XII-M1 5A

First Mid Term Test - 2019

Standard XII

MATHEMATICS

Time: 1.30 hrs. Marks: 50

PART - A

10x1=10

Note: I) Answer all the questions.

II) Choose and write the correct answer.

1. If $A=\begin{pmatrix}2 & 0\\ 1 & 5\end{pmatrix}$ and $B=\begin{pmatrix}1 & 4\\ 2 & 0\end{pmatrix}$, then $|\text{adj}(AB)|=$

a) -40 b) -80 c) -60 d) -20

2. The rank of the matrix $\begin{pmatrix}6 & 0 & -9\\ 0 & 2 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$ is

a) 1 b) 3 c) 2 d) 4

3. If $A=\begin{pmatrix}2 & 3\\ 5 & -2\end{pmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

a) 17 b) 14 c) 19 d) 21

4. If $\left|z-\frac{3}{z}\right|=2$, then the least value of $|z|$ is

a) 1 b) 2 c) 3 d) 5

5. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is

a) -2 b) -1 c) 1 d) 2

6. A zero of x^3+8 is

a) 0 b) 2 c) $2i$ d) -2

7. The polynomial $x^3 + 2x + 3$ has
   a) one negative and two imaginary zeros
   b) one positive and two imaginary zeros
   c) three real zeros
   d) no zeros

8. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$
   c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

9. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, then the value of $\cos\theta$ is

   a) $\frac{1}{2\sqrt{5}}$ b) $\frac{1}{5\sqrt{2}}$ c) $\frac{1}{5}$ d) $\frac{1}{\sqrt{2}}$

10. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

   a) no solution b) unique solution
   c) two solutions d) infinite number of solutions

PART - B

Note: i) Answer any four questions. **4x2=8**
ii) Question No.15 is compulsory.

11. If $\text{adj } A = \begin{pmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find A^{-1}.

12. Find z^{-1}, if $z = (2+3i)(1-i)$.

13. Solve : $\sin^2 x - 5\sin x + 4 = 0$

14. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

15. Find the value of

i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ ii) $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

PART - C

4x3=12

Note: i) Answer any four questions.
ii) Question No.20 is compulsory.

16. Find the rank of $\begin{pmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{pmatrix}$.

17. Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

18. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3$.

19. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$.

20. Simplify : $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

PART - D

Note: Answer all the questions. 4x5=20

21. a) Determine the values of λ for which the following system of equation $x+y+3z=0$, $4x+3y+\lambda z=0$, $2x+y+2z=0$ has

i) a unique solution ii) a non-trivial solution

(OR)

b) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method?

22. a) If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$.

(OR)

b) If $\omega \neq 1$ is a cube root of unity, show that

i) $(1-\omega-\omega^2)^6+(1+\omega-\omega^2)^6=128$.

ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots(1+\omega^{2^{11}})=1$.

23. a) Solve the equation $x^4-14x^2+45=0$.

(OR)

b) If p and q are the roots of the equation $\ell x^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}}=0$.

24. a) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}.$$

(OR)

b) Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$

$$=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$$

~~~
~~~