XII-M1 5B

First Mid Term Test - 2019

Standard XII

MATHEMATICS

Time: 1.30 hrs. Marks: 50

PART - A

Note: i) Answer all the questions. 10x1=10

ii) Choose and write the correct answer.

1. If $|\text{adj}(\text{adj}A)| = |A|^9$, then the order of the square matrix A is

   a) 3 b) 4 c) 2 d) 5

2. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $A(\text{adj}A) = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ then $k =$

   a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

3. $\dfrac{1+2i+3i^2}{1-2i+3i^2}$ equals

   a) i b) -1 c) $-i$ d) 4

4. If $z = 1-\cos\theta+i\sin\theta$ then $|z| =$

   a) $2\sin\dfrac{1}{3}$ b) $2\cos\dfrac{\theta}{2}$ c) $2\left|\sin\dfrac{\theta}{2}\right|$ d) $2\left|\cos\dfrac{\theta}{2}\right|$

5. The least positive integer such that $\left(\dfrac{2i}{1+i}\right)^n$ is a positive integer is

   a) 16 b) 8 c) 4 d) 2

6. $\tan^{-1}\left(\dfrac{1}{4}\right)+\tan^{-1}\left(\dfrac{2}{9}\right)$ is equal to

   a) $\dfrac{1}{2}\cos^{-1}\left(\dfrac{3}{5}\right)$ b) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{3}{5}\right)$

   c) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{3}{5}\right)$ d) $\tan^{-1}\left(\dfrac{1}{2}\right)$

7. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of $x^{2017}+y^{2018}+z^{2019}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is

a) 0 b) 1 c) 2 d) 3

8. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x-2\sin^2 x+1$ is

a) 2 b) 4 c) 1 d) ∞

9. The equation $\tan^{-1}(x)-\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

a) no solution b) unique solution
c) two solutions d) infinite number of solutions

10. $\sin^{-1}(2\cos^2 x-1)+\cos^{-1}(1-2\sin^2 x)=$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

PART - B

Note: i) Answer any four questions. 4x2=8
ii) Question No.15 is compulsory.

11. Find the inverse of $\begin{bmatrix}-2 & 4\\ 1 & -3\end{bmatrix}$.

12. Simplify: $i^{1947}+i^{1950}$

13. Find the domain of $\tan^{-1}\sqrt{9-x^2}$.

14. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}(x), |x|>1$.

15. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

PART - C

Note: i) Answer any four questions. 4x3=12
ii) Question No.20 is compulsory.

16. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

17. Evaluate: $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

18. Solve: $(x-1)^4+(x-5)^4=82$

19. If $z=x+iy$ then show that $z\bar{z}+2(z+\bar{z})+a=0$, where $a\in R$, represents a circle.

20. Solve: $2x+3y=10$, $x+6y=4$ using Cramer's rule.

PART - D

Note: Answer all the questions. 4x5=20

21. a) Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$ have

i) no solution
ii) a unique solution
iii) an infinite number of solutions.

(OR)

b) If $z=x+iy$ is a complex number such that $\text{Im}\left[\frac{2z+1}{iz+1}\right]=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$.

22. a) Find all the roots of $(2-2i)^{\frac{1}{3}}$ and also find the product of its roots.

(OR)

b) If p and q are the roots of the equation $\ell x^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{\ell}} = 0$.

23. a) Let z_1, z_2, z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$, prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$.

(OR)

b) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

24. a) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

(OR)

b) Solve : $\tan^{-1}\left[\frac{x-1}{x-2}\right] - \tan^{-1}\left[\frac{x+1}{x+2}\right] = \frac{\pi}{4}$

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