let $A, B, C \subseteq \mathbb{N}$ and a function $f: A \to B$ be defined by $f(x) = 2x + 1$ and $g: B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.
6. Let $f(x) = x^2 - 1$. Find (i) $f \circ f$ (ii) $f \circ f \circ f$
7. If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?
8. Consider the functions $f(x), g(x), h(x)$ as given below. Show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.
   (i) $f(x) = x - 1, g(x) = 3x + 1$ and $h(x) = x^2$
   (ii) $f(x) = x^2, g(x) = 2x$ and $h(x) = x + 4$
   (iii) $f(x) = x - 4, g(x) = x^2$ and $h(x) = 3x - 5$
9. Let $f = \{(-1,3), (0,-1), (2,-9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.
10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit $C(t) = 3t$ is linear.

Exercise 1.6

Choose the Correct Answer.

1. If $n(A \times B) = 6$ and $A = \{1,3\}$ then $n(B)$ is
   (A) 1 (B) 2 (C) 3 (D) 6
2. $A = \{a, b, p\}, B = \{2,3\}, C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$
   (A) 8 (B) 20 (C) 12 (D) 16
3. If $A = \{1,2\}, B = \{1,2,3,4\}, C = \{5,6\}$ and $D = \{5,6,7,8\}$ then state which of the following statement is true.
   (A) $(A \times C) \subset (B \times D)$ (B) $(B \times D) \subset (A \times C)$
   (C) $(A \times B) \subset (A \times D)$ (D) $(D \times A) \subset (B \times A)$
4. If there are 1024 relations from a set $A = \{1,2,3,4,5\}$ to a set B, then the number of elements in B is
   (A) 3 (B) 2 (C) 4 (D) 8
5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is
   (A) $\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$ (C) $\{4,9,25,49,121\}$ (D) $\{1,4,9,25,49,121\}$
6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
   (A) $(2,-2)$ (B) $(5,1)$ (C) $(2,3)$ (D) $(3,-2)$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) n^m (C) $2^{mn} - 1$ (D) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively

(A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)

9. Let $A = \{1,2,3,4\}$ and $B = \{4,8,9,10\}$. A function $f: A \to B$ given by $f = \{(1,4), (2,8), (3,9), (4,10)\}$ is a (A) Many-one function (B) Identity function

(C) One-to-one function (D) Into function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is

(A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ (C) $\frac{2}{9x^2}$ (D) $\frac{1}{6x^2}$

11. If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to

(A) 7 (B) 49 (C) 1 (D) 14

12. Let f and g be two functions given by $f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$
$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ then the range of $f \circ g$ is

(A) $\{0,2,3,4,5\}$ (B) $\{-4,1,0,2,7\}$ (C) $\{1,2,3,4,5\}$ (D) $\{0,1,2\}$

13. Let $f(x) = \sqrt{1 + x^2}$ then

(A) $f(xy) = f(x) \cdot f(y)$ (B) $f(xy) \geq f(x) \cdot f(y)$

(C) $f(xy) \leq f(x) \cdot f(y)$ (D) None of these

14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are

(A) $(-1,2)$ (B) $(2,-1)$ (C) $(-1,-2)$ (D) $(1,2)$

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is

(A) linear (B) cubic (C) reciprocal (D) quadratic

Unit Exercise - 1

1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2,5)$ are equal, then find x and y.
2. The cartesian product $A \times A$ has 9 elements among which $(-1,0)$ and $(0,1)$ are found. Find the set A and the remaining elements of $A \times A$.
3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$. Find

   (i) $f(0)$ (ii) $f(3)$ (iii) $f(a + 1)$ in terms of a. (Given that $a \geq 0$)
4. Let $A = \{9,10,11,12,13,14,15,16,17\}$ and let $f: A \to N$ be defined by $f(n) =$ the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.
5. Find the domain of the function $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$
6. If $f(x) = x^2, g(x) = 3x$ and $h(x) = x - 2$, Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.
7. Let $A = \{1,2\}$ and $B = \{1,2,3,4\}, C = \{5,6\}$ and $D = \{5,6,7,8\}$. Verify whether A × C is a subset of B × D?
8. If $f(x) = \frac{x-1}{x+1}, x \neq -1$ show that $f(f(x)) = -\frac{1}{x}$, provided $x \neq 0$.
9. The functions f and g are defined by $f(x) = 6x + 8; g(x) = \frac{x-2}{3}$

   (i) Calculate the value of $gg\left(\frac{1}{2}\right)$ (ii) Write an expression for $gf(x)$ in its simplest form.
10. Write the domain of the following real functions

    (i) $f(x) = \frac{2x+1}{x-9}$ (ii) $p(x) = \frac{-5}{4x^2+1}$ (iii) $g(x) = \sqrt{x-2}$ (iv) $h(x) = x + 6$

CHAPTER - 2. NUMBERS AND SEQUENCES

Example 2.1 We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked?

Example 2.2 Find the quotient and remainder when a is divided by b in the following cases
(i) $a = -12, b = 5$ (ii) $a = 17, b = -3$ (iii) $a = -19, b = -4$

Example 2.3 Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Example 2.4 If the Highest Common Factor of 210 and 55 is expressible in the form $55x - 325$, find x.

Example 2.5 Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

Example 2.6 Find the HCF of 396, 504, 636.

Exercise 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2 .
2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.
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3. Prove that the product of two consecutive positive integers is divisible by 2 .
4. When the positive integers a, b and c are divided by 13 , the respective remainders are 9,7 and 10 . Show that $a + b + c$ is divisible by 13 .
5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4 .
6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of
   (i) 340 and 412 (ii) 867 and 255
   (iii) 10224 and 9648 (iv) 84, 90 and 120
7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.
8. If d is the Highest Common Factor of 32 and 60 , find x and y satisfying $d = 32x + 60y$.
9. A positive integer when divided by 88 gives the remainder 61 . What will be the remainder when the same number is divided by 11 ?
10. Prove that two consecutive positive integers are always coprime.

Example 2.7: In the given factorisation, find the numbers m and n.

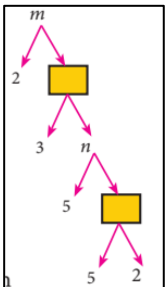


Fig.2.3

Example 2.8: Can the number 6^n, n being a natural number end with the digit 5 ? Give reason for your answer.

Example 2.10: ' a ' and ' b ' are two positive integers such that $a^b \times b^a = 800$. Find ' a ' and ' b '.

Exercise 2.2

1. For what values of natural number n, 4^n can end with the digit 6 ?
2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5 ?
3. Find the HCF of 252525 and 363636.
4. If $13824 = 2^a \times 3^b$ then find a and b.
5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{r_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.
6. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.
7. Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36 ?
8. What is the smallest number that when divided by three numbers such as 35,56 and 91 leaves remainder 7 in each case?
9. Find the least number that is divisible by the first ten natural numbers.

Example 2.11: Find the remainders when 70004 and 778 is divided by 7 .

Example 2.12: Determine the value of d such that $15 \equiv 3(\text{mod}\, d)$.

Example 2.13 Find the least positive value of x such that

(i) $67 + x \equiv 1 (\text{mod} 4)$ (ii) $98 \equiv (x + 4)(\text{mod} 5)$

Example 2.14 Solve $8x \equiv 1(\text{mod} 11)$

Example 2.15 Compute x, such that $10^4 \equiv x(\text{mod} 19)$

Example 2.16 Find the number of integer solutions of $3x \equiv 1(\text{mod} 15)$.

Example 2.17 A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

Example 2.18 Kala and Vani are friends. Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

Exercise 2.3

1. Find the least positive value of x such that
   (i) $71 \equiv x(\text{mod} 8)$ (ii) $78 + x \equiv 3(\text{mod} 5)$
   (iii) $89 \equiv (x + 3)(\text{mod} 4)$ (iv) $96 \equiv \frac{x}{7}(\text{mod} 5)$ (v) $5x \equiv 4(\text{mod} 6)$
2. If x is congruent to 13 modulo 17 then $7x - 3$ is congruent to which number modulo 17 ?
3. Solve $5x \equiv 4(\text{mod} 6)$
4. Solve $3x - 2 \equiv 0(\text{mod} 11)$
5. What is the time 100 hours after 7 a.m.?
6. What is the time 15 hours before 11 p.m.?
7. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?
8. Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.
9. Find the remainder when 2^{81} is divided by 17 .
10. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

Example 2.19 Find the next three terms of the sequences

(i) $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14},$ (ii) $5, 2, -1, -4, \ldots$ (iii) $1, 0.1, 0.01, \ldots$

Example 2.20 Find the general term for the following sequences

(i) $3, 6, 9, \ldots$ (ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ (iii) $5, -25, 125, \ldots$

Example 2.21 The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3) & ; n \in \mathbb{N} \text{ is odd} \\ n^2 + 1 & ; n \in \mathbb{N} \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

Example 2.22 Find the first five terms of the following sequence.

$$a_1 = 1, a_2 = 1, a_n = \frac{a_{n-1}}{a_{n-2} + 3}; n \geq 3, n \in \mathbb{N}$$

Exercise 2.4

1. Find the next three terms of the following sequence.
   (i) $8, 24, 72, \ldots$ (ii) $5, 1, -3, \ldots$ (iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$
2. Find the first four terms of the sequences whose n^{th} terms are given by
   (i) $a_n = n^3 - 2$ (ii) $a_n = (-1)^{n+1} n(n+1)$ (iii) $a_n = 2n^2 - 6$
3. Find the n^{th} term of the following sequences
   (i) $2, 5, 10, 17, \ldots$ (ii) $0, \frac{1}{2}, \frac{2}{3}, \ldots$ (iii) $3, 8, 13, 18, \ldots$

4. Find the indicated terms of the sequences whose n^{th} terms are given by
   (i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13} (ii) $a_n = -(n^2-4)$; a_4 and a_{11}
5. Find a_8 and a_{15} whose n^{th} term is $a_n = \begin{cases} \frac{n^2-1}{n+3} & ; n \text{ is even}, n \in \mathbb{N} \\ \frac{n^2}{2n+1} & ; n \text{ is odd}, n \in \mathbb{N} \end{cases}$
6. If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2} n \geq 3, n \in \mathbb{N}$, then find the first six terms of the sequence.

Example 2.23 Check whether the following sequences are in A.P. or not?
(i) $x+2, 2x+3, 3x+4, \ldots$ (ii) $2,4,8,16, \ldots$ (iii) $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \ldots$

Example 2.24 Write an A.P. whose first term is 20 and common difference is 8.

Example 2.25 Find the 15^{th}, 24^{th} and n^{th} term (general term) of an A.P. given by 3,15 , 27, 39,...

Example 2.26 Find the number of terms in the A.P. 3, 6, 9, 12, ..., 111.

Example 2.27 Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} term is 17 .

Example 2.28 If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that
(i) $x(m-n) + y(n-l) + z(l-m) = 0$ (ii) $(x-y)n + (y-z)l + (z-x)m = 0$

Example 2.29 In an A.P., sum of four consecutive terms is 28 and the sum of their squares is 276 . Find the four numbers.

Example 2.30 A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹ 4623 . Find the amount received by each child.

Exercise 2.5

1. Check whether the following sequences are in A.P.
   (i) $a-3, a-5, a-7, \ldots$ (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ (iii) $9,13,17,21,25, \ldots$
   (iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$ (v) $1, -1, 1, -1, 1, -1, \ldots$
2. First term a and common difference d are given below. Find the corresponding A.P.
   (i) $a = 5, d = 6$ (ii) $a = 7, d = -5$ (iii) $a = \frac{3}{4}, d = \frac{1}{2}$
3. Find the first term and common difference of the Arithmetic Progressions whose n^{th} terms are given below
   (i) $t_n = -3 + 2n$ (ii) $t_n = 4 - 7n$
4. Find the 19^{th} term of an A.P. $-11, -15, -19, \ldots$
5. Which term of an A.P. $16,11,6,1, \ldots$ is -54 ?
6. Find the middle term(s) of an A.P. 9, 15, 21, 27,...,183.
7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.
8. If $3+k, 18-k, 5k+1$ are in A.P. then find k.
9. Find x, y and z, given that the numbers $x, 10, y, 24, z$ are in A.P.
10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?
11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288 . Find the three terms.
12. The ratio of 6^{th} and 8^{th} term of an A.P. is $7:9$. Find the ratio of 9^{th} term to 13^{th} term.
13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0°C and the sum of the temperatures from Wednesday to Friday is 18°C. Find the temperature on each of the five days.
14. Priya earned ₹ 15,000 in the first month. Thereafter her salary increased by ₹ 1500 per year. Her expenses are ₹ 13,000 during the first month and the expenses increases by ₹ 900 per year. How long will it take for her to save ₹ 20,000 per month.

Example 2.31 Find the sum of first 15 terms of the A. P. 8, $7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \ldots$

Example 2.32 Find the sum of $0.40 + 0.43 + 0.46 + \cdots + 1$.

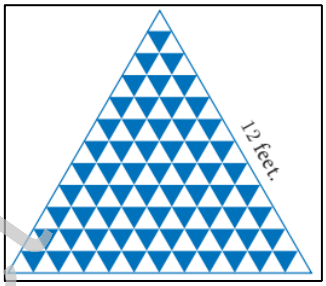
Example 2.33 How many terms of the series $1 + 5 + 9 + \cdots$ must be taken so that their sum is 190?

Example 2.34 The 13^{th} term of an A.P. is 3 and the sum of first 13 terms is 234 . Find the common difference and the sum of first 21 terms.

Example 2.35 In an A.P. the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$. Find the 17^{th} term.

Example 2.36 Find the sum of all natural numbers between 300 and 600 which are divisible by 7 .

Example 2.37 A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



Example 2.38 The houses of a street are numbered from 1 to 49 . Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number?

Example 2.39 The sum of first $n, 2n$ and $3n$ terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Exercise 2.6

1. Find the sum of the following

(i) 3,7,11, ... up to 40 terms. (ii) 102,97,92, ... up to 27 terms. (iii) $6 + 13 + 20 + \cdots + 97$

2. How many consecutive odd integers beginning with 5 will sum to 480 ?
3. Find the sum of first 28 terms of an A.P. whose n^{th} term is $4n - 3$.
4. The sum of first n terms of a certain series is given as $2n^2 - 3n$. Show that the series is an A.P.
5. The 104^{th} term and 4^{th} term of an A.P. are 125 and 0 . Find the sum of first 35 terms.
6. Find the sum of all odd positive integers less than 450.
7. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4 .
8. Raghu wish to buy a laptop. He can buy it by paying ₹ 40,000 cash or by giving it in 10 installments as ₹ 4800 in the first month, ₹ 4750 in the second month, ₹ 4700 in the third month and so on. If he pays the money in this fashion, find
   (i) total amount paid in 10 installments.
   (ii) how much extra amount that he has to pay than the cost?
9. A man repays a loan of ₹ 65,000 by paying ₹ 400 in the first month and then increasing the payment by ₹ 300 every month. How long will it take for him to clear the loan?
10. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.
    (i) How many bricks are required for the top most step?
    (ii) How many bricks are required to build the stair case?
11. If $S_1, S_2, S_3, \ldots, S_m$ are the sums of n terms of m A.P.'s whose first terms are $1,2,3, \ldots, m$ and whose common differences are $1,3,5, \ldots, (2m - 1)$ respectively, then

    show that $S_1 + S_2 + S_3 + \cdots + S_m = \frac{1}{2}mn(mn + 1)$.
12. Find the sum $\left[\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots \text{ to 12 terms }\right]$.

Example 2.40 Which of the following sequences form a Geometric Progression?

(i) 7,14,21,28, ... (ii) $\frac{1}{2}, 1,2,4, \ldots$ (iii) 5,25,50,75, ...

Example 2.42 Find the 8^{th} term of the G.P. 9, 3, 1, ...

Example 2.43 In a Geometric progression, the 4^{th} term is $\frac{8}{9}$ and the 7^{th} term is $\frac{64}{243}$. Find the Geometric Progression.

Example 2.44 The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Example 2.45 The present value of a machine is ₹ 40,000 and its value depreciates each year by 10%. Find the estimated value of the machine in the 6^{th} year.

Exercise 2.7

1. Which of the following sequences are in G.P.?
   (i) $3,9,27,81,\ldots$ (ii) $4,44,444,4444,\ldots$ (iii) $0.5,0.05,0.005,\ldots$ (iv) $\frac{1}{3},\frac{1}{6},\frac{1}{12},\ldots$
   (v) $1,-5,25,-125,\ldots$ (vi) $120,60,30,18,\ldots$ (vii) $16,4,1,\frac{1}{4},\ldots$
2. Write the first three terms of the G.P. whose first term and the common ratio are given below.
   (i) $a=6, r=3$ (ii) $a=\sqrt{2}, r=\sqrt{2}$ (iii) $a=1000, r=\frac{2}{5}$
3. In a G.P. $729,243,81,\ldots$ find t_7.
4. Find x so that $x+6, x+12$ and $x+15$ are consecutive terms of a Geometric Progression.
5. Find the number of terms in the following G.P.
   (i) $4,8,16,\ldots,8192$? (ii) $\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots,\frac{1}{2187}$
6. In a G.P. the 9^{th} term is 32805 and 6^{th} term is 1215 . Find the 12^{th} term.
7. Find the 10^{th} term of a G.P. whose 8^{th} term is 768 and the common ratio is 2 .
8. If a,b,c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.
9. In a G.P. the product of three consecutive terms is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms.
10. A man joined a company as Assistant Manager. The company gave him a starting salary of ₹ 60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?
11. Sivamani is attending an interview for a job and the company gave two offers to him. Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years. Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years. What is his salary in the 4^{th} year with respect to the offers A and B ?
12. If a,b,c are three consecutive terms of an A.P. and x,y,z are three consecutive terms of a G.P. then prove that $x^{b-c}\times y^{c-a}\times z^{a-b}=1$.

Example 2.46 Find the sum of 8 terms of the G.P. $1,-3,9,-27\ldots$

Example 2.47 Find the first term of a G.P. in which $S_6=4095$ and $r=4$.

Example 2.48 How many terms of the series $1+4+16+\cdots$ make the sum 1365 ?

Example 2.49 Find the sum $3+1+\frac{1}{3}+\cdots\infty$

Example 2.50 Find the rational form of the number $0.6666\ldots$

Example 2.51 Find the sum to n terms of the series $5+55+555+\cdots$

Example 2.52 Find the least positive integer n such that $1+6+6^2+\cdots+6^n>5000$.

Example 2.53 A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year?

Exercise 2.8

1. Find the sum of first n terms of the G.P. (i) $5,-3,\frac{9}{5},-\frac{27}{25},\ldots$ (ii) $256,64,16,\ldots$
2. Find the sum of first six terms of the G.P. $5,15,45,\ldots$
3. Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872 .

4. Find the sum to infinity of (i) $9+3+1+\cdots$ (ii) $21+14+\frac{28}{3}+\cdots$
5. If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.
6. Find the sum to n terms of the series
(i) $0.4+0.44+0.444+\cdots$ to n terms (ii) $3+33+333+\cdots$ to n terms
7. Find the sum of the Geometric series $3+6+12+\cdots+1536$.
8. Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹2 to mail one letter, find the amount spent on postage when 8^{th} set of letters is mailed.
9. Find the rational form of the number $0.\overline{123}$.
10. If $S_n=(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots n$ terms then prove that
$$(x-y)S_n=\left[\frac{x^2(x^n-1)}{x-1}-\frac{y^2(y^n-1)}{y-1}\right]$$

Example 2.54 Find the value of (i) $1+2+3+\cdots+50$ (ii) $16+17+18+\cdots+75$

Example 2.55 Find the sum of (i) $1+3+5+\cdots$ to 40 terms (ii) $2+4+6+\cdots+80$ (iii) $1+3+5+\cdots+55$

Example 2.56 Find the sum of (i) $1^2+2^2+\cdots+19^2$ (ii) $5^2+10^2+15^2+\cdots+105^2$ (iii) $15^2+16^2+17^2+\cdots+28^2$

Example 2.57 Find the sum of (i) $1^3+2^3+3^3+\cdots+16^3$ (ii) $9^3+10^3+\cdots+21^3$

Example 2.58 If $1+2+3+\cdots+n=666$ then find n.

Exercise 2.9

1. Find the sum of the following series
(i) $1+2+3+\cdots+60$ (ii) $3+6+9+\cdots+96$ (iii) $51+52+53+\cdots+92$
(iv) $1+4+9+16+\cdots+225$ (v) $6^2+7^2+8^2+\cdots+21^2$
(vi) $10^3+11^3+12^3+\cdots+20^3$ (vii) $1+3+5+\cdots+71$
2. If $1+2+3+\cdots+k=325$, then find $1^3+2^3+3^3+\cdots+k^3$.
3. If $1^3+2^3+3^3+\cdots+k^3=44100$ then find $1+2+3+\cdots+k$.
4. How many terms of the series $1^3+2^3+3^3+\cdots$ should be taken to get the sum 14400 ?
5. The sum of the cubes of the first n natural numbers is 2025 , then find the value of n.
6. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, … ,24 cm. How much area can be decorated with these colour papers?
7. Find the sum of the series $(2^3-1^3)+(4^3-3^3)+(6^3-5^3)+\cdots$ to
(i) n terms (ii) 8 terms

Exercise 2.10

Multiple choice questions

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a=bq+r$, where r must satisfy.
(A) $1<r<b$ (B) $0<r<b$ (C) $0\le r<b$ (D) $0<r\le b$
2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are
(A) 0,1,8 (B) 1,4,8 (C) 0,1,3 (D) 1,3,5
3. If the HCF of 65 and 117 is expressible in the form of $65m-117$, then the value of m is
(A) 4 (B) 2 (C) 1 (D) 3
4. The sum of the exponents of the prime factors in the prime factorization of 1729 is
(A) 1 (B) 2 (C) 3 (D) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
(A) 2025 (B) 5220 (C) 5025 (D) 2520

6. $7^{4k} \equiv$ ______(mod100)
(A) 1 (B) 2 (C) 3 (D) 4

7. Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
(A) 3 (B) 5 (C) 8 (D) 11

8. The first term of an arithmetic progression is unity and the common difference is 4 . Which of the following will be a term of this A.P.
(A) 4551 (B) 10091 (C) 7881 (D) 13531

9. If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is
(A) 0 (B) 6 (C) 7 (D) 13

10. An A.P. consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is
(A) 16 m (B) 62 m (C) 31 m (D) $\frac{31}{2}$ m

11. In an A.P., the first term is 1 and the common difference is 4 . How many terms of the A.P. must be taken for their sum to be equal to 120 ?
(A) 6 (B) 7 (C) 8 (D) 9

12. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \cdots + 2^0$ which of the following is true?
(A) B is 2^{64} more than A (B) A and B are equal
(C) B is larger than A by 1 (D) A is larger than B by 1

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \cdots$ is
(A) $\frac{1}{24}$ (B) $\frac{1}{27}$ (C) $\frac{2}{3}$ (D) $\frac{1}{81}$

14. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, \ldots$ is
(A) a Geometric Progression (B) an Arithmetic Progression
(C) neither an Arithmetic Progression nor a Geometric Progression
(D) a constant sequence

15. The value of $(1^3 + 2^3 + 3^3 + \cdots + 15^3) - (1 + 2 + 3 + \cdots + 15)$ is
(A) 14400 (B) 14200 (C) 14280 (D) 14520

Unit Exercise - 2

1. Prove that $n^2 - n$ divisible by 2 for every positive integer n.
2. A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.
3. When the positive integers a, b and c are divided by 13 the respective remainders are 9,7 and 10 . Find the remainder when $a + 2b + 3c$ is divided by 13 .
4. Show that 107 is of the form $4q + 3$ for any integer q.
5. If $(m+1)^{\text{th}}$ term of an A.P. is twice the $(n+1)^{\text{th}}$ term, then prove that $(3m+1)^{\text{th}}$ term is twice the $(m+n+1)^{th}$ term.
6. Find the 12^{th} term from the last term of the A. P $-2, -4, -6, \ldots -100$.
7. Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7 . Show that the difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms.
8. A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?
9. Find the G.P. in which the 2^{nd} term is $\sqrt{6}$ and the 6^{th} term is $9\sqrt{6}$.
10. The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000 ?

CHAPTER - 3. ALGEBRA

Example 3.1 The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

Example 3.2 Solve $2x-3y=6, x+y=1$

Example 3.3 Solve the following system of linear equations in three variables

$3x-2y+z=2, 2x+3y-z=5, x+y+z=6.$

Example 3.4 In an interschool atheletic meet, with total of 24 individual prizes, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

Example 3.5 Solve $x+2y-z=5; x-y+z=-2; -5x-4y+z=-11$

Example 3.6 Solve $3x+y-3z=1; -2x-y+2z=1; -x-y+z=2.$

Example 3.7 Solve $\frac{x}{2}-1=\frac{y}{6}+1=\frac{z}{7}+2; \frac{y}{3}+\frac{z}{2}=13$

Example 3.8 Solve : $\frac{1}{2x}+\frac{1}{4y}-\frac{1}{3z}=\frac{1}{4}; \frac{1}{x}=\frac{1}{3y}; \frac{1}{x}-\frac{1}{5y}+\frac{4}{z}=2\frac{2}{15}$

Example 3.9 The sum of thrice the first number, second number and twice the third number is 5 . If thrice the second number is subtracted from the sum of first number and thrice the third we get 2 . If the third number is subtracted from the sum of twice the first, thrice the second, we get 1 . Find the numbers.

Exercise 3.1

1. Solve the following system of linear equations in three variables
   (i) $x+y+z=5; 2x-y+z=9; x-2y+3z=16$
   (ii) $\frac{1}{x}-\frac{2}{y}+4=0;\ \frac{1}{y}-\frac{1}{z}+1=0;\ \frac{2}{z}+\frac{3}{x}=14$
   (iii) $x+20=\frac{3y}{2}+10=2z+5=110-(y+z)$
2. Discuss the nature of solutions of the following system of equations
   (i) $x+2y-z=6; -3x-2y+5z=-12; x-2z=3$
   (ii) $2y+z=3(-x+1); -x+3y-z=-4; 3x+2y+z=-\frac{1}{2}$
   (iii) $\frac{y+z}{4}=\frac{z+x}{3}=\frac{x+y}{2}; x+y+z=27$
3. Vani, her father and her grand father have an average age of 53 . One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now?
4. The sum of the digits of a three-digit number is 11 . If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number?
5. There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹ 105 . When first 2 sorts are interchanged in their numbers its value will be increased by ₹ 20 . Find the number of currencies in each sort.

Example 3.10 Find the GCD of the polynomials x^3+x^2-x+2 and $2x^3-5x^2+5x-3$.

Example 3.11 Find the GCD of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$.

Example 3.12 Find the LCM of the following

(i) $8x^4y^2, 48x^2y^4$ (ii) $5x-10, 5x^2-20$
(iii) x^4-1, x^2-2x+1 (iv) $x^3-27, (x-3)^2, x^2-9$

Exercise 3.2

1. Find the GCD of the given polynomials
   (i) $x^4+3x^3-x-3, x^3+x^2-5x+3$
   (ii) x^4-1, x^3-11x^2+x-11
   (iii) $3x^4+6x^3-12x^2-24x, 4x^4+14x^3+8x^2-8x$
   (iv) $3x^3+3x^2+3x+3, 6x^3+12x^2+6x+12$
2. Find the LCM of the given expressions.
   (i) $4x^2y, 8x^3y^2$ (ii) $9a^3b^2, 12a^2b^2c$ (iii) $16m, 12m^2n^2, 8n^2$ (iv) p^2-3p+2, p^2-4
   (v) $2x^2-5x-3, 4x^2-36$ (vi) $(2x^2-3xy)^2, (4x-6y)^3, 8x^3-27y^3$

Exercise 3.3

1. Find the LCM and GCD for the following and verify that $f(x)\times g(x)=LCM\times GCD$
   (i) $21x^2y, 35xy^2$ (ii) $(x^3-1)(x+1), (x^3+1)$ (iii) $(x^2y+xy^2), (x^2+xy)$
2. Find the LCM of each pair of the following polynomials
   (i) $a^2+4a-12, a^2-5a+6$ whose GCD is $a-2$
   (ii) $x^4-27a^3x, (x-3a)^2$ whose GCD is $(x-3a)$
3. Find the GCD of each pair of the following polynomials
   (i) $12(x^4-x^3), 8(x^4-3x^3+2x^2)$ whose LCM is $24x^3(x-1)(x-2)$
   (ii) $(x^3+y^3), (x^4+x^2y^2+y^4)$ whose LCM is $(x^3+y^3)(x^2+xy+y^2)$
4. Given the LCM and GCD of the two polynomials $p(x)$ and $q(x)$ find the unknown polynomial in the following table

S.No.	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3-10a^2+11a+70$	$a-7$	$a^2-12a+35$	
(ii)	$(x^4-y^4)(x^4+x^2y^2+y^4)$	(x^2-y^2)		$(x^4-y^4)(x^2+y^2-xy)$

Example 3.13 Reduce the rational expressions to its lowest form
(i) $\frac{x-3}{x^2-9}$ (ii) $\frac{x^2-16}{x^2+8x+16}$

Example 3.14 Find the excluded values of the following expressions (if any).
(i) $\frac{x+10}{8x}$ (ii) $\frac{7p+2}{8p^2+13p+5}$ (iii) $\frac{x}{x^2+1}$.

Exercise 3.4

1. Reduce each of the following rational expressions to its lowest form.
   (i) $\frac{x^2-1}{x^2+x}$ (ii) $\frac{x^2-11x+18}{x^2-4x+4}$ (iii) $\frac{9x^2+81x}{x^3+8x^2-9x}$ (iv) $\frac{p^2-3p-40}{2p^3-24p^2+64p}$
2. Find the excluded values, if any of the following expressions.
   (i) $\frac{y}{y^2-25}$ (ii) $\frac{t}{t^2-5t+6}$ (iii) $\frac{x^2+6x+8}{x^2+x-2}$ (iv) $\frac{x^3-27}{x^3+x^2-6x}$

Example 3.15 (i) Multiply $\frac{x^3}{9y^2}$ by $\frac{27y}{x^5}$ (ii) Multiply $\frac{x^4b^2}{x-1}$ by $\frac{x^2-1}{a^4b^3}$

Example 3.16 Find
(i) $\frac{14x^4}{y}\surd\frac{7x}{3y^4}$ (ii) $\frac{x^2-16}{x+4}\div\frac{x-4}{x+4}$ (iii) $\frac{16x^2-2x-3}{3x^2-2x-1}\div\frac{8x^2+11x+3}{3x^2-11x-4}$

Exercise 3.5

1. Simplify
   (i) $\frac{4x^2y}{2z^2}\times\frac{6xz^3}{20y^4}$ (ii) $\frac{p^2-10p+21}{p-7}\times\frac{p^2+p-12}{(p-3)^2}$ (iii) $\frac{5t^3}{4t-8}\times\frac{6t-12}{10t}$

2. Simplify

(i) $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$ (ii) $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$

3. Simplify

(i) $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$ (ii) $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$

(iii) $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$ (iv) $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

4. If $x = \frac{a^2+3a-4}{3a^2-3}$ and $y = \frac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2}.

5. If a polynomial $p(x) = x^2 - 5x - 14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$, find $q(x)$.

Example 3.17 Find $\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28}$

Example 3.18 Simplify $\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$

Exercise 3.6

1. Simplify (i) $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$ (ii) $\frac{x+2}{x+3} + \frac{x-1}{x-2}$ (iii) $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

2. Simplify (i) $\frac{(2x+1)(x-2)}{x-4} - \frac{(2x^2-5x+2)}{x-4}$ (ii) $\frac{4x}{x^2-1} - \frac{x+1}{x-1}$

3. Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

4. Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to get $\frac{3}{x^2-2x+4}$

5. If $A = \frac{2x+1}{2x-1}, B = \frac{2x-1}{2x+1}$ find $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$

6. If $A = \frac{x}{x+1}, B = \frac{1}{x+1}$, prove that $\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}$

7. Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?

8. Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy?

Example 3.19 Find the square root of the following expressions

(i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$ (ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Example 3.20 Find the square root of the following expressions

(i) $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$ (ii) $(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$

(iii) $[\sqrt{15}x^2 + (\sqrt{3} + \sqrt{10})x + \sqrt{2}][\sqrt{5}x^2 + (2\sqrt{5} + 1)x + 2][\sqrt{3}x^2 + (\sqrt{2} + 2\sqrt{3})x + 2\sqrt{2}]$

Exercise 3.7

1. Find the square root of the following rational expressions.

(i) $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$ (ii) $\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$ (iii) $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

2. Find the square root of the following

(i) $4x^2 + 20x + 25$ (ii) $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

(iii) $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

(iv) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Example 3.21 Find the square root of $64x^4-16x^3+17x^2-2x+1$

Example 3.22 If $9x^4+12x^3+28x^2+ax+b$ is a perfect square, find the values of a and b.

Exercise 3.8

1. Find the square root of the following polynomials by division method
   (i) $x^4-12x^3+42x^2-36x+9$ (ii) $37x^2-28x^3+4x^4+42x+9$
   (iii) $16x^4+8x^2+1$ (iv) $121x^4-198x^3-183x^2+216x+144$
2. Find the values of a and b if the following polynomials are perfect squares
   (i) $4x^4-12x^3+37x^2+bx+a$ (ii) $ax^4+bx^3+361x^2+220x+100$
3. Find the values of m and n if the following polynomials are perfect sqaures
   (i) $36x^4-60x^3+61x^2-mx+n$ (ii) $x^4-8x^3+mx^2+nx+16$

Example 3.23 Find the zeroes of the quadratic expression $x^2+8x+12$.

Example 3.24 Write down the quadratic equation in general form for which sum and product of the roots are given below.

(i) 9,14 (ii) $-\frac{7}{2},\frac{5}{2}$ (iii) $-\frac{3}{5},-\frac{1}{2}$

Example 3.25 Find the sum and product of the roots for each of the following quadratic equations:

(i) $x^2+8x-65=0$ (ii) $2x^2+5x+7=0$ (iii) $kx^2-k^2x-2k^3=0$

Exercise 3.9

1. Determine the quadratic equations, whose sum and product of roots are
   (i) $-9,20$ (ii) $\frac{5}{3},4$ (iii) $\frac{-3}{2},-1$ (iv) $-(2-a)^2,(a+5)^2$
2. Find the sum and product of the roots for each of the following quadratic equations
   (i) $x^2+3x-28=0$ (ii) $x^2+3x=0$ (iii) $3+\frac{1}{a}=\frac{10}{a^2}$ (iv) $3y^2-y-4=0$

Example 3.26 Solve $2x^2-2\sqrt{6}x+3=0$

Example 3.27 Solve $2m^2+19m+30=0$

Example 3.28 Solve $x^4-13x^2+42=0$

Example 3.29 Solve $\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$

Exercise 3.10

1. Solve the following quadratic equations by factorization method
   (i) $4x^2-7x-2=0$ (ii) $3(p^2-6)=p(p+5)$ (iii) $\sqrt{a(a-7)}=3\sqrt{2}$
   (iv) $\sqrt{2}x^2+7x+5\sqrt{2}=0$ (v) $2x^2-x+\frac{1}{8}=0$
2. The number of volleyball games that must be scheduled in a league with n teams is given by $G(n)=\frac{n^2-n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

Example 3.30 Solve $x^2-3x-2=0$

Example 3.31 Solve $2x^2-x-1=0$

Example 3.32 Solve $x^2+2x-2=0$ by formula method

Example 3.33 Solve $2x^2-3x-3=0$ by formula method.

Example 3.34 Solve $3p^2+2\sqrt{5}p-5=0$ by formula method.

Example 3.35 Solve $pqx^2 - (p+q)^2x + (p+q)^2 = 0$

Exercise 3.11

1. Solve the following quadratic equations by completing the square method

   (i) $9x^2 - 12x + 4 = 0$ (ii) $\frac{5x+7}{x-1} = 3x + 2$

2. Solve the following quadratic equations by formula method

   (i) $2x^2 - 5x + 2 = 0$ (ii) $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

   (iii) $3y^2 - 20y - 23 = 0$ (iv) $36y^2 - 12ay + (a^2 - b^2) = 0$

3. A ball rolls down a slope and travels a distance $d = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.

Example 3.36 The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Example 3.37 A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets if the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?

Example 3.38 A flock of swans contained x^2 members. As the clouds gathered, $10x$ went to a lake and one-eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

Example 3.39 A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of the express train is more than that of the passenger train by 20 km per hour. Find the average speed of both the trains.

Exercise 3.12

1. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.
2. A garden measuring 12 m by 16 m is to have a pedestrian pathway that is ' w ' meters wide installed all the way around so that it increases the total area to 285 m^2. What is the width of the pathway?
3. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/ hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.
4. A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375 . Find their present ages.
5. A pole has to be erected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4 m . Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?
6. From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?
7. Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m . Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).
8. There is a square field whose side is 10 m . A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square metre respectively is ₹ 364 . Find the width of the gravel path.

9. The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm . Find the length of the smallest side.

Example 3.40 Determine the nature of roots for the following quadratic equations
(i) $x^2 - x - 20 = 0$ (ii) $9x^2 - 24x + 16 = 0$ (iii) $2x^2 - 2x + 9 = 0$

Example 3.41 (i) Find the values of ' k ', for which the quadratic equation $kx^2 - (8k + 4)x + 81 = 0$ has real and equal roots?
(ii) Find the values of ' k ' such that quadratic equation $(k + 9)x^2 + (k + 1)x + 1 = 0$ has no real roots?

Example 3.42 Prove that the equation $x^2(p^2 + q^2) + 2x(pr + qs) + r^2 + s^2 = 0$ has no real roots. If $ps = qr$, then show that the roots are real and equal.

Exercise 3.13

1. Determine the nature of the roots for the following quadratic equations
(i) $15x^2 + 11x + 2 = 0$ (ii) $x^2 - x - 1 = 0$ (iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$
(iv) $9y^2 - 6\sqrt{2}y + 2 = 0$ (v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$
2. Find the value(s) of ' k ' for which the roots of the following equations are real and equal.
(i) $(5k - 6)x^2 + 2kx + 1 = 0$ (ii) $kx^2 + (6k + 2)x + 16 = 0$
3. If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression.
4. If a, b are real then show that the roots of the equation $(a - b)x^2 - 6(a + b)x - 9(a - b) = 0$ are real and unequal.
5. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$

Example 3.43 If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Example 3.44 If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of
(i) $(\alpha - \beta)$ (ii) $\alpha^2 + \beta^2$ (iii) $\alpha^3 - \beta^3$
(iv) $\alpha^4 + \beta^4$ (v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Example 3.45 If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of
(i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Example 3.46 If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are (i) $\frac{1}{\alpha}, \frac{1}{\beta}$ (ii) $\alpha^2\beta, \beta^2\alpha$ (iii) $2\alpha + \beta, 2\beta + \alpha$

Exercise 3.14

1. Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.
(i) $\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha}$ (ii) $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$ (iii) $(3\alpha - 1)(3\beta - 1)$ (iv) $\frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha}$
2. The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. Without solving for the roots, find
(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (iii) $\frac{\alpha+2}{\beta+2} + \frac{\beta+2}{\alpha+2}$
3. The roots of the equation $x^2 + 6x - 4 = 0$ are α, β. Find the quadratic equation whose roots are
(i) α^2 and β^2 (ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ (iii) $\alpha^2\beta$ and $\beta^2\alpha$
4. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$. Find the values of a.
5. If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a.
6. If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.

Example 3.47 Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm .

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

Example 3.48 A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

(i) the constant of variation

(ii) how far will it travel in 90 minutes?

(iii) the time required to cover a distance of 300 km from the graph..

Example 3.49 A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation.

(ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers?

(iii) If the work has to be completed by 200 days, how many workers are required?

Example 3.50 Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2hrs, 3hrs, 4hrs and 6 hours respectively.

Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

Exercise 3.15

1. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find
   (i) the marked price when a customer gets a discount of ₹3250 (from graph)
   (ii) the discount when the marked price is ₹2500
2. Draw the graph of $xy = 24, x, y > 0$. Using the graph find,
   (i) y when $x = 3$ and (ii) x when $y = 6$.
3. Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$ (ii) find x when $y = 7.5$.
4. The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

(ii) Find the number of pipes when the time is 9 minutes.

5. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

6. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr ; (ii) find the parking duration when the amount paid is ₹ 150 .

Example 3.51 Discuss the nature of solutions of the following quadratic equations.

(i) $x^2 + x - 12 = 0$ (ii) $x^2 - 8x + 16 = 0$ (iii) $x^2 + 2x + 5 = 0$

Example 3.52 Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$

Example 3.53 Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$

Example 3.54 Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

Example 3.55 Draw the graph of $y = x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$

Exercise 3.16

1. Graph the following quadratic equations and state their nature of solutions.

(i) $x^2 - 9x + 20 = 0$ (ii) $x^2 - 4x + 4 = 0$ (iii) $x^2 + x + 7 = 0$

(iv) $x^2 - 9 = 0$ (v) $x^2 - 6x + 9 = 0$ (vi) $(2x - 3)(x + 2) = 0$

2. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$

3. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$

4. Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

5. Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$

6. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$

7. Draw the graph of $y = 2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$

8. Draw the graph of $y = (x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$

Example 3.56 Consider the following information regarding the number of men and women workers in three factories I, II and III.

Factory	Men	Women
I	23	18
II	47	36
III	15	16

Example 3.57 If a matrix has 16 elements, what are the possible orders it can have?

Example 3.58 Construct a 3 × 3 matrix whose elements are $a_{ij} = i^2 j^2$

Example 3.59 Find the value of a, b, c, d from the equation $\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$

Exercise 3.17

1. In the matrix $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$, write (i) The number of elements

(ii) The order of the matrix (iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$.

2. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

3. Construct a 3×3 matrix whose elements are given by

(i) $a_{ij} = |i - 2j|$ (ii) $a_{ij} = \frac{(i+j)^3}{3}$

4. If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

5. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of $-A$.

6. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

7. Find the values of x, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$ (ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$ (iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Example 3.60 If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find $A + B$.

Example 3.61 Two examinations were conducted for three groups of students namely group 1, group 2, group 3 and their data on average of marks for the subjects Tamil, English, Science and Mathematics are given below in the form of matrices A and B. Find the total marks of both the examinations for all the three groups.

$A =$

	Tamil	English	Science	Mathematics
Group 1	22	15	14	23
Group 2	50	62	21	30
Group 3	53	80	32	40

$A =$

	Tamil	English	Science	Mathematics
Group 1	20	38	15	40
Group 2	18	12	17	80
Group 3	81	47	52	18

Example 3.62 If $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}, B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$, find $A + B$.

Example 3.63 If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}, B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find $2A + B$.

Example 3.64 If $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}, B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$, find $4A - 3B$.

Example 3.65 Find the value of a, b, c, d from the following matrix equation.

$$\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$$

Example 3.66 If $A = \begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}, B = \begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute the following :

(i) $3A + 2B - C$ (ii) $\frac{1}{2}A - \frac{3}{2}B$

Exercise 3.18

1. If $A = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}, B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ then verify that
   (i) $A + B = B + A$ (ii) $A + (-A) = (-A) + A = O$.
2. If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ then verify that $A + (B + C) = (A + B) + C$.
3. Find X and Y if $X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$
4. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find the value of (i) $B - 5A$ (ii) $3A - 9B$
5. Find the values of x, y, z if (i) $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$
   (ii) $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$
6. Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
7. Find the non-zero values of x satisfying the matrix equation
   $$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$
8. Solve for x, y: $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

Example 3.67 If $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$, find AB.

Example 3.68 If $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ find AB and BA. Verify $AB = BA$?

Example 3.69 If $A = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$

Show that A and B satisfy commutative property with respect to matrix multiplication.

Example 3.70 Solve $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Example 3.71 If $A = (1 \quad -1 \quad 2), B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ show that $(AB)C = A(BC)$.

Example 3.72 If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}, C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ verify that $A(B + C) = AB + AC$.

Example 3.73 If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T A^T$

Exercise 3.19

1. Find the order of the product matrix AB if

	(i)	(ii)	(iii)	(iv)	(v)
Orders of A	3×3	4×3	4×2	4×5	1×1
Orders of B	3×3	3×2	2×2	5×1	1×3

2. If A is of order $p \times q$ and B is of order $q \times r$ what is the order of AB and BA ?

3. A has ' a ' rows and ' $a+3$ ' columns. B has ' b ' rows and ' $17-b$ ' columns, and if both products AB and BA exist, find a, b ?
4. If $A=\begin{pmatrix}2 & 5\\4 & 3\end{pmatrix}, B=\begin{pmatrix}1 & -3\\2 & 5\end{pmatrix}$ find AB, BA and verify $AB=BA$?
5. Given that $A=\begin{pmatrix}1 & 3\\5 & -1\end{pmatrix}, B=\begin{pmatrix}1 & -1 & 2\\3 & 5 & 2\end{pmatrix}, C=\begin{pmatrix}1 & 3 & 2\\-4 & 1 & 3\end{pmatrix}$ verify that $A(B+C)=AB+AC$.
6. Show that the matrices $A=\begin{pmatrix}1 & 2\\3 & 1\end{pmatrix}, B=\begin{pmatrix}1 & -2\\-3 & 1\end{pmatrix}$ satisfy commutative property $AB=BA$
7. Let $A=\begin{pmatrix}1 & 2\\1 & 3\end{pmatrix}, B=\begin{pmatrix}4 & 0\\1 & 5\end{pmatrix}, C=\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$ Show that (i) $A(BC)=(AB)C$ (ii) $(A-B)C=AC-BC$ (iii) $(A-B)^T=A^T-B^T$
8. If $A=\begin{pmatrix}\cos\theta & 0\\0 & \cos\theta\end{pmatrix}, B=\begin{pmatrix}\sin\theta & 0\\0 & \sin\theta\end{pmatrix}$ then show that $A^2+B^2=I$.
9. If $A=\begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix}$ prove that $AA^T=I$.
10. Verify that $A^2=I$ when $A=\begin{pmatrix}5 & -4\\6 & -5\end{pmatrix}$
11. If $A=\begin{pmatrix}a & b\\c & d\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ show that $A^2-(a+d)A=(bc-ad)I_2$
12. If $A=\begin{pmatrix}5 & 2 & 9\\1 & 2 & 8\end{pmatrix}, B=\begin{pmatrix}1 & 7\\1 & 2\\5 & -1\end{pmatrix}$ verify that $(AB)^T=B^TA^T$
13. If $A=\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$ show that $A^2-5A+7I_2=0$

Exercise 3.20

Multiple choice questions

1. A system of three linear equations in three variables is inconsistent if their planes
   (A) intersect only at a point (B) intersect in a line
   (C) coincides with each other (D) do not intersect
2. The solution of the system $x+y-3z=-6, -7y+7z=7, 3z=9$ is
   (A) $x=1, y=2, z=3$ (B) $x=-1, y=2, z=3$
   (C) $x=-1, y=-2, z=3$ (D) $x=1, y=-2, z=3$
3. If $(x-6)$ is the HCF of $x^2-2x-24$ and x^2-kx-6 then the value of k is
   (A) 3 (B) 5 (C) 6 (D) 8
4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is
   (A) $\frac{9y}{7}$ (B) $\frac{9y^3}{(21y-21)}$ (C) $\frac{21y^2-42y+21}{3y^3}$ (D) $\frac{7(y^2-2y+1)}{y^2}$
5. $y^2+\frac{1}{y^2}$ is not equal to
   (A) $\frac{y^4+1}{y^2}$ (B) $\left(y+\frac{1}{y}\right)^2$ (C) $\left(y-\frac{1}{y}\right)^2+2$ (D) $\left(y+\frac{1}{y}\right)^2-2$
6. $\frac{x}{x^2-25}-\frac{8}{x^2+6x+5}$ gives
   (A) $\frac{x^2-7x+40}{(x-5)(x+5)}$ (B) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$ (C) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$ (D) $\frac{x^2+10}{(x^2-25)(x+1)}$
7. The square root of $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to
   (A) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (B) $16\left|\frac{y^2}{x^2z^4}\right|$ (C) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ (D) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$
8. Which of the following should be added to make x^4+64 a perfect square
   (A) $4x^2$ (B) $16x^2$ (C) $8x^2$ (D) $-8x^2$
9. The solution of $(2x-1)^2=9$ is equal to
   (A) -1 (B) 2 (C) $-1, 2$ (D) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
(A) 100,120 (B) 10,12 (C) $-120,100$ (D) 12,10

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in
(A) A.P (B) G.P (C) Both A.P and G.P (D) none of these

12. Graph of a linear equation is a
(A) straight line (B) circle C) parabola (D) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is
(A) 0 (B) 1 (C) 0 or 1 (D) 2

14. For the given matrix A $= \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ the order of the matrix A^T is
(A) 2×3 (B) 3×2 (C) 3×4 (D) 4×3

15. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have
(A) 3 (B) 4 (C) 2 (D) 5

16. If number of columns and rows are not equal in a matrix then it is said to be a
(A) diagonal matrix (B) rectangular matrix
(C) square matrix (D) identity matrix

17. Transpose of a column matrix is
(A) unit matrix (B) diagonal matrix (C) column matrix (D) row matrix

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$
(A) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. Which of the following can be calculated from the given matrices
$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, (i) A^2 (ii) B^2 (iii) AB (iv) BA
(A) (i) and (ii) only (B) (ii) and (iii) only
(C) (ii) and (iv) only (D) all of these

20. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?
(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$ (iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$
(A) (i) and (ii) only (B) (ii) and (iii) only
(C) (iii) and (iv) only (D) all of these

Unit Exercise - 3

1. Solve $\frac{1}{3}(x + y - 5) = y - z = 2x - 11 = 9 - (x + 2z)$
2. One hundred and fifty students are admitted to a school. They are distrbuted over three sections A, B and C. If 6 students are shifted from section A to section C, the sections will have equal number of students. If 4 times of students of section C exceeds the number of students of section A by the number of students in section B, find the number of students in the three sections.
3. In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.
4. Find the least common multiple of $xy(k^2 + 1) + k(x^2 + y^2)$ and $xy(k^2 - 1) + k(x^2 - y^2)$

5. Find the GCD of the following by division algorithm
$2x^4+13x^3+27x^2+23x+7, x^3+3x^2+3x+1, x^2+2x+1$

6. Reduce the given Rational expressions to its lowest form
(i) $\frac{x^{3a}-8}{x^{2a}+2x^a+4}$ (ii) $\frac{10x^3-25x^2+4x-10}{-4-10x^2}$

7. Simplify $\frac{\frac{1}{p}+\frac{1}{q+r}}{\frac{1}{p}-\frac{1}{q+r}} \times \left(1+\frac{q^2+r^2-p^2}{2qr}\right)$

8. Arul, Madan and Ram working together can clean a store in 6 hours. Working alone, Madan takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?

9. Find the square root of $289x^4-612x^3+970x^2-684x+361$.

10. Solve $\sqrt{y+1}+\sqrt{2y-5}=3$

11. A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr , what is the speed of the boat in still water?

12. Is it possible to design a rectangular park of perimeter 320 m and area 4800 m^2 ? If so find its length and breadth.

13. At t minutes past 2 pm , the time needed to 3 pm is 3 minutes less than $\frac{t^2}{4}$. Find t.

14. The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5 . Find the number of rows in the hall at the beginning.

15. If α and β are the roots of the polynomial $f(x)=x^2-2x+3$, find the polynomial whose roots are
(i) $\alpha+2, \beta+2$ (ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$.

16. If -4 is a root of the equation $x^2+px-4=0$ and if the equation $x^2+px+q=0$ has equal roots, find the values of p and q.

17. Two farmers Thilagan and Kausigan cultivates three varieties of grains namely rice, wheat and ragi. If the sale (in ₹) of three varieties of grains by both the farmers in the month of April is given by the matrix.and the May month sale (in ₹) is exactly twice as that of the April month sale for each variety.

April Slae in ₹

$$A=\begin{pmatrix} \text{Rice} & \text{Wheat} & \text{Ragi} \\ 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{pmatrix} \begin{matrix} \text{Thilagan} \\ \text{Kausigan} \end{matrix}$$

(i) What is the average sales of the months April and May.
(ii) If the sales continues to increase in the same way in the successive months, what will be sales in the month of August?

18. If $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta \\ \cos\theta & x\end{pmatrix}=I_2$, find x.

19. Given $A=\begin{pmatrix}p & 0 \\ 0 & 2\end{pmatrix}, B=\begin{pmatrix}0 & -q \\ 1 & 0\end{pmatrix}, C=\begin{pmatrix}2 & -2 \\ 2 & 2\end{pmatrix}$ and if $BA=C^2$, find p and q.

20. $A=\begin{pmatrix}3 & 0 \\ 4 & 5\end{pmatrix}, B=\begin{pmatrix}6 & 3 \\ 8 & 5\end{pmatrix}, C=\begin{pmatrix}3 & 6 \\ 1 & 1\end{pmatrix}$ find the matrix D, such that $CD-AB=0$

CHAPTER - 4.GEOMETRY

Example 4.1 Show that $\triangle PST \sim \triangle PQR$

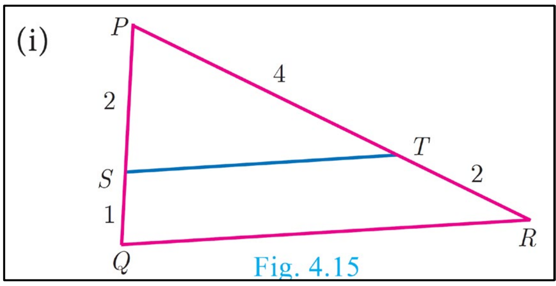


Fig. 4.15

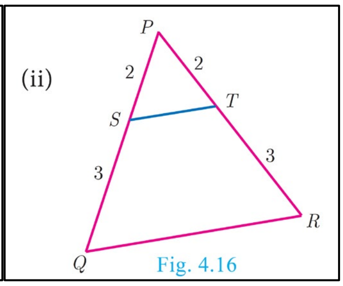
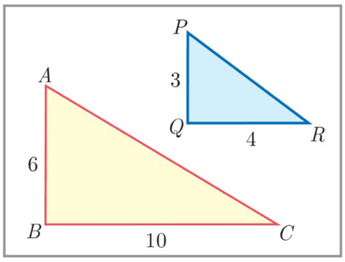
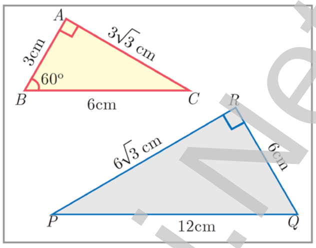


Fig. 4.16

Example 4.2 Is $\triangle ABC \sim \triangle PQR$?



Example 4.3 Observe Fig.4.18 and find $\angle P$.



Example 4.4 A boy of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamppost is 3.6 m above the ground, find the length of his shadow cast after 4 seconds.

Example 4.5 In Fig.4.20 $\angle A = \angle CED$ prove that $\triangle CAB \sim \triangle CED$. Also find the value of x.

Example 4.6 In Fig.4.21, QA and PB are perpendiculars to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ.

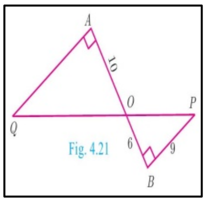


Fig 4.21

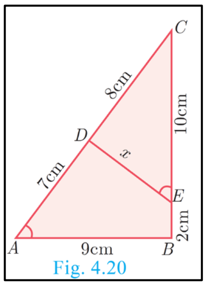


Fig. 4.20

Example 4.7 The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm . If $PQ = 10$ cm, find AB.

Example 4.8 If $\triangle ABC$ is similar to $\triangle DEF$ such that $BC = 3$ cm, $EF = 4$ cm and area of $\triangle ABC = 54$ cm^2. Find the area of $\triangle DEF$.

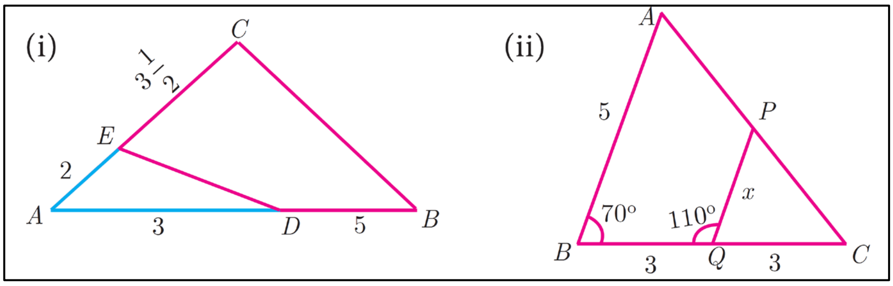
Example 4.9 Two poles of height ' a ' metres and ' b ' metres are ' p ' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Example 4.10 Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR(scale factor $\frac{3}{5} < 1$)

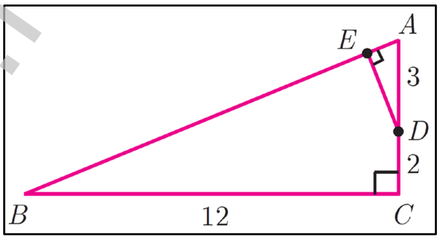
Example 4.11 Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)

Exercise 4.1

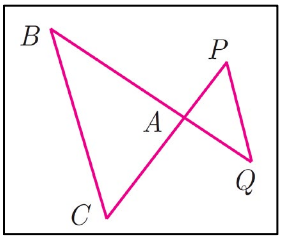
1. Check whether the which triangles are similar and find the value of x.



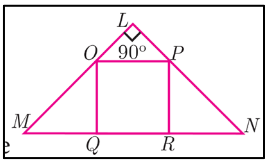
2. A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamppost. The girl whose height is 1.25 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamppost are in a same line, find the height of the lamp post.
3. A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.
4. Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.
5. In the adjacent figure, $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$ and hence find the lengths of AE and DE.



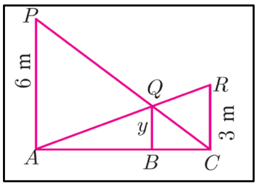
6. In the adjacent figure, $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, find CA and AQ.



7. If figure $OPRQ$ is a square and $\angle MLN = 90°$. Prove that
   (i) $\triangle LOP \sim \triangle QMO$ (ii) $\triangle LOP \sim \triangle RPN$
   (iii) $\triangle QMO \sim \triangle RPN$ (iv) $QR^2 = MQ \times RN$



8. If $\triangle ABC \sim \triangle DEF$ such that area of $\triangle ABC$ is 9 cm^2 and the area of $\triangle DEF$ is 16 cm^2 and $BC = 2.1$ cm. Find the length of EF.
9. Two vertical poles of heights 6 m and 3 m are erected above a horizontal ground AC. Find the value of y.



10. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3} < 1$).
11. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).
12. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$).
13. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3} > 1$).

Example 4.12 In $\triangle ABC$, if $DE \| BC, AD = x, DB = x - 2, AE = x + 2$ and $EC = x - 1$ then find the lengths of the sides AB and AC.

Example 4.13 D and E are respectively the points on the sides AB and AC of a $\triangle ABC$ such that $AB =$ 5.6 cm, AD $= 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm, show that $DE \| BC$.

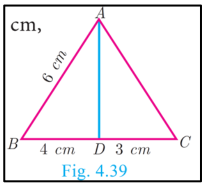
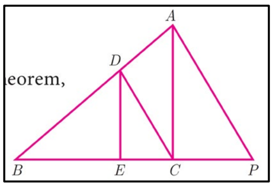


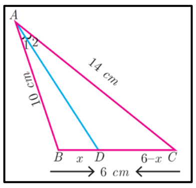
Fig. 4.39



Example 4.14 In the Fig.4.38, $DE \| AC$ and $DC \| AP$.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.

Example 4.15 In the Fig.4.39, AD is the bisector of $\angle A$. If $BD = 4$ cm, $DC = 3$ cm and $AB = 6$ cm, find AC.



Example 4.16 In the Fig. 4.40, AD is the bisector of $\angle BAC$, if $AB = 10$ cm, $AC = 14$ cm and BC = 6 cm. Find BD and DC.

Example 4.17 Construct a $\triangle PQR$ in which $PQ = 8$ cm, $\angle R = 60°$ and the median RG from R to PQ is 5.8 cm . Find the length of the altitude from R to PQ.

Example 4.18 Construct a triangle $\triangle PQR$ such that $QR = 5$ cm, $\angle P = 30°$ and the altitude from P to QR is of length 4.2 cm .

Example 4.19 Draw a triangle ABC of base $BC = 8$ cm, $\angle A = 60°$ and the bisector of $\angle A$ meets BC at D such that $BD = 6$ cm.

Exercise 4.2

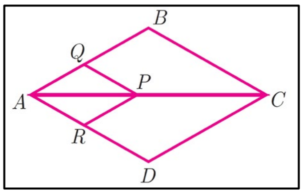
1. In $\triangle ABC, D$ and E are points on the sides AB and AC respectively such that $DE \| BC$

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm find AE.

(ii) If $AD = 8x - 7, DB = 5x - 3, AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

2. ABCD is a trapezium in which $AB \| DC$ and P, Q are points on AD and BC respectively, such that $PQ \| DC$ if $PD = 18$ cm, $BQ = 35$ cm and $QC = 15$ cm, find AD.

3. In $\triangle ABC, D$ and E are points on the sides AB and AC respectively. Show that $DE \| BC$ if $AB = 12$ cm, $AD = 8$ cm, $AE = 12$ cm and $AC = 18$ cm.

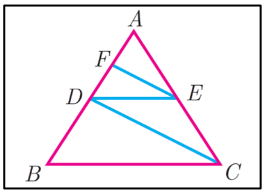


4. In fig. if $PQ \| BC$ and $PR \| CD$ prove that

(i) $\frac{AR}{AD} = \frac{AQ}{AB}$ (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$.

5. Rhombus PQRB is inscribed in $\triangle ABC$ such that $\angle B$ is one of its angle. P, Q and R lie on AB, AC and BC respectively. If $AB = 12$ cm and $BC = 6$ cm, find the sides PQ, RB of the rhombus.

6. In trapezium $ABCD, AB \| DC, E$ and F are points on non-parallel sides AD and BC respectively, such that $EF \| AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

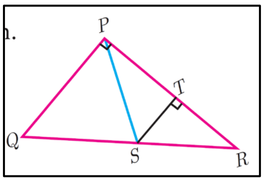


7. In figure $DE \| BC$ and $CD \| EF$. Prove that $AD^2 = AB \times AF$.

8. Check whether AD is bisector of $\angle A$ of $\triangle ABC$ in each of the following

(i) $AB = 5$ cm, $AC = 10$ cm, $BD = 1.5$ cm and $CD = 3.5$ cm.

(ii) $AB = 4$ cm, $AC = 6$ cm, $BD = 1.6$ cm and $CD = 2.4$ cm.



9. In figure $\angle QPR = 90°$, PS is its bisector.If $ST \perp PR$,

prove that $ST \times (PQ + PR) = PQ \times PR$.

10. $ABCD$ is a quadrilateral in which AB = AD, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively. Prove that $EF \| BD$.

11. Construct a $\triangle PQR$ which the base $PQ = 4.5$ cm, $\angle R = 35°$ and the median RG from R to PQ is 6 cm .

12. Construct a $\triangle PQR$ in which $QR = 5$ cm, $\angle P = 40°$ and the median PG from P to QR is 4.4 cm . Find the length of the altitude from P to QR.

13. Construct a $\triangle PQR$ such that $QR = 6.5$ cm, $\angle P = 60°$ and the altitude from P to QR is of length 4.5 cm.

14. Construct a $\triangle ABC$ such that $AB = 5.5$ cm, $\angle C = 25°$ and the altitude from C to AB is 4 cm.

15. Draw a triangle ABC of base $BC = 5.6$ cm, $\angle A = 40°$ and the bisector of $\angle A$ meets BC at D such that $CD = 4$ cm.

16. Draw $\triangle PQR$ such that $PQ = 6.8$ cm, vertical angle is $50°$ and the bisector of the vertical angle meets the base at D where $PD = 5.2$ cm.

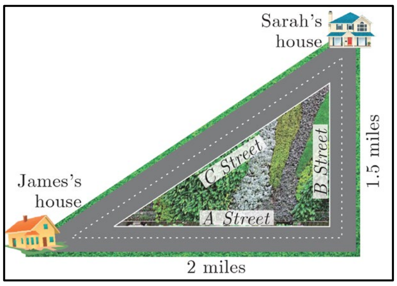
Example 4.20 An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?

Example 4.21 P and Q are the mid-points of the sides CA and CB respectively of a $\triangle ABC$, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.

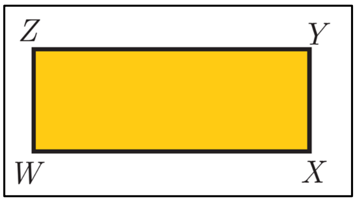
Example 4.22 What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Example 4.23 An Aeroplane after take off from an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane take off from the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1/2$ hours?

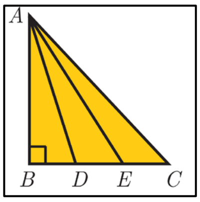
Exercise 4.3



1. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?
2. There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take B street and then A street. How much shorter is the direct path along C street? (Using figure).
3. To get from point A to point B you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?



4. In the rectangle $WXYZ, XY + YZ = 17$ cm, and $XZ + YW = 26$ cm. Calculate the length and breadth of the rectangle?
5. The hypotenuse of a right triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle.
6. 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.



7. The perpendicular PS on the base QR of a $\triangle PQR$ intersects QR at S, such that $QS = 3SR$. Prove that $2PQ^2 = 2PR^2 + QR^2$
8. In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.

Example 4.24 Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm .

Example 4.25 PQ is a chord of length 8 cm to a circle of radius 5 cm . The tangents at P and Q intersect at a point T . Find the length of the tangent TP.

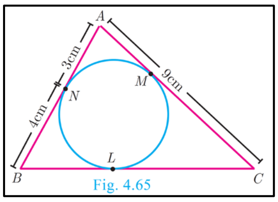


Fig. 4.65

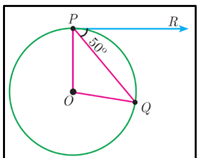


Fig. 4.64

Example 4.26 In Fig.4.64, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of $50°$ with PQ. Find $\angle POQ$.

Example 4.27 In Fig.4.65, $\triangle ABC$ is circumscribing a circle. Find the length of BC.

Example 4.28 If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

Example 4.29 Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Example 4.30 Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Example 4.31 Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

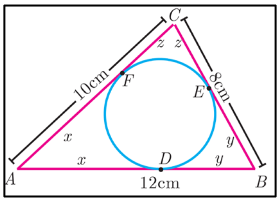
Example 4.32 Show that in a triangle, the medians are concurrent.

Example 4.33 In $\triangle ABC$, points D, E, F lies on BC, CA, AB respectively. Suppose AB, AC and BC have lengths 13,14 and 15 respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$. Find BD and DC.

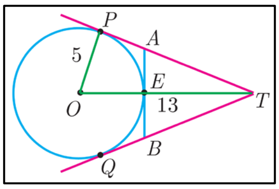
Example 4.34 In a garden containing several trees, three particular trees P, Q, R are located in the following way, $BP = 2$ m, $CQ = 3$ m, $RA = 10$ m, $PC = 6$ m, $QA = 5$ m, $RB = 2$ m, where A, B, C are points such that P lies on BC, Q lies on AC and R lies on AB. Check whether the trees P, Q, R lie on a same straight line.

Exercise 4.4

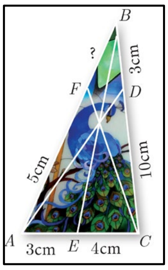
1. The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm . What is the radius of the circle?
2. $\triangle LMN$ is a right angled triangle with $\angle L = 90°$. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm . Find the radius of the circle.



3. A circle is inscribed in $\triangle ABC$ having sides 8 cm , 10 cm and 12 cm as shown in figure, Find AD, BE and CF.
4. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120°$. Find $\angle OPQ$.
5. A tangent ST to a circle touches it at B. AB is a chord such that $\angle ABT = 65°$. Find $\angle AOB$, where " O " is the centre of the circle.



6. In figure, O is the centre of the circle with radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects the circle E, if AB is the tangent to the circle at E, find the lenght of AB.
7. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm . Find the radius of the larger circle.
8. Two circles with centres O and O' of radii 3 cm and 4 cm , respectively intersect at two points P and Q, such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ.



9. Show that the angle bisectors of a triangle are concurrent.
10. An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.
11. Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P ?
12. Draw a circle of radius 4.5 cm . Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.
13. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm . Also, measure the lengths of the tangents.
14. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

15. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm . Also, measure the lengths of the tangents.

16. Draw a tangent to the circle from the point P having radius 3.6 cm , and centre at O. Point P is at a distance 7.2 cm from the centre.

Exercise 4.5

Multiple choice questions

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

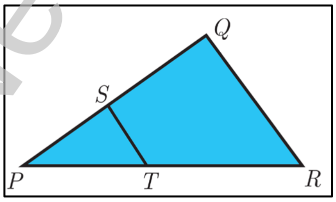
(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ ` (C) $\angle B = \angle D$ (D) $\angle A = \angle F$

2. In $\triangle LMN, \angle L = 60°, \angle M = 50°$. If $\triangle LMN \sim \triangle PQR$ then the value of $\angle R$ is

(A) 40° (B) 70° (C) 30° (D) 110°

3. If $\triangle ABC$ is an isosceles triangle with $\angle C = 90°$ and $AC = 5$ cm, then AB is

(A) 2.5 cm (B) 5 cm (C) 10 cm (D) $5\sqrt{2}$ cm



4. In a given figure $ST \| QR, PS = 2$ cm and $SQ = 3$ cm. Then the ratio of the area of $\triangle PQR$ to the area of $\triangle PST$ is

(A) 25: 4 (B) 25: 7 (C) 25: 11 (D) 25: 13

5. The perimeters of two similar triangles $\triangle ABC$ and $\triangle PQR$ are 36 cm and 24 cm respectively. If $PQ = 10$ cm, then the length of AB is

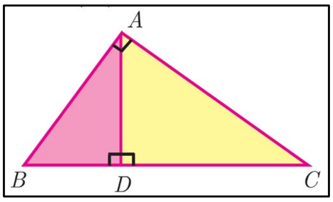
(A) $6\frac{2}{3}$ cm (B) $\frac{10\sqrt{6}}{3}$ cm (C) $66\frac{2}{3}$ cm (D) 15 cm

6. If in $\triangle ABC, DE \| BC. AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm then the length of AE is

(A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm

7. In a $\triangle ABC, AD$ is the bisector of $\angle BAC$. If $AB = 8$ cm, $BD = 6$ cm and $DC = 3$ cm. The length of the side AC is

(A) 6 cm (B) 4 cm (C) 3 cm (D) 8 cm



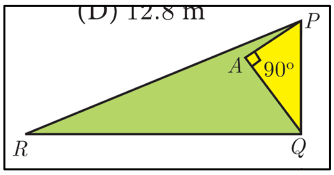
8. In the adjacent figure $\angle BAC = 90°$ and $AD \perp BC$ then

(A) $BD \cdot CD = BC^2$ (B) $AB \cdot AC = BC^2$

(C) $BD \cdot CD = AD^2$ (D) $AB \cdot AC = AD^2$

9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m , what is the distance between their tops?

(A) 13 m (B) 14 m (C) 15 m (D) 12.8 m



10. In the given figure, $PR = 26$ cm, $QR = 24$ cm, $\angle PAQ = 90°, PA = 6$ cm and $QA = 8$ cm. Find $\angle PQR$

(A) 80° (B) 85° (C) 75° (D) 90°

11. A tangent is perpendicular to the radius at the

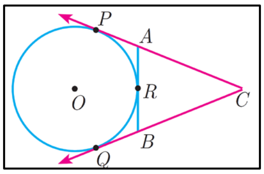
(A) centre (B) point of contact (C) infinity (D) chord

12. How many tangents can be drawn to the circle from an exterior point?

(A) one (B) two (C) infinite (D) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is

(A) 100° (B) 110° (C) 120° (D) 130°

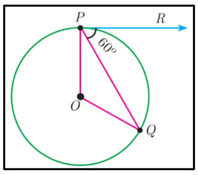


14. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11$ cm and $BC = 7$ cm, then the length of BR is

(A) 6 cm (B) 5 cm (C) 8 cm (D) 4 cm

15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is

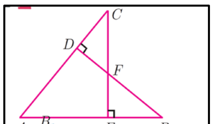
(A) $120°$ (B) $100°$ (C) $110°$ (D) $90°$



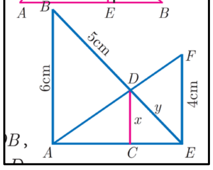
Unit Exercise - 4

1. In the figure, if $BD \perp AC$ and $CE \perp AB$, prove that

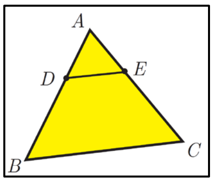
(i) $\triangle AEC \sim \triangle ADB$ (ii) $\frac{CA}{AB} = \frac{CE}{DB}$



2. In the given figure $AB \| CD \| EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 4$ cm, $BD = 5$ cm and $DE = y$ cm. Find x and y.



3. O is any point inside a triangle ABC. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in point D, E and F respectively. Show that $AD \times BE \times CF = DB \times EC \times FA$.
4. In the figure, ABC is a triangle in which $AB = AC$. Points D and E are points on the side AB and AC respectively such that $AD = AE$. Show that the points B, C, E and D lie on a same circle.



5. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 km/hr. After 2 hours, what is the distance between them?.
6. D is the mid point of side BC and $AE \perp BC$. If $BC = a, AC = b, AB = c, ED = x, AD = p$ and $AE = h$, Prove that (i) $b^2 = p^2 + ax + \frac{a^2}{4}$ (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$ (iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$
7. A man whose eye-level is 2 m above the ground wishes to find the height of a tree. He places a mirror horizontally on the ground 20 m from the tree and finds that if he stands at a point C which is 4 m from the mirror B, he can see the reflection of the top of the tree. How height is the tree?
8. An Emu which is 8 feet tall is standing at the foot of a pillar which is 30 feet high. It walks away from the pillar. The shadow of the Emu falls beyond Emu. What is the relation between the length of the shadow and the distance from the Emu to the pillar?.
9. Two circles intersect at A and B. From a point P on one of the circles lines PAC and PBD are drawn intersecting the second circle at C and D. Prove that CD is parallel to the tangent at P.
10. Let ABC be a triangle and D, E, F are points on the respective sides AB, BC, AC (or their extensions). Let $AD:DB = 5:3, BE:EC = 3:2$ and $AC = 21$. Find the length of the line segment CF.

CHAPTER - 5. COORDINATE GEOMETRY

Example 5.1 Find the area of the triangle whose vertices are $(-3,5),(5,6)$ and $(5,-2)$

Example 5.2 Show that the points $P(-1.5,3), Q(6,-2), R(-3,4)$ are collinear.

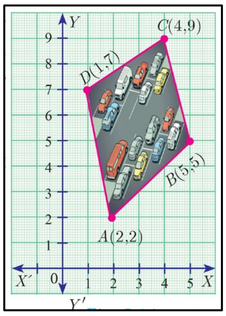
Example 5.3 If the area of the triangle formed by the vertices $A(-1,2), B(k,-2)$ and $C(7,4)$ (taken in order) is 22 sq. units, find the value of k.

Example 5.4 If the points $P(-1,-4), Q(b,c)$ and $R(5,-1)$ are collinear and if $2b+c=4$, then find the values of b and c.

Example 5.5 The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at $(-3,2),(-1,-1)$ and $(1,2)$. If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Example 5.6 Find the area of the quadrilateral formed by the points $(8,6),(5,11),(-5,12)$ and $(-4,3)$.

Example 5.7 The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

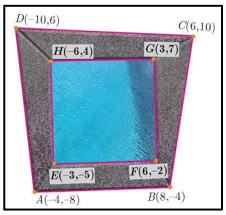


Exercise 5.1

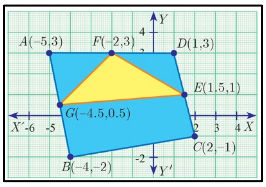
1. Find the area of the triangle formed by the points
(i) $(1,-1),(-4,6)$ and $(-3,-5)$ (ii) $(-10,-4),(-8,-1)$ and $(-3,-5)$
2. Determine whether the sets of points are collinear?
(i) $\left(-\frac{1}{2},3\right),(-5,6)$ and $(-8,8)$ (ii) $(a,b+c),(b,c+a)$ and $(c,a+b)$
3. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of ' p '.

S.No.	Vertices	Area (sq.units)
(i)	$(0,0),(p,8),(6,2)$	20
(ii)	$(p,p),(5,6),(5,-2)$	32

4. In each of the following, find the value of ' a ' for which the given points are collinear.
(i) $(2,3),(4,a)$ and $(6,-3)$ (ii) $(a,2-2a),(-a+1,2a)$ and $(-4-a,6-2a)$
5. Find the area of the quadrilateral whose vertices are at
(i) $(-9,-2),(-8,-4),(2,2)$ and $(1,-3)$ (ii) $(-9,0),(-8,6),(-1,-2)$ and $(-6,-3)$
6. Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are taken in the order $(-4,-2),(-3,k),(3,-2)$ and $(2,3)$
7. If the points $A(-3,9), B(a,b)$ and $C(4,-5)$ are collinear and if $a+b=1$, then find a and b.
8. Let $P(11,7), Q(13.5,4)$ and $R(9.5,4)$ be the midpoints of the sides AB, BC and AC respectively of $\triangle ABC$. Find the coordinates of the vertices A, B and C. Hence find the area of $\triangle ABC$ and compare this with area of $\triangle PQR$.
9. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.



10. A triangular shaped glass with vertices at $A(-5,-4), B(1,6)$ and $C(7,-4)$ has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.
11. In the figure, find the area of (i) triangle AGF (ii) triangle FED (iii) quadrilateral $BCEG$.



Example 5.8 (i) What is the slope of a line whose inclination is 30° ?

(ii) What is the inclination of a line whose slope is $\sqrt{3}$?

Example 5.9 Find the slope of a line joining the given points

(i) $(-6,1)$ and $(-3,2)$ (ii) $\left(-\frac{1}{3},\frac{1}{2}\right)$ and $\left(\frac{2}{7},\frac{3}{7}\right)$ (iii) $(14,10)$ and $(14,-6)$

Example 5.10 The line r passes through the points $(-2,2)$ and $(5,8)$ and the line s passes through the points $(-8,7)$ and $(-2,0)$. Is the line r perpendicular to s ?

Example 5.11 The line p passes through the points $(3,-2),(12,4)$ and the line q passes through the points $(6,-2)$ and $(12,2)$. Is p parallel to q ?

Example 5.12 Show that the points $(-2,5),(6,-1)$ and $(2,2)$ are collinear.

Example 5.13 Let $A(1,-2),B(6,-2),C(5,1)$ and $D(2,1)$ be four points

(i) Find the slope of the line segments (a) AB (b) CD

(ii) Find the slope of the line segments (a) BC (b) AD

(iii) What can you deduce from your answer.

Example 5.14 Consider the graph representing growth of population (in crores). Find the slope of the line AB and hence estimate the population in the year 2030?

Example 5.15 Without using Pythagoras theorem, show that the points $(1,-4),(2,-3)$ and $(4,-7)$ form a right angled triangle.

Example 5.16 Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.

Exercise 5.2

1. What is the slope of a line whose inclination with positive direction of x-axis is
   (i) 90° (ii) 0°
2. What is the inclination of a line whose slope is
   (i) 0 (ii) 1
3. Find the slope of a line joining the points
   (i) $(5,\sqrt{5})$ with the origin (ii) $(\sin\theta,-\cos\theta)$ and $(-\sin\theta,\cos\theta)$
4. What is the slope of a line perpendicular to the line joining $A(5,1)$ and P where P is the mid-point of the segment joining $(4,2)$ and $(-6,4)$.
5. Show that the given points are collinear: $(-3,-4),(7,2)$ and $(12,5)$
6. If the three points $(3,-1),(a,3)$ and $(1,-3)$ are collinear, find the value of a.
7. The line through the points $(-2,a)$ and $(9,3)$ has slope $-\frac{1}{2}$. Find the value of a.
8. The line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(8,12)$ and $(x,24)$. Find the value of x.
9. Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem
   (i) $A(1,-4),B(2,-3)$ and $C(4,-7)$ (ii) $L(0,5),M(9,12)$ and $N(3,14)$
10. Show that the given points form a parallelogram :$A(2.5,3.5),B(10,-4),C(2.5,-2.5)$ and $D(-5,5)$.
11. If the points $A(2,2),B(-2,-3),C(1,-3)$ and $D(x,y)$ form a parallelogram then find the value of x and y.
12. Let $A(3,-4),B(9,-4),C(5,-7)$ and $D(7,-7)$. Show that $ABCD$ is a trapezium.
13. A quadrilateral has vertices at $A(-4,-2),B(5,-1),C(6,5)$ and $D(-7,6)$. Show that the mid-points of its sides form a parallelogram.

Example 5.17 Find the equation of a straight line passing through (5,7) and is

(i) parallel to X axis (ii) parallel to Y axis.

Example 5.18 Find the equation of a straight line whose

(i) Slope is 5 and y intercept is -9 (ii) Inclination is 45° and y intercept is 11

Example 5.19 Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$

Solution Equation of the given straight line is $8x - 7y + 6 = 0$ $7y = 8x + 6$ (bringing it to

Example 5.20 The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree) (a) Find the slope and y intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius?

Example 5.21 Find the equation of a line passing through the point $(3, -4)$ and having slope $\frac{-5}{7}$

Example 5.22 Find the equation of a line passing through the point $A(1,4)$ and perpendicular to the line joining points $(2,5)$ and $(4,7)$.

Example 5.23 Find the equation of a straight line passing through $(5, -3)$ and $(7, -4)$.

Example 5.24 Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from $(6,10)$ to $(14,12)$, find the equation of the rod joining the buildings?

Example 5.25 Find the equation of a line which passes through $(5,7)$ and makes intercepts on the axes equal in magnitude but opposite in sign.

Example 5.26 Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.

Example 5.27 A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as

$$y = -0.25x + 1$$

(i) Find the number of hours elapsed if the battery power is 40%.

(ii) How much time does it take so that the battery has no power?

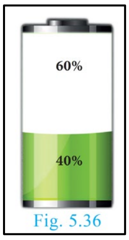


Fig. 5.36

Example 5.28 A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through $(-3,8)$. Find its equation.

Example 5.29 A circular garden is bounded by East Avenue and Cross Road. Cross Road intersects North Street at D and East Avenue at E. AD is tangential to the circular garden at $A(3,10)$. Using the figure.

(a) Find the equation of

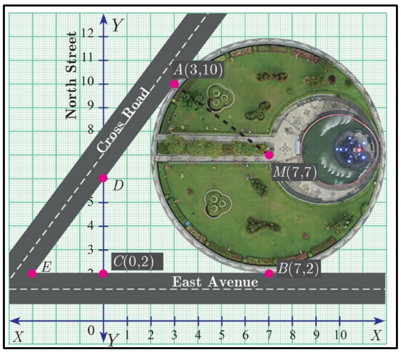
(i) East Avenue.

(ii) North Street

(iii) Cross Road

(b) Where does the Cross Road intersect?(i) North Street

(ii) East Avenue



Exercise 5.3

1. Find the equation of a straight line passing through the mid-point of a line segment joining the points $(1, -5), (4,2)$ and parallel to (i) X axis (ii) Y axis
2. The equation of a straight line is $2(x - y) + 5 = 0$. Find its slope, inclination and intercept on the Y axis.
3. Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.
4. Find the slope and y intercept of $\sqrt{3}x + (1 - \sqrt{3})y = 3$.
5. Find the value of 'a', if the line through $(-2,3)$ and $(8,5)$ is perpendicular to $y = ax + 2$
6. The hill in the form of a right triangle has its foot at $(19,3)$. The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

7. Find the equation of a line through the given pair of points

(i) $\left(2,\frac{2}{3}\right)$ and $\left(\frac{-1}{2},-2\right)$ (ii) $(2,3)$ and $(-7,-1)$

8. A cat is located at the point $(-6,-4)$ in xy plane. A bottle of milk is kept at $(5,11)$. The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

9. If the vertices of a $\triangle ABC$ are $A(6,2), B(-5,-1)$ and $C(1,9)$

(i) find the equation of median (ii) find the equation of altitude

10. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point $(-1,2)$.

11. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$.

(i) find the total MB of the song.

(ii) after how many seconds will 75% of the song gets downloaded?

(iii) after how many seconds the song will be downloaded completely?

12. Find the equation of a line whose intercepts on the x and y axes are given below.

(i) $4, -6$ (ii) $-5, \frac{3}{4}$

13. Find the intercepts made by the following lines on the coordinate axes.

(i) $3x - 2y - 6 = 0$ (ii) $4x + 3y + 12 = 0$

14. Find the equation of a straight line

(i) passing through $(1,-4)$ and has intercepts which are in the ratio $2:5$

(ii) passing through $(-8,4)$ and making equal intercepts on the coordinate axes

Example 5.30 Find the slope of the straight line $6x + 8y + 7 = 0$.

Example 5.31 Find the slope of the line which is

(i) parallel to $3x - 7y = 11$ (ii) perpendicular to $2x - 3y + 8 = 0$

Example 5.32 Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Example 5.33 Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Example 5.34 Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point $(6,4)$.

Example 5.35 Find the equation of a straight line perpendicular to the line $y = \frac{4}{3}x - 7$ and passing through the point $(7,-1)$.

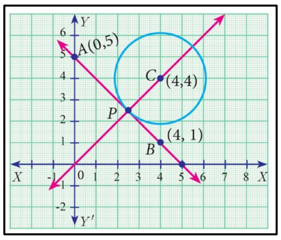
Example 5.36 Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $4x + 5y = 13$ and $x - 8y + 9 = 0$.

Example 5.37 The line joining the points $A(0,5)$ and $B(4,1)$ is a tangent to a circle whose centre C is at the point $(4,4)$ find

(i) the equation of the line AB.

(ii) the equation of the line through C which is perpendicular to the line AB.

(iii) the coordinates of the point of contact of tangent line AB with the circle.



Exercise 5.4

1. Find the slope of the following straight lines

(i) $5y - 3 = 0$ (ii) $7x - \frac{3}{17} = 0$

2. Find the slope of the line which is

(i) parallel to $y = 0.7x - 11$ (ii) perpendicular to the line $x = -11$

3. Check whether the given lines are parellel or perpendicular
   (i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ and $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$
   (ii) $5x+23y+14=0$ and $23x-5y+9=0$
4. If the straight lines $12y=-(p+3)x+12, 12x-7y=16$ are perpendicular then find 'p'.
5. Find the equation of a straight line passing through the point $P(-5,2)$ and parallel to the line joining the points $Q(3,-2)$ and $R(-5,4)$.
6. Find the equation of a line passing through $(6,-2)$ and perpendicular to the line joining the points $(6,7)$ and $(2,-3)$.
7. $A(-3,0)$ $B(10,-2)$ and $C(12,3)$ are the vertices of $\triangle ABC$. Find the equation of the altitude through A and B.
8. Find the equation of the perpendicular bisector of the line joining the points $A(-4,2)$ and $B(6,-4)$.
9. Find the equation of a straight line through the intersection of lines $7x+3y=10, 5x-4y=1$ and parallel to the line $13x+5y+12=0$
10. Find the equation of a straight line through the intersection of lines $5x-6y=2, 3x+2y=10$ and perpendicular to the line $4x-7y+13=0$
11. Find the equation of a straight line joining the point of intersection of $3x+y+2=0$ and $x-2y-4=0$ to the point of intersection of $7x-3y=-12$ and $2y=x+3$
12. Find the equation of a straight line through the point of intersection of the lines $8x+3y=18, 4x+5y=9$ and bisecting the line segment joining the points $(5,-4)$ and $(-7,6)$.

Exercise 5.5

Multiple choice questions

1. The area of triangle formed by the points $(-5,0),(0,-5)$ and $(5,0)$ is
   A) 0 sq.units (B) 25 sq.units (C) 5 sq.units (D) none of these
2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is
   (A) $x=10$ (B) $y=10$ (C) $x=0$ D) $y=0$
3. The straight line given by the equation $x=11$ is
   (A) parallel to X axis (B) parallel to Y axis
   (C) passing through the origin (D) passing through the point $(0,11)$
4. If $(5,7),(3,p)$ and $(6,6)$ are collinear, then the value of p is
   (A) 3 (B) 6 (C) 9 (D) 12
5. The point of intersection of $3x-y=4$ and $x+y=8$ is
   (A) (5,3) (B) (2,4) (C) (3,5) (D) (4,4)
6. The slope of the line joining $(12,3),(4,a)$ is $\frac{1}{8}$. The value of 'a' is
   (A) 1 (B) 4 (C) -5 (D) 2
7. The slope of the line which is perpendicular to a line joining the points $(0,0)$ and $(-8,8)$ is
   (A) -1 (B) 1 (C) $\frac{1}{3}$ (D) -8
8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is
   (A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 0
9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is
   (A) $8x+5y=40$ (B) $8x-5y=40$ (C) $x=8$ (D) $y=5$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is
(A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$ (C) $3x + 7y = 0$ (D) $7x - 3y = 0$

11. Consider four straight lines
(i) $l_1; 3y = 4x + 5$ (ii) $l_2; 4y = 3x - 1$ (iii) $l_3; 4y + 3x = 7$ (iv) $l_4; 4x + 3y = 2$
Which of the following statement is true ?
(A) l_1 and l_2 are perpendicular (B) l_1 and l_4 are parallel
(C) l_2 and l_4 are perpendicular (D) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21$. Which of the following is true
(A) The slope is 0.5 and the y intercept is 2.6
(B) The slope is 5 and the y intercept is 1.6
(C) The slope is 0.5 and the y intercept is 1.6
(D) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium, it is necessary to show
(A) Two sides are parallel. (B) Two parallel and two non-parallel sides.
(C) Opposite sides are parallel. (D) All sides are of equal length.

14. When proving that a quadrilateral is a parallelogram by using slopes you must find
(A) The slopes of two sides (B) The slopes of two pair of opposite sides
(C) The lengths of all sides (D) Both the lengths and slopes of two sides

15. (2,1) is the point of intersection of two lines.
(A) $x - y - 3 = 0; 3x - y - 7 = 0$ (B) $x + y = 3; 3x + y = 7$
(C) $3x + y = 3; x + y = 7$ (D) $x + 3y - 3 = 0; x - y - 7 = 0$

Unit Exercise - 5

1. PQRS is a rectangle formed by joining the points $P(-1,-1), Q(-1,4), R(5,4)$ and $S(5,-1)$. A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Is the quadrilateral $ABCD$ a square, a rectangle or a rhombus? Justify your answer.
2. The area of a triangle is 5 sq.units. Two of its vertices are (2,1) and $(3,-2)$. The third vertex is (x, y) where $y = x + 3$. Find the coordinates of the third vertex.
3. Find the area of a triangle formed by the lines $3x + y - 2 = 0, 5x + 2y - 3 = 0$ and $2x - y - 3 = 0$
4. If vertices of a quadrilateral are at $A(-5,7), B(-4,k), C(-1,-6)$ and $D(4,5)$ and its area is 72 sq. units. Find the value of k.
5. Without using distance formula, show that the points $(-2,-1), (4,0), (3,3)$ and $(-3,2)$ are vertices of a parallelogram.
6. Find the equations of the lines, whose sum and product of intercepts are 1 and -6 respectively.
7. The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹ 14 /litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹17/litre?
8. Find the image of the point (3,8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.
9. Find the equation of a line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.
10. A person standing at a junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ seek to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the equation of the path that he should follow.

CHAPTER - 6. TRIGONOMETRY

Example 6.1 Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

Example 6.2 Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Example 6.3 Prove that $1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

Example 6.4 Prove that $\sec\theta - \cos\theta = \tan\theta\sin\theta$

Example 6.5 Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Example 6.6 Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

Example 6.7 Prove that $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$

Example 6.8 If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Example 6.9 Prove that $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Example 6.10 Prove that $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A} = 2\operatorname{cosec}A$.

Example 6.11 If $\operatorname{cosec}\theta + \cot\theta = P$, then prove that $\cos\theta = \frac{P^2-1}{P^2+1}$.

Example 6.12 Prove that $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$.

Example 6.13 Prove that $\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$.

Example 6.14 Prove that $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$.

Example 6.15 Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$.

Example 6.16 Prove that $\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A - \operatorname{cosec}A} = \sin^2 A\cos^2 A$.

Example 6.17 If $\frac{\cos^2\theta}{\sin\theta} = p$ and $\frac{\sin^2\theta}{\cos\theta} = q$, then prove that $p^2q^2(p^2+q^2+3) = 1$.

Exercise 6.1

1. Prove the following identities.
   (i) $\cot\theta + \tan\theta = \sec\theta\operatorname{cosec}\theta$ (ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$
2. Prove the following identities.
   (i) $\frac{1-\tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$ (ii) $\frac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$
3. Prove the following identities.
   (i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$ (ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$
4. Prove the following identities.
   (i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$
   (ii) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + (\sec\theta + \operatorname{cosec}\theta)^2$
5. Prove the following identities.
   (i) $\sec^4\theta(1-\sin^4\theta) - 2\tan^2\theta = 1$ (ii) $\frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$
6. Prove the following identities.
   (i) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$ (ii) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$
7. (i) If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.
   (ii) If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

8. (i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2+n^2)\cos^2\beta = n^2$.

(ii) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$.

9. (i) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\theta = q$, then prove that $q(p^2-1) = 2p$.

(ii) If $\sin\theta(1+\sin^2\theta) = \cos^2\theta$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$.

10. If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that $\frac{a^2-1}{a^2+1} = \sin\theta$.

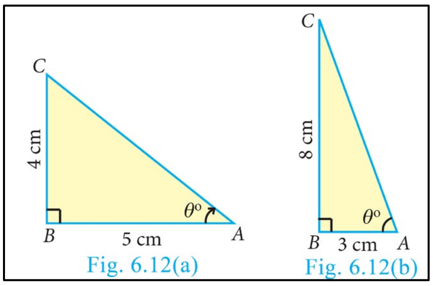


Fig. 6.12(a) Fig. 6.12(b)

Example 6.18 Calculate $\angle BAC$ in the given triangles. ($\tan 38.7^\circ = 0.8011, \tan 69.4^\circ = 2.6604$)

Example 6.19 A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Example 6.20 A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Example 6.21 Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)

Example 6.22 From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3} = 1.732$)

Example 6.23 A TV tower stands vertically on a bank of a canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is 58°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. ($\tan 58^\circ = 1.6003$)

Example 6.24 An aeroplane sets off from G on a bearing of 24° towards H, a point 250 km away. At H it changes course and heads towards J deviates further by 55° and a distance of 180 km away.

(i) How far is H to the North of G ?

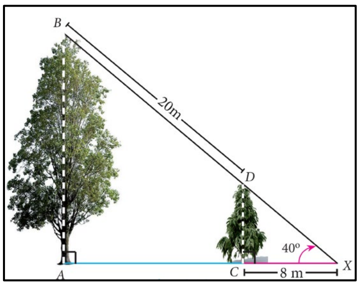
(iii)How far is J to the North of H ? $\begin{pmatrix}\sin 24^\circ = 0.4067 & \sin 11^\circ = 0.1908 \\ \cos 24^\circ = 0.9135 & \cos 11^\circ = 0.9816\end{pmatrix}$

Example 6.25 As shown in the figure, two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is 40°. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m , calculate

(i) the distance between the point X and the top of the smaller tree.

(ii) the horizontal distance between the two trees.

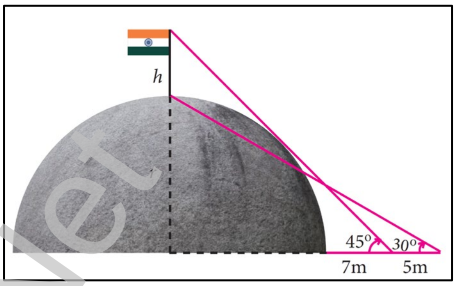
($\cos 40^\circ = 0.7660$).



Exercise 6.2

1. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

2. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is $30°$. Find the width of the road.
3. To a man standing outside his house, the angles of elevation of the top and bottom of a window are $60°$ and $45°$ respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? $(\sqrt{3} = 1.732)$
4. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60 Y and from the same point the angle of elevation of the top of the pedestal is 40 r . Find the height of the pedestal. $\left(\tan 40° = 0.8391, \sqrt{3} = 1.732\right)$
5. A flag pole of height ' h ' metres is on the top of the hemispherical dome of radius ' r ' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle $45°$ and moving 5 m away from the dome and seeing the bottom of the pole at an angle $30°$. Find (i) the height of the pole
(ii) radius of the dome. $(\sqrt{3} = 1.732)$



6. The top of a 15 m high tower makes an angle of elevation of $60°$ with the bottom of an electronic pole and angle of elevation of $30°$ with the top of the pole. What is the height of the electric pole?

Example 6.26 A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as $60°$. Find the distance between the foot of the tower and the ball. $(\sqrt{3} = 1.732)$

Example 6.27 The horizontal distance between two buildings is 140 m . The angle of depression of the top of the first building when seen from the top of the second building is $30°$. If the height of the first building is 60 m , find the height of the second building. $(\sqrt{3} = 1.732)$

Example 6.28 From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be $30°$ and $45°$ respectively. Find the height of the tree. $(\sqrt{3} = 1.732)$

Example 6.29 As observed from the top of a 60 m high lighthouse from the sea level, the angles of depression of two ships are $28°$ and $45°$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. $(\tan 28° = 0.5317)$

Example 6.30 A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of $60°$ with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes $45°$. What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water? $(\sqrt{3} = 1.732)$

Exercise 6.3

1. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be $30°$. Find the distance of the car from the rock.
2. The horizontal distance between two buildings is 70 m . The angle of depression of the top of the first building when seen from the top of the second building is $45°$. If the height of the second building is 120 m , find the height of the first building.
3. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be $38°$ and $60°$ respectively. Find the height of the lamp post. $\left(\tan 38° = 0.7813, \sqrt{3} = 1.732\right)$

4. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)
5. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.
6. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.

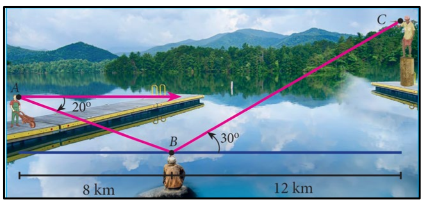
Example 6.31 From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Example 6.32 A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point ' A ' on the ground is 60° and the angle of depression to the point ' A ' from the top of the tower is 45°. Find the height of the tower. ($\sqrt{3} = 1.732$)

Example 6.33 From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ_1 and θ_2 respectively. Show that the height of the opposite house is $h\left(1 + \frac{\cot\theta_2}{\cot\theta_1}\right)$.

Exercise 6.4

1. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3} = 1.732$)
2. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3} = 1.732$)
3. If the angle of elevation of a cloud from a point ' h ' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\frac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2-\tan\theta_1}$.
4. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m , find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m . State if the height of the above mentioned cell phone tower meets the radiation norms.
5. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find
(i) The height of the lamp post.(ii) The difference between height of the lamp post and the apartment.
(iii) The distance between the lamp post and the apartment. ($\sqrt{3} = 1.732$).
6. Three villagers A, B and C can see each other using telescope across a valley. The horizontal distance between A and B is 8 km and the horizontal distance between B and C is 12 km . The angle of depression of B from A is 20° and the angle of elevation of C from B is 30°. Calculate: (i) the vertical height between A and B.
(ii) the vertical height between B and C. $\left(\tan 20° = 0.3640, \sqrt{3} = 1.732\right)$



Exercise 6.5

Multiple choice questions

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to

(A) $\tan^2\theta$ (B) 1 (C) $\cot^2\theta$ (D) 0

2. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ is equal to

(A) $\sec\theta$ (B) $\cot^2\theta$ (C) $\sin\theta$ (D) $\cot\theta$

3. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to

(A) 9 (B) 7 (C) 5 (D) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to

(A) $2a$ (B) $3a$ (C) 0 (D) $2ab$

5. If $5x = \sec\theta$ and $\frac{5}{y} = \tan\theta$, then $x^2 - \frac{1}{y^2}$ is equal to

(A) 25 (B) $\frac{1}{25}$ (C) 5 (D) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to

(A) $\frac{-3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then

(A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to

(A) 0 (B) 1 (C) 2 (D) -1

9. $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$ then $p^2 - q^2$ is equal to

(A) $a^2 - b^2$ (B) $b^2 - a^2$ (C) $a^2 + b^2$ (D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure

(A) 45° (B) 30° (C) 90° (D) 60°

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point ' b ' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to

(A) $\sqrt{3}b$ (B) $\frac{b}{3}$ (C) $\frac{b}{2}$ (D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m heigh. Its shadow reduces by x metres when the angle of elevation of the sun increases from 30° to 45° then x is equal to

(A) 41.92 m (B) 43.92 m (C) 43 m (D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is

(A) $20, 10\sqrt{3}$ (B) $30, 5\sqrt{3}$ (C) 20,10 (D) $30, 10\sqrt{3}$

14. Two persons are standing ' x ' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(A) $\sqrt{2}x$ (B) $\frac{x}{2\sqrt{2}}$ (C) $\frac{x}{\sqrt{2}}$ (D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is

(A) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ (B) $\frac{h(1-\tan\beta)}{1+\tan\beta}$ (C) $h\tan(45° - \beta)$ (D) none of these

Unit Exercise - 6

1. Prove that

(i) $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)=0$ (ii) $\frac{\tan^2\theta-1}{\tan^2\theta+1}=1-2\cos^2\theta$

2. Prove that $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$

3. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$, then prove that $x^2+y^2=1$.

4. If $a\cos\theta-b\sin\theta=c$, then prove that $(a\sin\theta+b\cos\theta)=\pm\sqrt{a^2+b^2-c^2}$.

5. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Determine the speed at which the bird flies. $(\sqrt{3}=1.732)$

6. An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m . The angle of elevation of the aeroplane from a point on the Earth's surface is 37°. After what period of time does the angle of elevation increase to 53° ? $(\tan 53°=1.3270, \tan 37°=0.7536)$

7. A bird is flying from A towards B at an angle of 35°, a point 30 km away from A. At B it changes its course of flight and heads towards C on a bearing of 48° and distance 32 km away.

(i) How far is B to the North of A ? (ii) How far is B to the West of A ?

(iii) How far is C to the North of B ? (iv) How far is C to the East of B ?

$(\sin 55°=0.8192, \cos 55°=0.5736, \sin 42°=0.6691, \cos 42°=0.7431)$

8. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the lighthouse.

9. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of base of the statue is 34°. Find the height of the statue. $(\tan 24°=0.4452, \tan 34°=0.6745)$.

CHAPTER - 7. MENSURATION

Example 7.1 A cylindrical drum has a height of 20 cm and base radius of 14 cm . Find its curved surface area and the total surface area.

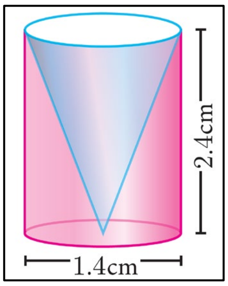
Example 7.2 The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Example 7.3 A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Example 7.4 If one litre of paint covers 10 m^2, how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m , internal radius is 6 m and height is 25 m .

Example 7.5 The radius of a conical tent is 7 m and the height is 24 m . Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m ?

Example 7.6 If the total surface area of a cone of radius 7 cm is 704 cm^2, then find its slant height.



Example 7.7 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm , a conical cavity of the same height and base is hollowed out (Fig.7.13). Find the total surface area of the remaining solid.

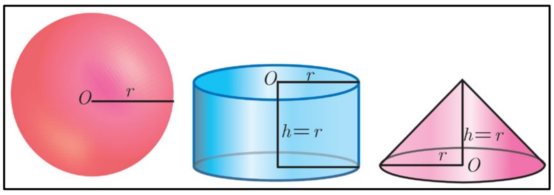
Example 7.8 Find the diameter of a sphere whose surface area is 154 m^2.

Example 7.9 The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Example 7.10 If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Example 7.11 The internal and external radii of a hollow hemispherical shell are 3 m and 5 m respectively. Find the T.S.A. and C.S.A. of the shell.

Example 7.12 A sphere, a cylinder and a cone are of the same height which is equal to its radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.



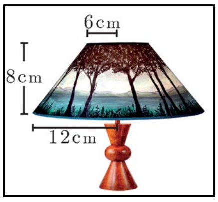
Example 7.13 The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm . Find its curved surface area.

Example 7.14 An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m . Find the curved and total surface area of the bucket.

Exercise 7.1

1. The radius and height of a cylinder are in the ratio 5: 7 and its curved surface area is 5500sq. cm. Find its radius and height.
2. A solid iron cylinder has total surface area of 1848 sq.cm. Its curved surface area is five - sixth of its total surface area. Find the radius and height of the iron cylinder.
3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.
4. A right angled triangle PQR where $\angle Q = 90^\circ$ is rotated about QR and PQ. If $QR = 16$ cm and $PR = 20$ cm, compare the curved surface areas of the right circular cones so formed by the triangle.
5. 4 persons live in a conical tent whose slant height is 19 m . If each person require 22 m^2 of the floor area, then find the height of the tent.

6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is $5720\ cm^2$, how many caps can be made with radius 5 cm and height 12 cm .
7. The ratio of the radii of two right circular cones of same height is $1:3$. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.
8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.
9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm^2.
10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1sq. cm is ₹ 2 .

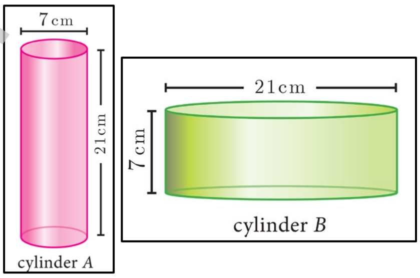


Example 7.15 Find the volume of a cylinder whose height is 2 m and whose base area is $250\ m^2$.

Example 7.16 The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7 m , find its height.

Example 7.17 Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

Example 7.18 For the cylinders A and B (Fig. 7.27),
(i) find out the cylinder whose volume is greater.
(ii) verify whether the cylinder with greater volume has greater total surface area.
(iii) find the ratios of the volumes of the cylinders A and B.



Example 7.19 The volume of a solid right circular cone is $11088\ cm^3$. If its height is 24 cm then find the radius of the cone.

Example 7.20 The ratio of the volumes of two cones is $2:3$. Find the ratio of their radii if the height of second cone is double the height of the first.

Example 7.21 The volume of a solid hemisphere is $29106\ cm^3$. Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere.

Example 7.22 Calculate the mass of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1 mm , and whose density is $17.3\ g/cm^3$. (Hint: mass = density × volume)

Example 7.23 If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm , find the volume of the frustum.

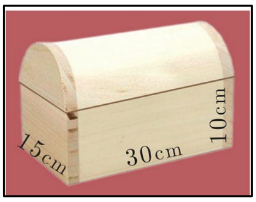
Exercise 7.2

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m . Find the height of the embankment.
2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm . A small cylindrical metal of radius 5 cm and height 4 cm is immersed completely. Calculate the raise of the water in the glass?
3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm .
4. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m . If the container can release the petrol through its bottom at the rate of 25 cu . meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.
5. A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.
6. The volumes of two cones of same base radius are $3600\ cm^3$ and $5040\ cm^3$. Find the ratio of heights.
7. If the ratio of radii of two spheres is $4:7$, find the ratio of their volumes.

8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}:4$.
9. The outer and the inner surface areas of a spherical copper shell are 576π cm^2 and 324π cm^2 respectively. Find the volume of the material required to make the shell.
10. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre.

Example 7.24 A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm Find the total surface area of the toy if its common diameter is 12 cm .

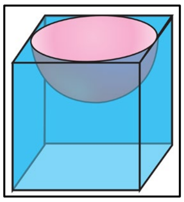
Example 7.25 A jewel box (Fig. 7.39) is in the shape of a cuboid of dimensions 30 cm × 15 cm × 10 cm surmounted by a half part of a cylinder as shown in the figure. Find the volume of the box.



Example 7.26 Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m ?

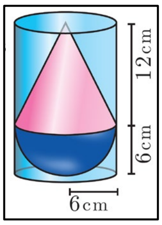
Example 7.27 A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm , diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm . Find the outer surface area of the funnel.

Example 7.28 A hemispherical section is cut out from one face of a cubical block (Fig.7.42) such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.

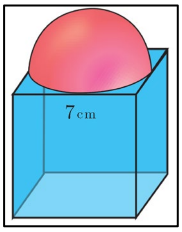


Exercise 7.3

1. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm . Find the capacity of the vessel.
2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm . If each cone has a height of 2 cm , find the volume of the model that Nathan made.
3. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm , a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.
4. A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm .



5. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm , how much medicine it can hold?
6. As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.
7. A right circular cylinder just enclose a sphere of radius r units. Calculate
   (i) the surface area of the sphere
   (ii) the curved surface area of the cylinder
   (iii) the ratio of the areas obtained in (i) and (ii).



Example 7.29 A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm . How many small spheres can be obtained?

Example 7.30 A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Example 7.31 A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm , having a hemispherical cap. Find the number of cones needed to empty the container.

Exercise 7.4

1. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm . Find the height of the cylinder.
2. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm .
3. A conical flask is full of water. The flask has base radius r units and height h units, the water is poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask.
4. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm , find the internal diameter.
5. Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm . Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.
6. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm , then find the height of the cylinder.
7. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm , then find the thickness of the cylinder.
8. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Exercise 7.5

Multiple choice questions

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(A) 60π cm^2 (B) 68π cm^2 (C) 120π cm^2 (D) 136π cm^2
2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
(A) $4\pi r^2$ sq. Units (B) $6\pi r^2$ sq. Units (C) $3\pi r^2$ sq. Units (D) $8\pi r^2$ sq. units
3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be
(A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm
4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(A) 1: 2 (B) 1: 4 (C) 1:6 (D) 1: 8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is
(A) $\frac{9\pi h^2}{8}$ sq.units (B) $24\pi h^2$ sq.units (C) $\frac{8\pi h^2}{9}$ sq.units (D) $\frac{56\pi h^2}{9}$ sq.units
6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm . If its height is 20 cm , the volume of the material in it is
(A) 5600π cm^3 (B) 1120π cm^3 (C) 56π cm^3 (D) 3600π cm^3
7. If the radius of the base of a cone is tripled and the height is doubled then the volume is
(A) made 6 times (B) made 18 times (C) made 12 times (D) unchanged
8. The total surface area of a hemi-sphere is how much times the square of its radius.
(A) π (B) 4π (C) 3π (D) 2π
9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is
(A) $3x$ cm (B) x cm (C) $4x$ cm (D) $2x$ cm
10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm . Then, the volume of the frustum is
(A) 3328π cm^3 (B) 3228π cm^3 (C) 3240π cm^3 (D) 3340π cm^3
11. A shuttle cock used for playing badminton has the shape of the combination of
(A) a cylinder and a sphere (B) a hemisphere and a cone
(C) a sphere and a cone (D) frustum of a cone and a hemisphere
12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is
(A) $2:1$ (B) $1:2$ (C) $4:1$ (D) $1:4$
13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is
(A) $\frac{4}{3}\pi$ (B) $\frac{10}{3}\pi$ (C) 5π (D) $\frac{20}{3}\pi$
14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1:2$ then $r_2 : r_1$ is
(A) $1:3$ (B) $1:2$ (C) $2:1$ (D) $3:1$
15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is
(A) 1:2:3 (B) $2:1:3$ (C) 1:3:2 (D) 3:1:2

Unit Exercise - 7

1. The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre?
2. A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?
3. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units.
4. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm , the diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm , then find the area of the tin sheet required to make the funnel.
5. Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm .

6. A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.
7. The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m . Find the cost of painting its curved surface area at ₹100 per sq. m .
8. A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm . Its external diameter is 14 cm . Find its thickness.
9. The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm . Find the slant height of the cone.
10. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

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CHAPTER - 8. STATISTICS AND PROBABILITY

Example 8.1 Find the range and coefficient of range of the following data: 25,67,48,53,18,39,44.

Example 8.2 Find the range of the following distribution.

Age (in years)	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26	26 – 28
Number of students	0	4	6	8	2	2

Example 8.3 The range of a set of data is 13.67 and the largest value is 70.08 . Find the smallest value.

Example 8.4 The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10 . Find its standard deviation.

Example 8.5 The amount of rainfall in a particular season for 6 days are given as 17.8 cm , 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm . Find its standard deviation.

Example 8.6 The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Example 8.7 The amount that the children have spent for purchasing some eatables in one day trip of a school are 5,10,15,20,25,30,35,40. Using step deviation method, find the standard deviation of the amount they have spent.

Example 8.8 Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Example 8.9 Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4 . Find the standard deviation of the new values.

Example 8.10 Find the mean and variance of the first n natural numbers.

Example 8.11 48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Example 8.12 The marks scored by the students in a slip test are given below. Find the standard deviation of their marks.

x	4	6	8	10	12
f	7	3	5	9	5

Example 8.13 Marks of the students in a particular subject of a class are given below. Find its standard deviation.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of students	8	12	17	14	9	7	4

Example 8.14 The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23 ?

Exercise 8.1

1. Find the range and coefficient of range of the following data.
   (i) 63,89,98,125,79,108,117,68 (ii) 43.5,13.6,18.9,38.4,61.4,29.8
2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
3. Calculate the range of the following data.

Income	400 – 450	450 – 500	500 – 550	550 – 600	600 – 650
Number of workers	8	12	30	21	6

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32,35,37,30,33,36,35 and 37 pages. Find the standard deviation of the pages completed by them.
5. Find the variance and standard deviation of the wages of 9 workers given below: ₹ 310, ₹ 290, ₹ 320 , ₹ 280 , ₹ 300 , ₹ 290 , ₹320, ₹310, ₹ 280.
6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2o′ clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.
7. Find the standard deviation of first 21 natural numbers.
8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5 , then find the new standard deviation.
9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3 , then find the new variance and new standard deviation.
10. The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

11. In a study about viral fever, the number of people affected in a town were noted as Find its standard deviation.

Age in years	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of people affected	3	5	16	18	12	7	4

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21 – 24	25 – 28	29 – 32	33 – 36	37 – 40	41 – 44
Number of plates	15	18	20	16	8	7

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken (sec)	8.5 – 9.5	9.5 – 10.5	10.5 – 11.5	11.5 – 12.5	12.5 – 13.5
Number of students	6	8	17	10	9

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27 . Find the correct mean and standard deviation.

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2,4,10,12 and 14 , then find the remaining two observations.

Example 8.15 The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Example 8.16 The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 kg
Variance	72.25 cm^2	28.09 kg

Which is more varying than the other?

Exercise 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.
2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.
3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.
4. If $n = 5, \bar{x} = 6, \Sigma x^2 = 765$, then calculate the coefficient of variation.
5. Find the coefficient of variation of 24,26,33,37,29,31.

6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38,40,47,44,46,43,49,53. Find the coefficient of variation.
7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?
8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows more consistent and which shows less consistent in marks?

Example 8.17 Express the sample space for rolling two dice using tree diagram.

Example 8.18 A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Example 8.19 Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

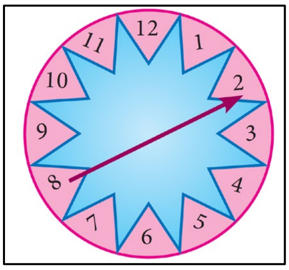
Example 8.20 Two coins are tossed together. What is the probability of getting different faces on the coins?

Example 8.21 What is the probability that a leap year selected at random will contain 53 saturdays.

Example 8.22 A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Example 8.23 A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Example 8.24 A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1,2,3, ... 12. What is the probability that it will point to
(i) 7
(ii) a prime number
(iii) a composite number?



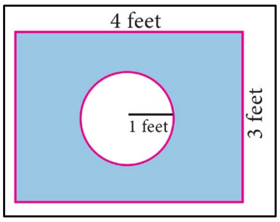
Exercise 8.3

1. Write the sample space for tossing three coins using tree diagram.
2. Write the sample space for selecting two balls at a time from a bag containing 6 balls numbered 1 to 6 (using tree diagram).
3. If A is an event of a random experiment such that $P(A):P(\bar{A}) = 17:15$ and $n(S) = 640$ then find (i) $P(\bar{A})$ (ii) $n(A)$.
4. A coin is tossed thrice. What is the probability of getting two consecutive tails?
5. At a fete, cards bearing numbers 1 to 1000 , one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500 , the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?
6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

7. Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number (iii) the sum as a prime number (iv) the sum as 1

8. Three fair coins are tossed together. Find the probability of getting
(i) all heads (ii) atleast one tail
(iii) atmost one head (iv) atmost two tails

9. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) white (ii) black or red
(iii) not white (iv) neither white nor black



10. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

11. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?($\pi = 3.14$)

12. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on
(i) the same day (ii) different days (iii) consecutive days?

13. In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Example 8.25 If $P(A) = 0.37, P(B) = 0.42, P(A \cap B) = 0.09$ then find $P(A \cup B)$.

Example 8.26 A flower is selected at random from a basket containing 80 yellow, 70 red and 50 white flowers. Find the probability of selecting a yellow or red flower?

Example 8.27 Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Example 8.28 If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{8}$, find (i) $P(A$ or $B)$ (ii) $P($ not A and not $B)$.

Example 8.29 In an apartment, in selecting a house from door numbers 1 to 100 randomly, find the probability of getting the door number of the house to be an even number or a perfect square number or a perfect cube number

Example 8.30 In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
(i) The student opted for NCC but not NSS.
(ii) The student opted for NSS but not NCC.
(iii) The student opted for exactly one of them.

Example 8.31 A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3 . Prove that the probability of B being selected is atmost 0.8 .

Exercise 8.4

1. If $P(A)=\frac{2}{3}, P(B)=\frac{2}{5}, P(A\cup B)=\frac{1}{3}$ then find $P(A\cap B)$.
2. A and B are two events such that, $P(A)=0.42, P(B)=0.48$, and $P(A\cap B)=0.16$. Find
(i) $P(\text{ not } A)$ (ii) $P(\text{not } B)$ (iii) $P(A \text{ or } B)$
3. If A and B are two mutually exclusive events of a random experiment and $P(\text{ not } A)=0.45$, $P(A\cup B)=0.65$, then find $P(B)$.
4. The probability that atleast one of A and B occur is 0.6 . If A and B occur simultaneously with probability 0.2 , then find $P(\bar{A})+P(\bar{B})$.
5. The probability of happening of an event A is 0.5 and that of B is 0.3 . If A and B are mutually exclusive events, then find the probability that neither A nor B happen.
6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.
7. A box contains cards numbered 3,5,7,9, ... 35,37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.
8. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.
9. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?
10. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?
11. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.
12. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$, $P(A\cup B\cup C)=\frac{9}{10}$, $P(A\cap B\cap C)=\frac{1}{15}$, then find $P(A), P(B)$ and $P(C)$?
13. In a class of 35 , students are numbered from 1 to 35 . The ratio of boys to girls is 4: 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

Exercise 8.5

Multiple choice questions

1. Which of the following is not a measure of dispersion?
(A) Range (B) Standard deviation (C) Arithmetic mean (D) Variance
2. The range of the data 8,8,8,8,8 ... 8 is
(A) 0 (B) 1 (C) 8 (D) 3
3. The sum of all deviations of the data from its mean is
(A) Always positive (B) always negative (C) zero (D) non-zero integer
4. The mean of 100 observations is 40 and their standard deviation is 3 . The sum of squares of all observations is
(A) 40000 (B) 160900 (C) 160000 (D) 30000

5. Variance of first 20 natural numbers is
(A) 32.25 (B) 44.25 (C) 33.25 (D) 30

6. The standard deviation of a data is 3 . If each value is multiplied by 5 then the new variance is
(A) 3 (B) 15 (C) 5 (D) 225

7. If the standard deviation of x, y, z is p then the standard deviation of $3x + 5, 3y + 5, 3z + 5$ is
(A) $3p + 5$ (B) $3p$ (C) $p + 5$ (D) $9p + 15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
(A) 3.5 (B) 3 (C) 4.5 (D) 2.5

9. Which of the following is incorrect?
(A) $P(A) > 1$ (B) $0 \leq P(A) \leq 1$ (C) $P(\phi) = 0$ (D) $P(A) + P(\bar{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is
(A) $\frac{q}{p+q+r}$ (B) $\frac{p}{p+q+r}$ (C) $\frac{p+q}{p+q+r}$ (D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
(A) $\frac{3}{10}$ (B) $\frac{7}{10}$ (C) $\frac{3}{9}$ (D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is
(A) 2 (B) 1 (C) 3 (D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is
(A) 5 (B) 10 (C) 15 (D) 20

14. If a letter is chosen at random from the English alphabets $\{a, b, \ldots, z\}$, then the probability that the letter chosen precedes x
(A) $\frac{12}{13}$ (B) $\frac{1}{13}$ (C) $\frac{23}{26}$ (D) $\frac{3}{26}$

15. A purse contains 10 notes of ₹ 2000,15 notes of ₹ 500 , and 25 notes of ₹ 200 . One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?
(A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$

Unit Exercise - 8

1. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50 . Compute the missing frequencies f_1 and f_2.

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	f_1	10	f_2	7	8

2. The diameter of circles (in mm) drawn in a design are given below.

Diameters	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
Number of circles	15	17	21	22	25

`
Calculate the standard deviation.

3. The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a varience of 160 . Determine the value of k.

4. The standard deviation of some temperature data in degree celsius ($^\circ$C) is 5 . If the data were converted into degree Farenheit ($^\circ$F) then what is the variance?

5. If for a distribution, $\sum(x-5)=3, \sum(x-5)^2=43$, and total number of observations is 18 , find the mean and standard deviation.

6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in city A	20	22	19	23	16
Prices in city B	10	20	18	12	15

7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5 .

9. In a two children family, find the probability that there is at least one girl in a family.

10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1 . If the probability of passing the English examination is 0.75 , what is the probability of passing the Tamil examination?

ALL THE BEST STUDENTS

EFFORT NEVER FAILS

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10 TH MATHS GEOMETRY & GRAPH

COMPLETE QUESTION BANK EM 2024-2025

SIMILAR TRIANGLE

I. SCALE FACTOR < 1 MODEL SUM (IN SIDE TRIANGLE)

1. Construct a triangle similar to a given triangle **PQR** with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle **PQR** (**Scale Factor** $\frac{3}{5} < 1$).
2. Construct a triangle similar to a given triangle **PQR** with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle **PQR** (**Scale Factor** $\frac{2}{3} < 1$).
3. Construct a triangle similar to a given triangle **LMN** with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle **LMN** (**Scale Factor** $\frac{4}{5} < 1$).

II. SCALE FACTOR > 1 MODEL SUM (OUT SIDE TRIANGLE)

1. Construct a triangle similar to a given triangle **PQR** with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle **PQR** (**Scale Factor** $\frac{7}{4} > 1$).
2. Construct a triangle similar to a given triangle **ABC** with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle **ABC** (**Scale Factor** $\frac{6}{5} > 1$).
3. Construct a triangle similar to a given triangle **PQR** with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle **PQR** (**Scale Factor** $\frac{7}{3} > 1$).

SINGLE TANGENT & ALTERNATE SEGMENT & TWO TANGENT

III. SINGLE OR ONE TANGENT (RADIUS IS GIVEN)

1. Draw a circle of **radius 3 cm**. Take a point P on this circle and draw **a tangent at P**.
2. Draw a tangent at any point R on the circle of **radius 3.4 cm** and centre at **P**?.

IV. ALTERNATE SEGMENT OR CHORD TANGENT THEOREM USING (RADIUS IS GIVEN)

1. Draw a circle of **radius 4 cm**. At a point L on it draw a **tangent** to the circle using the **alternate segment**.
2. Draw a circle of **radius 4.5 cm**. Take a point on the circle. Draw the tangent at that point using the **Alternate Segment Theorem.**

V. TWO TANGENT (DIAMETER OR RADIUS IS GIVEN) MOST IMPORTANT

1. Draw a circle of **diameter 6 cm** from a point P, which is **8 cm** away from its centre. Draw the **two tangents PA** and **PB** to the circle and measure their lengths.
2. Draw the **two tangents** from a point which is **10 cm** away from the centre of a circle of **radius 5 cm**. Also, measure the lengths of the tangents.
3. Take a point which is **11 cm** away from the centre of a circle of **radius 4 cm** and draw the **two tangents** to the circle from that point.
4. Draw the **two tangents** from a point which is **5 cm** away from the centre of a circle of **diameter 6 cm**. Also , measure the **lengths of the tangents**.

5. Draw **a tangent** to the circle from the point P having **radius 3.6 cm.** and centre O point P is at a **distance 7.2 cm** from the centre.

CONSTRUCTION OF A TRIANGLE

VI. INCIRCLE TRIANGLE OR MEDIAN SUM (MEDIAN IS GIVEN)

1. Construct a $\Delta\, PQR$ in which $PQ = 8$ **cm**, $\angle R = 60^\circ$ and the **Median RG** from R to **PQ is 5.8 cm**. Find the length of the **altitude from R to PQ**.
2. Construct a $\Delta\, PQR$ in which base $PQ = 4.5$ **cm**, $\angle R = 35^\circ$ and the **Median RG** from R to **PQ is 6 cm**.
3. Construct a $\Delta\, PQR$ in which $QR = 5$ **cm**, $\angle P = 40^\circ$ and the **Median PG** from P to **QR is 4.4 cm**. Find the length of the Altitude from P to QR.

VII. INCIRCLE TRIANGLE OR ALTITUDE SUM (ALTITUDE IS GIVEN)

1. Construct a $\Delta\, PQR$ in which such that $QR = 5$ **cm**, $\angle P = 30^\circ$ and the **Altitude** from P to **QR is of length 4.2 cm**.
2. Construct a $\Delta\, PQR$ in which such that $QR = 6.5$ **cm**, $\angle P = 60^\circ$ and the **Altitude** from P to **QR is of length 4.5 cm**
3. Construct a $\Delta\, PQR$ in which such that $QR = 5.5$ **cm**, $\angle P = 25^\circ$ and the **Altitude** from P to **QR is of length 4 cm**.

VIII. INCIRCLE TRIANGLE OR BISECTOR SUM (BISECTOR IS GIVEN)

1. Draw a triangle $\Delta\, ABC$ of base $BC = 8$ **cm**, $\angle A = 60^\circ$ and the **Bisector** of $\angle A$ meets BC at D such that **BD = 6 cm.**
2. Draw a triangle $\Delta\, ABC$ of base $BC = 5.6$ **cm**, $\angle A = 40^\circ$ and the **Bisector** of $\angle A$ meets BC at D such that **BD = 4 cm.**
3. Draw a triangle $\Delta\, PQR$ such that $PQ = 6.8$ **cm**, **vertical angle** is 50° and the **Bisector** of vertical angle meets the base at D where **PD = 5.8 cm.**

GRAPH OF VARIATION

I. DIRECT VARIATION MODEL SUM (GRAPH STRAIGHT LINE)

1. **Varshika drew 6** circles with different sizes. Draw a graph for the relationship between the **diameter and circumference** (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its **diameter is 6 cm.**

Diameter (x) cm	**1**	**2**	**3**	**4**	**5**
Circumference (y) cm	**3.1**	**6.2**	**9.3**	**12.4**	**15.5**

2. A bus is travelling at a uniform speed of **50 km / hr**. Draw the **distance time graph** and hence find
   (i) The **constant of variation**.
   (ii) How far will it travel in **90 minutes or 1 ½ hrs?.**
   (iii)The time required to cover a distance of **300 km** from the graph.

3. A garment shop announces a flat **50 % discount** on every purchase of items for their customers. Draw the graph for the relation between the **Marked Price and the Discount**. Hence find

(i) The marked price when a customer gets a **discount of ₹ 3250 (from graph).**

(ii) The discount when the **marked price is ₹ 2500.**

4. Graph the following **linear function** $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also find

(i) y when $x = 9$ and (ii) x when $y = 7.5$.

5. A two wheeler parking zone near bus stand charges as below.

Time x (in hours)	4	8	12	24
Amount y (in ₹)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also

(i) Find the amount to be paid when parking **time is 6 hrs**.

(ii) Find the parking duration when the amount **paid is ₹ 150**.

II. INVERSE OR INDIRECT VARIATION MODEL SUM (GRAPH CURVE LINE)

1. A Company initially started with **40 workers** to complete the work by **150 days**. Later it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the **type of variation**.

(ii) From the graph, find the number of days required to complete the work if the company decides to opt for **120 workers?.**

(iii) If the work has to be completed by **200 days**, how many workers are required?.

2. **Nishanth** is the winner in a Marathon race **12 km distance**. He ran at the uniform speed of **12 km / hr** and reached the destination in **1 hour**. He was followed by Aradhana, Jeyanth, Sathya and Swetha with their respective speed of **6 km / hr, 4 km / hr, 3 km / hr and 2 km / hr**. And, they covered the distance in **2 hrs, 3 hrs, 4 hrs and 6 hours** respectively.

Draw the Speed- time graph and use it to find time taken to Kaushik with his speed **2.4 km / hr**.

3. Draw the graph of $xy = 24, x, y > 0$. Using the graph find,

(i) y when $x = 3$ and (ii) x when $y = 6$.

4. The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of. pipes x	2	3	6	9
Time Taken y (in mins)	45	30	15	10

Draw the graph for the above data and hence

(i) Find the time taken to fill the tank when **five pipes** are used.

(ii) Find the number of pipes when the **time is 9 minutes**.

5. A School announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of. Participants (x)	2	4	6	8	10
Amount for each Participants y (in ₹)	180	90	60	45	36

(i) Find the **constant of variation**.

(ii) Graph the above data and hence, find how will each participants get if the number of **participants are 12.**

QUADRATIC EQUATION GRAPH

I. NATURE OF SOLUTION SUM (GRAPH CURVE OR PARABOLA 'U' LINE)

1. Discuss the nature of solutions of the following quadratic equations.

(i) $x^2 + x - 12 = 0$ (ii) $x^2 - 8x + 16 = 0$ (iii) $x^2 + 2x + 5 = 0$

2. Graph the following quadratic equations and state the nature of solutions.

(i) $x^2 - 9x + 20 = 0$ (ii) $x^2 - 4x + 4 = 0$ (iii) $x^2 + x + 7 = 0$ (iv) $x^2 - 9 = 0$

(v) $x^2 - 6x + 9 = 0$ (vi) $(2x - 3)(x + 2) = 0$

II. GRAPHICALLY SOLVE THE EQUATION SUM (GRAPH CURVE OR PARABOLA 'U' LINE)

1. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$.

2. Draw the graph of $y = x^2 + 4x + 3$ and hence solve $x^2 + x + 1 = 0$.

3. Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

4. Draw the graph of $y = x^2 - 4x + 3$ and hence solve $x^2 - 6x + 9 = 0$.

5. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$.

6. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.

7. Draw the graph of $y = x^2 + 3x + 2$ and hence solve $x^2 + 2x + 1 = 0$.

8. Draw the graph of $y = x^2 + 3x - 4$ and hence solve $x^2 + 3x - 4 = 0$.

9. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.

10. Draw the graph of $y = 2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$.

11. Draw the graph of $y = (x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$.

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