FIRST MID - TERM TEST - JULY - 2019

MATHS

12 - STD

TIME : 1.30 Hrs. MARKS : 50

PART - A

Choose the correct answer. 10 X 1 = 10

1. If the matrix $\begin{pmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{pmatrix}$ has an inverse then the values of k.

   a) k is any real number b) k = - 4 c) $k \neq -4$ d) $k \neq 4$

2. If $0 \leq \theta \leq \pi$, then system of equations x + (sin θ) y - (cos θ) z = 0, (cos θ) x - y + z = 0, (sin θ) x + y - z = 0 has a non - trivial solution then θ is

   a) $\frac{3\pi}{4}$ b) $\frac{5\pi}{4}$ c) $\frac{\pi}{4}$ d) $\frac{2\pi}{3}$

3. If | adj (adj A)| = $|A|^9$ then the order of the square matrix A is

   a) 3 b) 5 c) 4 d) 2

4. If $\frac{z-1}{z+1}$ is purely imaginary, then |z| is

   a) 1 b) $\frac{1}{2}$ c) 2 d) 3

5. If z = 0 then the arg (z) is

   a) 0 b) indeterminate c) π d) $\frac{\pi}{2}$

6. If the amplitude of a complex number is $\frac{\pi}{2}$ then the number is

   a) purely imaginary b) purely real c) 0 d) neither real nor imaginary

7. The value of $i \, i^2 \, i^3 \ldots\ldots\ldots i^{40}$ is

   a) 1 b) 0 c) - 1 d) i

8. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satifies

   a) $|k| \leq 6$ b) k = 0 c) $k^2 \geq 36$ d) $|k| \geq 6$

9. If f and g are polynomials of degrees m and n respectively, and if h(x) = (fog) x, then the degree of h is

   a) mn b) n^m c) m^n d) m + n

10. A zero of $x^3 + 216$ is

   a) 0 b) 4i c) 4 d) - 6

PART - B

Answer any 4 questions. Question No. 16 is compulsory. 4 X 2 = 8

11. If $adj\, A = \begin{pmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find A^{-1}.

12. Solve the system of linear e equations by matrix inversion method : 2x + 5y = -1, x + 2y = -3.

13. Find the principal argument arg z, when $z = \frac{-2}{1+i\sqrt{3}}$.
14. If the area of the triangle formed by the vertices z, i z and z + iz is 50 square units, find the value of $|z|$.
15. Prove that the straight line and parabola cannot intersect at more than two points.
16. Solve : $\sin^2 x - 5 \sin x + 4 = 0$.

PART - C

Answer any 4 questions. Question No. 22 is compulsory. 4 X 3 = 12

17. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to take 10 litres of a 40% acid solution? (Use Cramer's rule)
18. Find the inverse of $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{pmatrix}$ by Gauss - Fordan méthod.
19. Solve the equation $Z^3 + 8i = 0$, where $z \in C$.
20. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has atleast six imaginary roots.
21. Form a polynomial equation with integer coefficients with $\sqrt{\sqrt{2}/\sqrt{3}}$ as a root.
22. Find the square root of $-7 + 24i$.

PART - D

Answer all the questions. 4 X 5 = 20

23. A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6, 8), (-2, 12) and (3,8). He wants to meet his friend at P (7, 60) will he meet his friend? (Use Gaussian elimination method) *(OR)*

    By using Gassian elimination method, balance the chemical reaction equation. $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$.
24. Suppose Z_1, Z_2 and Z_3 are the vertices of an equilateral triangle inscribed in the circle. $|Z| = 2$. If $Z_1 = + i\sqrt{3}$, then find Z_1 and Z_3 *(OR)*

    If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that a) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i \sin(m\alpha - n\beta)$

    b) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$
25. Determine K and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots. *(OR)*

    Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.
26. Test for consistency of the following system of linear equations and if possible solve : $2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$. *(OR)*

    Solve : i) $8x^{3/2n} - 8x^{-3/2n} = 63$ ii) Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.

1- mark

1) c) $k \neq -4$

2) c) $\pi/4$

3) c) 4

4) a) 1

5) b) indeter

6) a) purely ima

7) a) 1

8) d) $|k| \geq 6$

9) a) mn

10) d) -6

2 mark

11)

$|adj\,A| = 9$

$A^{-1} = \pm \frac{1}{\sqrt{|adj(A)|}} adj(A)$

$= \pm \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

12) $AX = B$

$|A| = -1$

$adj\,A = \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix}$

$X = A^{-1}B$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -13 \\ 5 \end{pmatrix}$

13) $z = \frac{-2}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

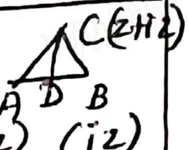
$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$\arg z = \tan^{-1}|y/x|$

$= \tan^{-1}|\sqrt{3}|$

$= 2\pi/3$

14) $|AB| = |z - iz|$



$|CD| = |z + \frac{iz}{2}|$

Area $= \frac{1}{2}|AB||CD|$

$50 = \frac{1}{2}|z - iz|\,|z + \frac{iz}{2}|$

$200 = |2z^2|$

$\therefore |z|^2 = 100$

$|z| = 10$

15) $y = mx + c$

$y^2 = 4ax$

$\therefore (mx+c)^2 = 4ax$

$m^2x^2 + x(2mc - 4a) + c^2 = 0$

Quadratic. It has two points

Hence proved

16) $y = \sin x$

$y^2 - 5y + 4 = 0$

$(y-4)(y-1) = 0$

$y = 4 \quad y = 1$

$\sin x = 4$ | $\sin x = \sin \pi/2$

No soln | $x = n\pi + (-1)^n \pi/2$

3 mark

17) $x + y = 10$ — (1)

$50\%\, x + 25\%\, y = 40\%\ 10$

$2x + y = 16$ — (2)

$\Delta = -1 \quad \Delta_x = -6 \quad \Delta_y = -4$

$x = 6 \quad y = 4$

18) $[A|I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$

$A^{-1} = \begin{pmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{pmatrix}$

19) $z^3 = -8i$

$z = 2\,cis\left(\frac{4k\pi - \pi}{6}\right)^{1/3}$ $k = 0, 1, 2$

$z = \sqrt{3} - i,\ 2i,\ -\sqrt{3} - i$

20) 2 sign change of $P(x)$ atmost +ve root

$\therefore$ 2 ~~+ve root~~

1 sign change of $P(-x)$

1 ~~+ve~~ -ve atmost -ve root

0 has no root

$\therefore$ real root $= 3$

atleast 6 imag root

21) $\left(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)$ is factor

$\left(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) \Rightarrow x^2 - \frac{\sqrt{2}}{\sqrt{3}}$

another factor $x^2 + \frac{\sqrt{2}}{\sqrt{3}}$

required polynomial

$x^4 - 2/3$

I - Mid Term – 2019
+2 – Maths – (Answer Key)

22) $|-7+24i| = 25$

$\sqrt{-7+24i} = \pm\left(\sqrt{\frac{25-7}{2}} + i\frac{24}{|24|}\sqrt{\frac{25+7}{2}}\right)$

$= \pm\left(\sqrt{9} + i\sqrt{16}\right)$

$= \pm(3+i4)$

5 mark

23) (a) $y = ax^2 + bx + c$

$(-6, 8)$ $36a - 6b + c = 8$

$(-2, -12)$ $4a - 2b + c = -12$

$(3, 8)$ $9a + 3b + c = 8$

$AX = B$

$(A, B) = \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & -116 \\ 0 & 0 & 30 & -300 \end{pmatrix}$

$c = -10$ $b = 3$ $a = 1$

$y = x^2 + 3x - 10$

(7, 60) is a point he will meet his friend

(b)

$x_1 C_5H_8 + x_2 O_2 = x_3 CO_2 + x_4 H_2O$

$5x_1 - x_3 = 0$, $4x_1 - x_4 = 0$

$2x_2 - 2x_3 - x_4 = 0$

$[A|B] = \left(\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right)$

$\sim \left(\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right)$

$\rho(A) = \rho(A|B) = 3 < 4$

$x_1 = 1$ $x_2 = 7$ $x_3 = 5$ $x_4 = 4$

24) $z_1 = 1 + i\sqrt{3}$

(a) $z_1 e^{i\frac{2\pi}{3}} = (1+i\sqrt{3})e^{i\frac{2\pi}{3}}$

$= -2$

$z_1 e^{i\frac{4\pi}{3}} = -2\left(-\frac{1}{2} + i\sqrt{3}/2\right)$

$= 1 - i\sqrt{3}$

$z_2 = -2$, $z_3 = 1 - i\sqrt{3}$

b) $x = \cos\alpha + i\sin\alpha$

$y = \cos\beta + i\sin\beta$

$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

25) α, β, γ be roots

a) $\alpha = 2(\beta + \gamma)$

$\alpha + \beta + \gamma = -b/a = -3$

~~2α~~ $2\alpha + 2(\beta+\gamma) = 6$

$2\alpha + \alpha = 6$

$3\alpha = 6$

$\alpha = 2$

$$\begin{array}{c|cccc} 2 & 2 & -6 & 3 & k \\ & 0 & 4 & -4 & -2 \\ \hline & 2 & -2 & -1 & 0 \end{array}$$

$2x^2 - 2x - 1 = 0$

$x = \frac{1 \pm \sqrt{3}}{2}$, $x = 2$

(b)

$$\begin{array}{c|ccccc} 1/3 & 6 & -5 & -38 & -5 & 6 \\ & 0 & 2 & -1 & -13 & -6 \\ \hline 3 & 6 & -3 & -39 & -18 & 0 \\ & 0 & 18 & 45 & 18 & \\ \hline & 6 & 15 & 6 & 0 & \end{array}$$

$6x^2 + 15x + 6 = 0$

$x = 1/2$, $x = -2$

26) (a) $AX = B$

$[A, B] = \begin{bmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{bmatrix}$

$\sim \begin{pmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\rho(A, B) = \rho(A) = 1 < n$

$2x - s + t = 2$ $y = s$, $z = t$

$x = \frac{1}{2}(2 + s - t)$ $s, t \in R$

(b) (i) $y = x^{3/2n}$

$8y - \frac{8}{y} = 63$

$8y^2 - 63y - 8 = 0$

$y = -1/8$ | $y = 8$

$x = \frac{1}{4^n}$ Not possible

$x = 4^n$

(ii) $(2^2)^x - 3(2^x \cdot 2^2) + 2^5 = 0$

$y = 2^x$ $y^2 - 12y + 32 = 0$

$(y-4)(y-8) = 0$

$y = 4$ | $y = 8$

$x = 2$ | $x = 3$