CO **COMMON FIRST MID - TERM TEST - 2019**

STANDARD - XII

MATHEMATICS

Reg.No. | | | | | | |

Time : 1.30 hours *Marks: 50*

PART - A

Answer all the questions. Each carries one mark: **10×1=10**

1. If A^TA^{-1} is symmetric, then A^2 =
   1) A^{-1} 2) $(A^T)^2$ 3) A^T 4) $(A^{-1})^2$
2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is
   1) 17 2) 14 3) 19 4) 21
3. Let A be a 3×3 matrix and |adj A| = 16, then |A| =
   1) ± 2 2) ± 4 3) ± 8 4) ± 1
4. The value of $i\, i^2\, i^3 \ldots i^{40}$ is
   1) 1 2) -i 3) 0 4) i
5. If z = 0, then arg z =
   1) 0 2) 1 3) undefined 4) p
6. If z is a non zero complex number such that $2iz^2 = \bar{z}$ then |z| is
   1) ½ 2) 1 3) 2 4) 3
7. If $\omega \neq 1$ is a cubic root of units and $(1+\omega)^7 = A + B\omega$, then (A, B) equals
   1) (1, 0) 2) (-1, 1) 3) (0, 1) 4) (1, 1)
8. A polynomial equation in x of degree n always has
   1) n distinct roots 2) n real roots 3) n imaginary roots 4) atmost one root
9. A zero of $x^3 + 64$ is
   1) 0 2) 4 3) 4i 4) -4
10. If a and b are rational numbers and c is irrational number such that a + bc is a rational number, then 'b' must be
    1) 1 2) -1 3) 0 4) $\sqrt{2}$

PART - B

Answer any 5 questions. Question No.17 is compulsory: **5×2=10**

11. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ then find A(adj A)
12. Find the modulus and principal argument of the complex number $-\sqrt{3} - i$
13. If $w \neq 1$ is a cubic root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$
14. Prove that a straight line and parabola cannot intersect at more than two points.
15. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$
16. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.
17. If $\text{adj}A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

PART - C

Answer any 5 questions. Question No.24 is compulsory: **5×3=15**

18. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

19. Find the rank of the matrix $\begin{bmatrix} 4 & 4 & 0 & 3 \\ -2 & 3 & -1 & 5 \\ 1 & 4 & 8 & 7 \end{bmatrix}$

20. Obtain the Cartesian equation for the locus of $Z = x + iy$ in the following case $|z + i| = |z - 1|$

21. Simplify : $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

22. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

23. Find a polynomial equation of minimum degree with ration coefficients having $2 + \sqrt{3}\,i$ as a root.

24. State and prove triangle inequality of complex number.

PART - D

Answer all the questions: **3×5=15**

25. a) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss - Jordan method. **(OR)**

b) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a$, $q \neq b$, $r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

26. a) Solve by Gramer's rule, the system of equations $3x + 3y - z = 11$, $2x - y + 2x = 9$, $4x + 3y + 2z = 25$ **(OR)**

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$, If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

27. a) $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$. **(OR)**

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

PART-A

1) $A^T A^{-1}$ is symmetric

$A^T A^{-1} = (A^T A^{-1})^T$

$= (A^{-1})^T (A^T)^T$

$A^T A^{-1} = (A^T)^{-1} A$

$A^T (A^T A^{-1}) A = (A^T)(A^T)^{-1} AA$

$(A^T)^2 I = I \cdot A^2$

$A^2 = (A^T)^2$

3) $(A^T)^2$

2) $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\lambda A^{-1} = A$

$\lambda \frac{1}{|A|} \text{adj} A = A$

$\lambda \left(-\frac{1}{19}\right) \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = A$

$\frac{\lambda}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = A$

$\boxed{\lambda = 19}$

3) 19

3. $|\text{adj} A| = |A|^{(n-1)^2}$

$n = 3$

$16 = |A|^{3-1}$

$4^2 = |A|^2$

$\boxed{\pm 4 = |A|}$

2) ± 4

4. 2) 1

$1, i^2, \ldots, i^{40}$

$n = 40$

$S_n = \frac{n(n+1)}{2}$

$= \frac{40^{20}(41)}{2}$

$S_n = 820$

$i^{820} = (i^4)^{205}$

$= 1$

5. $z = 0$

$\arg z = ?$

$a + ib = 0$

$\arg(a+ib) = 0$

$\tan^{-1} b/a = 0$

$\frac{b}{a} = \tan 0$

1) 0

6. $|2iz^2| = |\bar{z}|$

$2|i||z^2| = |z|$

$2|z|^2 = |z|$

$|z| = 1/2$

7. $1 + \omega + \omega^2 = 0$

$1 + \omega = -\omega^2$

$(1+\omega)^7 = (-\omega^2)^7$

$= -\omega^{14}$

$-\omega^{14} = A + B\omega$

$1 + \omega = A + B\omega$

$A = 1 \quad B = 1$

4) (1, 1)

8) 3) Imaginary 2006

9) $p(x) = x^3 + 64$

$p(-4) = 0$

4) -4

10) A) 1