10th STD

PUBLIC EXAMINATION - APRIL 2025

Reg. No.

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 100

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (i) Answer **all** the questions. **14 × 1 = 14**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. The range of the relation R = $\{(x, x^2) \mid x$ is a prime number less than 13$\}$ is :
(a) {2, 3, 5, 7} (b) {2, 3, 5, 7, 11}
(c) {4, 9, 25, 49, 121}
(d) {1, 4, 9, 25, 49, 121}

2. Let n(A) = m and n(B) = n, then the total number of functions that exist from B to A is :
(a) m^n (b) n^m (c) $2^{mn}-1$ (d) 2^{mn}

3. Euclid's division lemma states that for positive integers a and b, there exist unique positive integers q and r such that a = bq + 1, where r must satisfy.
(a) $1 < r < b$ (b) $0 < r < b$
(c) $0 \le r < b$ (d) $0 < r \le b$

4. If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is :
(a) 0 (b) 6 (c) 7 (d) 13

5. If $(x - 6)$ is the H.C.F of $x^2 - 2x - 24$ and $x^2 - kx - 6$, then the value of k is :
(a) 3 (b) 5 (c) 6 (d) 8

6. Transpose of a column matrix is :
(a) unit matrix (b) diagonal matrix
(c) column matrix (d) row matrix

7. A tangent is perpendicular to the radius of a circle at the :
(a) Centre (b) Point of Contact
(c) Infinity (d) Chord

8. The equation of a line passing through the origin and parallel to the line $7x - 3y + 4 = 0$ is :
(a) $7x - 3y + 4 = 0$ (b) $3x - 7y + 4 = 0$
(c) $3x + 7y = 0$ (d) $7x - 3y = 0$

9. The slope of the line which is perpendicular to a line joining the points (0,0) and (–8, 8) is :
(a) –1 (b) 1 (c) $\frac{1}{3}$ (d) –8

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure :
(a) 45° (b) 30° (c) 90° (d) 60°

11. The curved surface area of a right circular cone of height 15cm and base diameter 16cm is :
(a) 60π cm^2 (b) 68π cm^2
(c) 120π cm^2 (d) 136π cm^2

12. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is :
(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

13. Which of the following is incorrect?
(a) P(A) > 1 (b) $0 \le P(A) \le 1$
(c) $P(\phi) = 0$ (d) $P(A) + P(\bar{A}) = 1$

14. A purse contains 10 notes of ₹ 2000, 15 notes of ₹ 500 and 25 notes of ₹ 200. One note is drawn at random. What is the probability that the note is either a ₹ 500 note or ₹ 200 note?
(a) $\frac{1}{5}$ (b) $\frac{3}{10}$ (c) $\frac{2}{3}$ (d) $\frac{4}{5}$

Part - II

Note : Answer **any ten** questions. Question No.28 is **compulsory**. **10 × 2 = 20**

15. Let A = {1, 2, 3} and B = $\{x \mid x$ is a prime number less than 10$\}$. Find A × B.

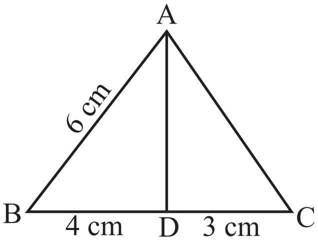
16. Find fog and gof when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

17. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

18. Add : $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

19. If A = $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ find AB and BA. Verify if AB = BA.

20. In the given diagram, AB is the bisector of $\angle$A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC.



21. The line through the points (–2, a) and (9,3) has slope $-\frac{1}{2}$. Find the value of 'a'.

22. Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

23. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30º. Find the distance of the car from the rock.

24. If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.

25. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

26. Find the range of the following distribution :

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of Students	0	4	6	8	2	2

27. A flower is selected at random from a basket containing 80 yellow, 70 red and 50 white flowers. Find the probability of selecting a yellow coloured flower.

28. Check whether the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \sqrt{50}$,...... is an A.P or not? If it is an A.P, then find the common difference.

Part - III

Note : Answer **any ten** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime numbers. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

30. Let $f : A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where A = {2, 4, 6, 10, 12}. B = {0, 1, 2, 4, 5, 9}. Represent f by
(i) set of ordered pairs (ii) a table
(iii) an arrow diagram (iv) a graph

31. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.

32. Find the sum of the series $6^2+7^2+8^2+....+21^2$.

33. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$, show that $(AB)^T = B^T A^T$.

34. State and prove Angle Bisector Theorem.

35. Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are taken in the order (–4, –2), (–3, k), (3, –2) and (2, 3).

36. A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (–3, 8). Find its equation.

37. Prove that : $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

38. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.

39. A metallic sphere of radius 16cm is melted and recast into small spheres each of radius 2cm. How many small spheres can be obtained?

40. The total marks scored by two students, Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

41. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) white (b) black or red
(iii) not white (iv) neither white nor black

42. Determine the nature of the roots of the equation: $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$.

Part - IV

Note : Answer **all** the questions. **2 × 8 = 16**

43. (a) Draw a triangle ABC of base BC = 8 cm, $\angle$A = 60º and the bisector of $\angle$A meets BC at D such that BD = 6 cm.

(OR)

(b) Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

44. (a) Graph the quadratic equation $x^2 - 9x + 20 = 0$ and state the nature of solution.

(OR)

(b) A bus is travelling at a uniform speed of 50 km / hr. Draw the distance – time graph and hence find.

(i) the constant of variation

(ii) how far will it travel in 90 minutes?

(iii) the time required to cover a distance of 300 km.

☸☸☸☸

ANSWERS

Part - I

1. (c) {4, 9, 25, 49, 121}

2. (a) m^n

3. (c) $0 \le r < b$

4. (a) 0

5. (b) 5

6. (d) row matrix

7. (b) Point of Contact

8. (d) $7x - 3y = 0$

9. (b) 1

10. (d) 60^o

11. (d) $136\,\pi$ cm^2

12. (a) $\frac{4}{3}\pi$

13. (a) $P(A) > 1$

14. (d) $\frac{4}{5}$

Part - II

15. $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

16. $f(x) = 2x + 1$, $g(x) = x^2 - 2$

$fog(x) = f(g(x)) = f(x^2 - 2)$

$= 2(x^2 - 2) + 1 = 2x^2 - 3$

$gof(x) = g(f(x)) = g(2x + 1)$

$= (2x + 1)^2 - 2 = 4x^2 + 4x - 1$

Thus, $fog(x) = 2x^2 - 3$, $gof = 4x^2 + 4x - 1$.

From the above, we see that $fog \neq gof$.

17. By Euclid's division algorithm,

$a = bq + r$, $0 \le r < b$.

Here $532 = 21q + r$ ... (1)

$\Rightarrow$ $532 = 21(25) + 7$ ... (2)

$\therefore q = 25$, and $r = 7$

[Comparing (1) and (2)]

$$\begin{array}{r|l} \therefore & 21 \overline{)532}\;(25 \\ & \underline{42} \\ & 112 \\ & \underline{105} \\ & 7 \end{array}$$

$\therefore$ Number of completed rows = 25 and the leftover flower pots = 7.

18. $\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3}{x-y} - \frac{y^3}{x-y}$

[Taking (–1) common from the second term]

LCM of $(x - y), (x - y)$ is $x - y$

$= \frac{x^3 - y^3}{(x-y)} = \frac{\cancel{(x-y)}(x^2 + xy + y^2)}{\cancel{(x-y)}}$

$[\therefore a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= x^2 + xy + y^2$

19. We observe that A is a 2 × 2 matrix and B is a 2 × 2 matrix, hence AB is defined and it will be of the order 2 × 2.

$$AB = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+1 & 0+3 \\ 2+3 & 0+9 \end{pmatrix}$$

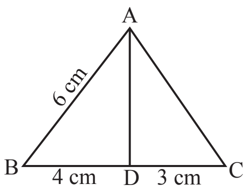
$$= \begin{pmatrix} 5 & 3 \\ 5 & 9 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+0 & 2+0 \\ 2+3 & 1+9 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 2 \\ 5 & 10 \end{pmatrix}$$

Therefore, $AB \neq BA$.

20. In Δ ABC, AD is the bisector of ∠A. Therefore by Angle Bisector Theorem



$\frac{BD}{DC} = \frac{AB}{AC}$

$\frac{4}{3} = \frac{6}{AC}$ gives

$4AC = 18$

Hence, $AC = \frac{18}{4} = 4.5$ cm

21. A line passing joining the points $(-2, a)$ and $(9, 3)$ has slope $m = \frac{-1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - a}{9 - (-2)} = \frac{-1}{2}$

$2(3 - a) = -1\,(11)$

$-2a = -11 - 6 = -17$

$a = \frac{17}{2}$.

22. Slope of the straight line is $x - 2y + 3 = 0$

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

Slope of the straight line $6x + 3y + 8 = 0$ is

$$m_2 = \frac{-6}{3} = -2$$

Now, $m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

Hence, the two straight lines are perpendicular.

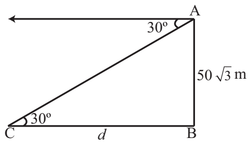
Aliter

$a_1 = 1,\ b_1 = -2;$
$a_2 = 6,\ b_2 = 3$
$a_1 a_2 + b_1 b_2 = 6 - 6 = 0$
The lines are perpendicular.

23. Let AB be the height of the rock and C be the position of car, let BC = d

Given AB = $50\sqrt{3}$ and $\angle ACB = 30^\circ$

In ΔABC, $\tan 30 = \frac{AB}{BC}$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{d}$$

$$\Rightarrow d = 50\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow d = 50 \times 3 = 150 \text{ m.}$$

24. Let r_1, r_2 be the radii of the two given spheres

Given $\frac{r_1}{r_2} = \frac{4}{7}$

Ratio of their volumes $= \dfrac{\frac{4}{3}\pi \times r_1^3}{\frac{4}{3}\pi \times r_2^3} = \dfrac{r_1^3}{r_2^3}$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{7}\right)^3 = \frac{4 \times 4 \times 4}{7 \times 7 \times 7} = \frac{64}{343}$$

Hence, ratio of their volumes = 64 : 343

25. Let l, R and r be the slant height, top radius and bottom radius of the frustum.

Given that, $l = 5$ cm, R = 4 cm, $r = 1$ cm

Now, C.S.A. of the frustum $= \pi(R + r)l$ sq. units

$$= \frac{22}{7} \times (4 + 1) \times 5$$

$$= \frac{550}{7}$$

Therefore, C.S.A. $= 78.57 \text{ cm}^2$

26. Her Largest value L = 28
Smallest value S = 16
Range R = L – S
R = 28 – 16
= 12 Years

27. Total number of flowers $n(S) = 80 + 70 + 50 = 200$

No. of yellow flowers $n(Y) = 80$

$$\therefore P(Y) = \frac{n(Y)}{n(S)} = \frac{80}{200}$$

No. of red flowers $n(R) = 70$

$$\therefore P(R) = \frac{n(R)}{n(S)} = \frac{70}{200}$$

Y and R are mutually exclusive $P(Y \cup R) = P(Y) + P(R)$

Probability of drawing either a yellow or red flower.

$$P(Y \cup R) = \frac{80}{200} + \frac{70}{200} = \frac{150}{200} = \frac{3}{4}$$

28. Simplify the sequences :

$\sqrt{2} = \sqrt{2}$

$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

The sequence becomes :

$\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2},\ 4\sqrt{2},\ 5\sqrt{2}...$

Check for a common difference :

$2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$9\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Since there is a common difference, the sequence is an arithmetic progression (AP). The common difference is $\sqrt{2}$.

Part - III

29. $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS $= (A \cap B) \times C$

$A \cap B = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(1)

RHS $= (A \times C) \cap (B \times C)$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(2)

(1) = (2) $\therefore$ LHS = RHS. Hence it is verified.

30. $f : A \rightarrow B$

A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}

$f(x) = \frac{x}{2} - 1,$

$f(2) = \frac{2}{2} - 1 = 0$

$f(4) = \frac{4}{2} - 1 = 1$

$f(6) = \frac{6}{2} - 1 = 2$

$f(10) = \frac{10}{2} - 1 = 4$

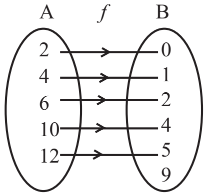
$f(12) = \frac{12}{2} - 1 = 5$

(i) Set of ordered pairs
={(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

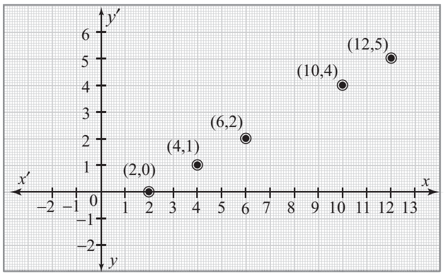
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) an arrow diagram;



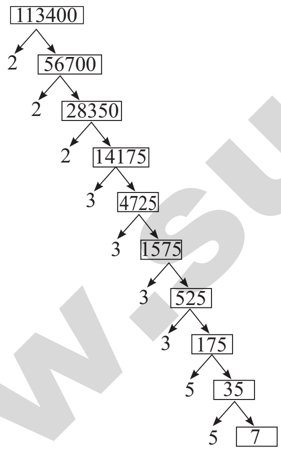
(iv) a graph



31. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

p_1, p_2, p_3, p_4 are primes in ascending order, x_1, x_2, x_3, x_4 are integers.

Using prime factorisation tree.



$113400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$

$= 2^3 \times 3^4 \times 5^2 \times 7^1$

$= p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$

$\therefore\ p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, x_1 = 3, x_2 = 4,$

$x_3 = 2, x_4 = 1.$

32. $6^2 + 7^2 + 8^2 + ... + 21^2$

$= (1^2 + 2^2 + ... + 21^2) - (1^2 + 2^2 + ... + 5^2)$

$= \sum_1^{21} n^2 - \sum_1^{5} n^2$

$= \left(\frac{n(n+1)(2n+1)}{6}\right)_{n=21} - \left(\frac{n(n+1)(2n+1)}{6}\right)_{n=5}$

$= \left(\frac{\overset{7}{\cancel{21}} \times \overset{11}{\cancel{22}} \times 43}{\cancel{6}_{\cancel{2}}}\right) - \left(\frac{5 \times \cancel{6} \times 11}{\cancel{6}}\right)$

$= 3311 - 55 = 3256$

33. LHS $= (AB)^T$

$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}_{2\times3} \times \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}_{3\times2}$

$= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$ ... (1)

RHS $= (B^T A^T)$

$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}, A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$

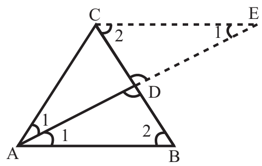
$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}_{2\times3} \times \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}_{3\times2}$

$= \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$ ... (2)

From (1) and (2), $(AB)^T = B^T A^T$. Hence proved.

34. Statement:



The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Proof : Given : In ΔABC, AD is the internal bisector.

To prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	Δ ACE is isosceles AC = CE ....(1)	In Δ ACE, $\angle CAE = \angle CEA$

3.	$\Delta ABD \sim \Delta ECD$ $\frac{-390}{-13}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC =CE. Hence proved.

35. $(-4, -2)$, $(-3, k)$ $(3, -2)$ and $(2, 3)$
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
(x_1, y_1) (x_2, y_2) (x_3, y_3) (x_4, y_4)

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq. units}$$

Given Area = 28

$$= \frac{1}{2}\begin{vmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{vmatrix} = 28 \text{ sq. units}$$

$\Rightarrow [(-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12)] = 56$

$\Rightarrow \quad 11 - 4k - (-10 + 3k) = 56$

$\Rightarrow \quad -7k + 21 = 56$

$-7k = 56 - 21 = 35$

$k = -5$

36. If a and b are the intercepts then $a + b = 7$ or $b = 7 - a$

By intercept form $\frac{x}{a} + \frac{y}{b} = 1$

We have $\frac{x}{a} + \frac{y}{7-a} = 1$

As this line pass through the point $(-3, 8)$, we have $\frac{-3}{a} + \frac{8}{7-a} = 1 \Rightarrow -3(7-a) + 8a = a(7-a)$

$-21 + 3a + 8a = 7a - a^2$

So, $a^2 + 4a - 21 = 0$

Solving this equation $(a-3)(a+7) = 0$

$a = 3$ or $a = -7$

Since a is positive, we have $a = 3$ and

$b = 7 - a = 7 - 3 = 4.$

Hence $\frac{x}{3} + \frac{y}{4} = 1$

Therefore, $4x+3y-12=0$ is the required equation.

37. $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$= \frac{(\sin A - \cos A)^3 - 3\sin A \cos A(\sin A + \cos A)}{\sin A + \cos A}$$

$$+ \frac{(\sin A + \cos A)^3 + 3\sin A \cos A(\sin A - \cos A)}{\sin A - \cos A}$$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$a^3 - b^3 = (a-b)^3 - 3ab(a-b)$

where $a = \sin A$ and $b = \cos A]$

$$= (\sin A + \cos A)\left[\frac{(\sin A + \cos A)^2 - 3\sin A \cos A}{\sin A + \cos A}\right]$$

$$+ (\sin A - \cos A)\left[\frac{(\sin A - \cos A)^2 + 3\sin A \cos A}{\sin A - \cos A}\right]$$

[$\because$ Taking (sinA + cos A) common from the first term and (sin A – cos A) common from the second term]

$= (\sin A + \cos A)^2 - 3\sin A\cos A +$
$(\sin A - \cos A)^2 + 3\sin A\cos A$

$= \sin^2 A + \cos^2 A) + 2\sin A\cos A$
$-3\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A$
$+ 3\sin A\cos A$

$= 1 - \sin A\cos A + 1 + \sin A\cos A = 1 + 1 = 2$

= RHS. Hence proved

38. Volume of the model = Volume of the cylinder + Volume of 2 cones.

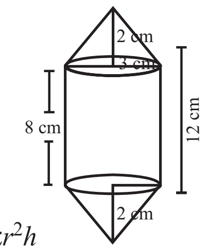
For cylinder $h = 12 - 2 - 2 = 8$ cm

diameter = 3 cm

$\Rightarrow \quad r = \frac{3}{2}$ cm

For cone $h = 2$ cm

$r = \frac{3}{2}$ cm



Volume of the cylinder part $= \pi r^2 h$

$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 8$

$= \frac{396}{7} = 56.57 \text{ cm}^3$

Volume of the 2 conical parts $= 2\left(\frac{1}{3}\pi r^2 h\right)$

$= 2 \times \frac{1}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 2 = 9.42 \text{ cm}^3$

$\therefore$ Total volume $= 56.57 + 9.42 = 65.99 \text{ cm}^3$

The volume of the model that Nathan made $= 66 \text{ cm}^3$

39. Let the number of small spheres obtained be n.

Let r be the radius of each small sphere and R be the radius of metallic sphere.

Here, R = 16 cm, $r = 2$ cm

Now, $n \times$ (Volume of a small sphere) = Volume of big metallic sphere

$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$n\left(\frac{4}{3}\pi \times 2^3\right) = \frac{4}{3}\pi \times 16^3$

$8n = 4096 \Rightarrow n = 512$

Therefore, there will be 512 small spheres.

40.

Sathya	Vidhya
Given $\Sigma x_1 = 460$	Given $\Sigma x_2 = 480$
$\sigma_1 = 4.6$	$\sigma_2 = 2.4$
$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{460}{5}$	$\bar{x}_2 = \frac{\Sigma x_2}{n} = \frac{480}{5}$
$= 92$	$= 96$
$\therefore \text{C.V}_1 = \frac{4.6}{92} \times 100$	$\text{C.V}_2 = \frac{2.4}{96} \times 100$
$= 5\%$	$= 2.5\%$

C.V of Vidhya is less than C.V. of Sathya.
∴ Vidhya is more consistent.

41. Sample space = {5 red, 6 white, 7 green, 8 black balls}

$\therefore n(S) = 26$

(i) Let A be the event of getting the white ball

Given $n(A) = 6$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$

(ii) Let B be the event of getting the black ball

$n(B) = 8$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{8}{26}$

Let C be the event of getting the red ball

$n(C) = 5$ $\quad \therefore P(C) = \frac{5}{26}$

∴ Probability of getting black or red balls

$= P(B) + P(C) = \frac{8}{26} + \frac{5}{26}$

$= \frac{8+5}{26} = \frac{13}{26} = \frac{1}{2}$

(iii) Probability of not getting a white ball = $P(\bar{A})$

$= 1 - P(A)$

$= 1 - \frac{3}{13} = \frac{13-3}{13} = \frac{10}{13}$

(iv) Probability of neither white nor black

= 1 – Probability of white or black

= 1 – [Probability of white ball + Probability of black ball]

$= 1 - \left[\frac{6}{26} + \frac{8}{26}\right]$

[∵ there are 6 white balls and 8 black balls]

$= 1 - \left(\frac{8+6}{26}\right) = 1 - \frac{14}{26} = \frac{26-14}{26} = \frac{12}{26}$

$= \frac{6}{13}$

42. To determine the nature of the roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$.

Expand the equation :

$(x^2 - 2x + 1) + (x^2 - 4x + 4) + (x^2 - 6x + 9) = 0$

Combine like terms : $3x^2 - 12x + 14 = 0$

The discriminant (D) is given by the formula $D = b^2 - 4ac$, where a, b and c are the coefficients of the quadratic equation $ax^2 + bx + c = 0$.

$a = 3, b = -12, c = 14$

$D = (-12)^2 - 4(3)(14)$

$= 144 - 168 \quad = -24$

Since $\Delta \angle 0$, the equation has two complex roots.

Part - IV

43. (a) Construction:

Step 1 : Draw a line segment BC = 8 cm.

Step 2 : At B, draw BE such that ∠CBE = 60° .

Step 3 : At B, draw BF such that ∠EBF = 90° .

Step 4 : Draw the perpendicular bisector to BC, which intersects BF at O and BC at G.

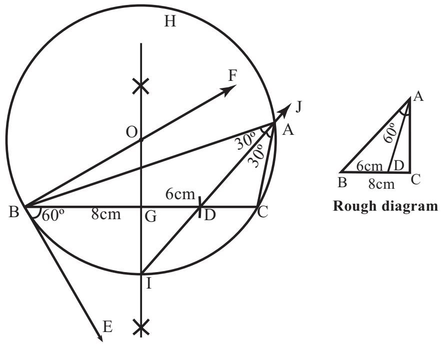
Step 5 : With O as centre and OB as radius draw a circle.

Step 6 : From B, mark an arc of 6cm on BC at D.

Step 7 : The perpendicular bisector intersects the circle at I. Joint ID.

Step 8 : ID produced meets the circle at A. Now join AB and AC.

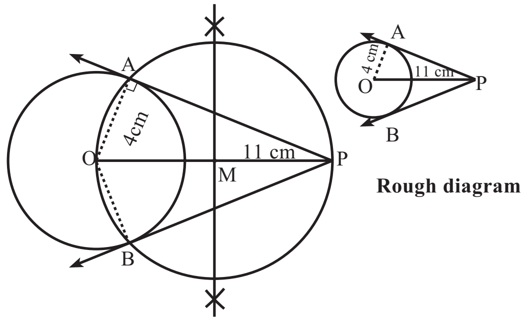
Then ΔABC is the required triangle.



(OR)

(b) Radius = 4 cm

The distance of a point from the center = 11 cm.



Construction:

Step 1 : With centre O, draw a circle of radius 4 cm.

Step 2 : Draw a line OP = 11 cm.

Step 3 : Draw a $\perp^r$ bisector of OP, which cuts at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 10.2 cm.

Verification:

In the right triangle

$$\angle OPA,\ PA = \sqrt{OP^2 - OA^2} = \sqrt{11^2 - 4^2}$$
$$= \sqrt{121-16} = \sqrt{105}$$
$$\cong 10.2 \text{ cm (approximately)}$$

44. (a)

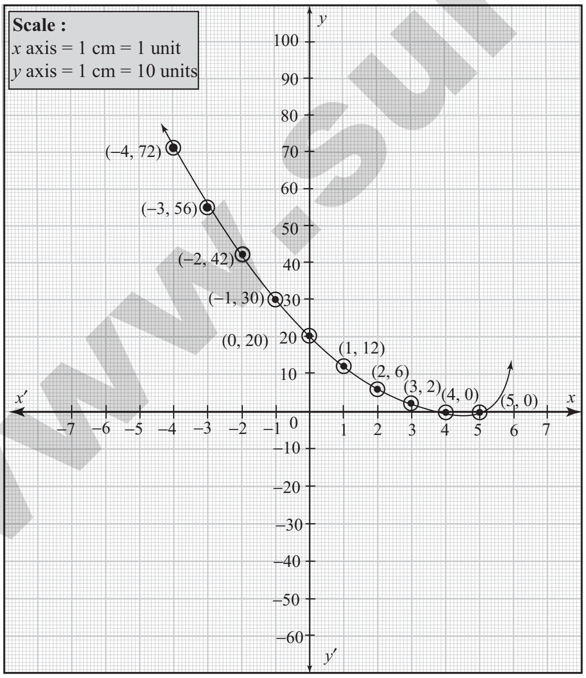
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-9x$	+36	27	18	9	0	–9	–18	–27	–36
20	20	20	20	20	20	20	20	20	20
	72	56	42	30	20	12	6	2	0

Step 1:

Points to be plotted : (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1, 12), (2, 6), (3, 2), (4, 0)

Step 2: The point of intersection of the curve with x axis is (4, 0) (5,0)

Step 3:



The roots are real & unequal

$\therefore$ Solution {4, 5}

(OR)

(b) Let x be the time taken in minutes and y be the distance travelled in km.

Time taken x (in minutes)	60	120	180	240
Distance y (in km)	50	100	150	200

(i) Observe that as time increases, the distance travelled also increases. Therefore, the variation is a direct variation. It is of the form $y = kx$.

Constant of variation

$$k = \frac{y}{x} = \frac{50}{60} = \frac{100}{120} = \frac{150}{180} = \frac{200}{240} = \frac{5}{6}$$

Hence, the relation may be given as

$$y = kx = y = \frac{5}{6}x$$

(ii) From the graph, $y = \frac{5x}{6}$, if $x = 90$,

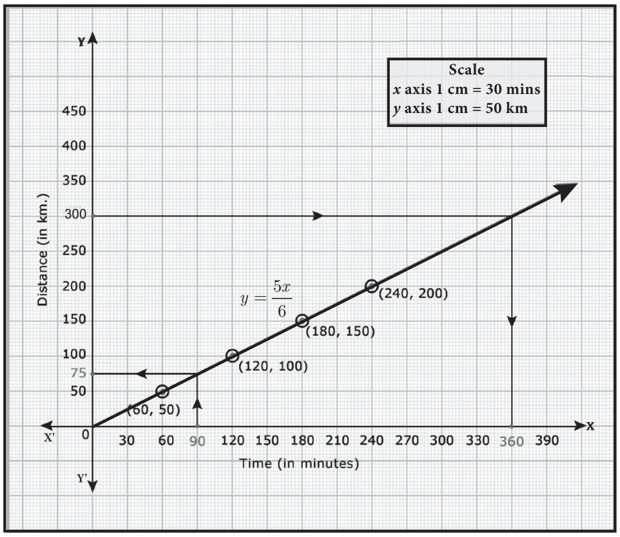
then $y = \frac{5}{6} \times 90 = 75\text{km}$

The distance travelled for 90 minutes is 75 km.

(iii) From the graph, $y = \frac{5x}{6}$, if $y = 300$

then $x = \frac{6y}{5} = \frac{6}{5} \times 300 = 360$ minutes (or) 6 hours.

The time taken to cover 300 km is 360 minutes (or) 6 hours.



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