SAIVEERA ACADEMY – REVOLUTION FOR LEARNING
PEELAMEDU,COIMBATORE – 641004
Higher secondary Second year
Mathematics
First mid term model test

Time : 2 hrs **Maximum marks :** 90

SECTION – I

Note . (i) All questions are compulsory **$20 \times 1 = 20$**

1.If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

a).$1 + \alpha^2 + \beta\gamma = 0$ b) $1 - \alpha^2 + \beta\gamma = 0$ c) $1 - \alpha^2 - \beta\gamma = 0$ d) $1 + \alpha^2 - \beta\gamma = 0$

2.If matrix A is both symmetric and skew symmetric , then

a)A is diagonal matrix b) A is zero matrix

c) A is square matrix d) None of these

3.If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to

a)6 b) -6 c)0 d)± 6

4.If $|adj(adj\ A)| = |A|^{16}$, then order of square matrix is

a)3 b) 4 c)2 d)5

5.If A = $\begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a)17 b) 14 c)19 d)16

6.Let A be 3×3 matrix B its adjoint matrix .If $|B| = 64$, then $|A| =$

a) ± 2 b) ± 14 c) ± 8 d)± 16

7.The system of equations $x + 2y = 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ has

a)One solution b) Two solution

c)No solution d)infinitely many solution

8.The value of $\sum_{n=1}^{13}(i^n + i^{n-1})$ is

a)1 b)i c)1 - i d)1 + i

9.If $|z| = 1$, the value of $\frac{1+\bar{z}}{1+z}$

a)z b)-z c)1/z d)1

10.The complex number z which satisfies condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on

a)circle b)y axis c) x axis d)line $x + y = 1$

11. The value of $e^{i\theta} - e^{-i\theta}$ is

a)sinθ b) - sinθ c) 2isinθ d)2cosθ

12. The polar form of the complex number $(i^{25})^3$ is

a) $\cos\pi/2 + i\sin\pi/2$ b) $\cos\pi + i\sin\pi$

c) $\cos\pi - i\sin\pi$ d) $\cos\pi/2 - i\sin\pi/2$

13. The locus of $|2z - 3 - i| = 3$ is

a) ellipse b) parabola c)hyperbola d)circle

14. The value of $(1+i)^{20}$ is
a)512 b)-512 c)512i d)-1024
15.If sinα , cosα are the roots of equation $ax^2 + bx + c = 0$ $c \neq 0$, then find which of the following is correct
a)$(a + c)^2 = b^2 - c^2$ b)$(a + c)^2 = b^2 + c^2$
c)$(a - c)^2 = b^2 - c^2$ d) a)$(a - c)^2 = b^2 + c^2$
16.If α,β,γ are roots of equation $x^3 + px^2 + r = 0$, then the value of $\sum \frac{1}{\beta\gamma}$ in term of coefficients
a)-p/r b) 1/r c)p/r d)1/p
17.What will be the three roots of equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1
a)2 , ½ , 2 b)3 . 1/3 , -2 c)3 , 1/3 , 2 d)-2 -1/2 , -2
18.A polynomial degree n≥ 1 has
a)atmost one roots b)atleast one roots c)Only real roots d) none of these
19.What will be the roots of equation if $3x^3 - 26x^2 + 52x - 24 = 0$ If its roots of the equation form a geometric equation
a)2 , 2/3 , 6 b)3 , 1/3 , -2 c)2 , -2/3 , 6 d)-2 -1/2 , -2
20.The number of real numbers in [0 , 2π] satisfying $\sin^4 x - 4\sin^2 x + 4$ is
a)3 b)2 c)1 d)4

SECTION – II

Note . (i) Answer any SEVEN questions are compulsory 7 × 2 = 14
(ii).Question number 30 is compulsory

21.If A & B are non singular matrices of same order , the product AB is also non singular then prove that $(AB)^{-1} = B^{-1} A^{-1}$
22.Reduce the matrix into row -echelon form $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$
23.Find the inverse of non singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss – Jordan method
24.Find the rank of matrix by minor method $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$
25.Which of the points 10 – 8i , 11+6i is closest to 1 + i
26.Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root
27.Solve the equation $2x^4 - 28x^2 + 90 = 0$
28.Discuss the maximum possible number of positive and negative roots of the polynomial $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$
29.Find the condition that the roots of $x^3 - ax^3 + bx - c = 0$ are in the ratio p : q : r

30. If z = $\frac{\sqrt{3}}{2} - i\frac{1}{2}$, find the rotation of z by Θ radians in counter clockwise direction about the origin when Θ = $\frac{2\pi}{3}$

SECTION – III

Note . (i) Answer any SEVEN questions are compulsory 7 × 3 = 21

(ii).Question number 40 is compulsory

31.Find matrix X so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

32. Find the inverse of each of the matrices $\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & sin\alpha \\ 0 & sin\alpha & -cos\alpha \end{bmatrix}$

33.Solve the system by rank method x +3 y - 2 z = 0 , 2x - y +4z = 0 , x -11 y +14 z = 0

34.If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ verify that A(adjA) = (adjA)A = $|A|$ I_3

35. $\frac{1+z}{1-z}$ = cos2θ+isin2θ , show that z = itanθ

36. Show that the points 7+9i , -3+7i , 3+3i form a right angled triangle on the argand diagram

37. If the area of the triangle formed by the vertices *z, iz* , , and *z*+ *iz* is 8 square units, find the value of modulus of z

38.Write in polar form $\frac{i-1}{\cos\frac{\pi}{3}+\sin\frac{\pi}{3}}$

39.Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

40.If a complex number z_o is a root of a polynomial equation with real coefficients , then its complex conjugate , $\overline{z_o}$ is also a root .

SECTION – IV

Note . (i) All questions are compulsory 7 × 5 = 35

41.(a).If A= $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ & B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB & BA & hence solve the equation x + y +2 z = 1 , 3x + 2y +z = 7 , 2x + y +3 z = 2

Or

(b).Solve $z^3 + 27 = 0$

42(a). find λ&μ values for x+y+z=6 , x+2y+ 3z=10, x +2y+ λ z= μ i) no soln ii)unique soln iii)infinite soln

Or

(b).Find the locus if Im $\left[\frac{2z+1}{iz+1}\right]$ = -2

43.(a). Find the value of *k* for which the equations kx-2y+z =1, x-2ky+z= -2 , x-2y+kz=1have (i) no solution (ii) unique solution (iii)infinitely many solution

Or

(b).Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i\,)$

44.(a).If the system of equations px+by+cz = 0 , ax + qy + cz = 0 , ax +qy +rz = 0 , where (p ≠ a , q ≠ b , r ≠ c) has non trivial soln , then prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$

Or

(b)Find all zeroes of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that 1+ 2i & $\sqrt{3}$

45.(a).The upward speed *v* (t)of a rocket at time *t* is approximated by v(t) = at^2+bt+c , $0 \leq t \leq 100$ where *a,b* and c are constants. It has been found that the speed at times *t* = 3 , t = 6 and *t* = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time *t* =20 seconds. (Gaussian elimination)

Or

(b).Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of other two roots

46.(a)Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ is given that two of its roots are in the ratio 3 : 2

Or

Find locus if arg (z-i / z + 2) = π/4

47.(a).A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of six dosai, four idlies and two vadais is Rs.280. The family has ` 350 in hand and they ate 3 dosai and six idlies and seven vadais. Will they be able to manage to pay the bill within the amount they had ? Using Cramar's method

Or

(b).2cosα = x + 1/x and 2cos β = y + 1/y then show that i) $x^m y^n + 1/ x^m y^n = 2\cos(m\alpha + n\beta)$ ii) $x^m/y^n - y^n/x^m = 2i\sin(m\alpha - n\beta)$

It's not about how bad you want
It's about how HARD you're willing to WORK for it

section - I

1. C. $1-\alpha^2-\beta\gamma=0$

$A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$A^2=A\cdot A=\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}$

$A^2=I$

$\begin{bmatrix}\alpha^2+\gamma & 0\\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\alpha^2+\beta\gamma=1$

$\boxed{1-\alpha^2-\beta\gamma=1}$

2. $A^T=A$ $\quad A^T=-A$ (B). A is a zero matrix

$A=-A$

$A+A=0$

$2A=0$

$\boxed{A=0}$

3. d) ± 6

$x^2-36=36-36$

$x^2=36$

$\boxed{x=\pm 6}$

4. d) 5

$|\text{adj}A|=|A|^{(n-1)^2}$

$|A|^{(n-1)^2}=|A|^{16}$

$(n-1)^2=16$

$n-1=4$

$\boxed{n=5}$

5) d) $\lambda=16$

$\lambda A^{-1}=A$

$\lambda\frac{1}{|A|}\text{adj}A=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}$

$\lambda\frac{1}{(-1-15)}\begin{bmatrix}-1 & -3\\ -5 & 1\end{bmatrix}=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}$

$\frac{\lambda}{(-16)}\begin{bmatrix}-1 & -3\\ -5 & 1\end{bmatrix}=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}$

$\frac{\lambda}{16}\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}$

$\boxed{\lambda=16}$

6) (c) ± 8

$\text{adj}A=B$

$A^{-1}=\frac{1}{|A|}\text{adj}A$.

$A^{-1}=\frac{1}{|A|}B$

$|A|A^{-1}=B$

$|A|^3|A^{-1}|=|B|=64$

$|A|^2=64$

$|A|=\pm 8$

7. d) infinitely many soln

$x+2y+3z=1 \rightarrow (1)$
$x-y+4z=0 \rightarrow (2)$
$2x+y+7z=1 \rightarrow (3)$

(1) (+) (2) we get (3)

$\therefore$ It is infinitely many solution

8. d) $1+i$

For example $[i+i^2+i^3+i^4 = 0]$

$= [i+\cdots+i^{13}]+[1+\cdots i^{12}]$

$= i+1$

9. c) $\frac{1}{z}$

$z\bar{z} = |z|^2 \qquad |z| = 1$

$\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}}$

$= \frac{z(1+z)}{(z+1)}$

$= \frac{1}{2}$

16. (c) x axis

$z = x+iy$

$\left|\frac{i+z}{i-z}\right| = 1 \Rightarrow \left|\frac{x+i(1+y)}{-x+i(1-y)}\right| = 1$

$\sqrt{x^2+(1+y)^2} = \sqrt{x^2+(1+y^2)}$

$x^2+(1-y)^2 = x^2 + (1+y^2)$

$1+y^2-2y = 1+y^2+2y$ $\boxed{y=0}$

11. c) $2i\sin\theta$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$e^{i\theta} - e^{-i\theta} = \cos\theta + i\sin\theta - \cos\theta + i\sin\theta$

$= 2i\sin\theta$

12. d) $\cos\pi/2 - i\sin\pi/2$

$i^{75} = (i^4)^{18} \cdot i^3$

$= (1)(-i)$

$= -i$

$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

13. d) circle

$z = x+iy$

$|2(x+iy)-3-i| = 3$

$|2x-3+i(y-1)| = 3$

$\sqrt{(2x-3)^2+(y-1)^2} = 3$

$(2x-3)^2+(y-1)^2 = 9$

when we expand we get equation of circle

14. d) -1024

polar form of $1+i = \sqrt{2}\,\text{cis}\,\frac{\pi}{4}$

$(1+i)^{20} = (\sqrt{2})^{20}\,\text{cis}\,\frac{20\pi}{4}$

$= 2^{10}\,\text{cis}\,5\pi$

$= -1024$

15. b) $(a+c)^2 = b^2+c^2$

sum of roots $= \sin\alpha + \cos\alpha = -\frac{b}{a}$

product of roots $= \sin\alpha\cos\alpha = \frac{c}{a}$

$\cos^2\alpha + \sin^2\alpha = 1$

$(\sin\alpha + \cos\alpha)^2 - 2\sin\alpha\cos\alpha = 1$

$\frac{b^2}{a^2} - \frac{2c}{a} = 1$

$b^2 - 2ac = a^2$

$b^2 - 2ac + c^2 = a^2 + c^2$

$b^2 + c^2 = a^2 + 2ac + c^2$

$\boxed{b^2 + c^2 = (a+c)^2}$

16. $\frac{p}{r}$

$\Sigma_1 \alpha+\beta+\gamma = -p$

$\Sigma_3 \alpha\beta\gamma = -r$

$\Sigma \frac{1}{\beta\gamma} = \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta}$

$= \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = p/r$

17. c) $3, \frac{1}{3}, 2$

Just apply roots in the equation & you get zero means it is the roots of given equation

18) b) atleast one root in C

19) a) $2, 2/3, 6$

20) b) $2 \quad y = \sin^2 x$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$

$y = 2, 2$

section - II

21. $|A| \neq 0 \quad |B| \neq 0$ [Theorem 1.7]

$|AB| \neq 0$

$AB(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$

$= AIA^{-1}$

$= AA^{-1}$

$= I$

$(B^{-1}A^{-1})AB = I$

$\therefore (AB)^{-1} = B^{-1}A^{-1}$

22. $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix} \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 + 2R_1} \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 4 \end{bmatrix}$

$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - \frac{1}{2}R_2$

[Example 1.13]

Example 1.20.

23. $[A \mid I_2] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{cc|cc} -1 & 6 & 0 & 1 \\ 0 & 5 & 1 & 0 \end{array}\right] R_1 \leftrightarrow R_2$

$\sim \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 5 & 1 & 0 \end{array}\right] R_1 \to -(R_1)$

$\sim \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 1 & 1/5 & 0 \end{array}\right] R_2 \to \frac{1}{5}R_2$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & 6/5 & -1 \\ 0 & 1 & 1/5 & 0 \end{array}\right] R_1 \to R_1 + 6R_2$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}$

24. A is a matrix of order 3×4

$\rho(A) \leq \min\{3, 4\} = 3$

Determinant of A i.e $|A| = 0$

for 3×3

$\rho(A) \neq 3$

Second order minor

non vanishes $\begin{vmatrix} -1 & -2 \\ 7 & -1 \end{vmatrix} = 1 + 14$
$= 15 \neq 0$

$\therefore \rho(A) = 2$

[Example 1.15 (ii)]

25. Ex. 2.5 – 3

$A(10 - 8i)$, $B(11 + 6i)$

$C(1 + i)$

Distance b/w A & C $= |10 - 8i - 1 - i|$

$= |9 - 9i|$

$= \sqrt{162} = 9\sqrt{2}$

Distance b/w B & C $= 5\sqrt{5}$

$9\sqrt{2} > 5\sqrt{5}$

$\therefore$ B is closest to C

26. Example 3.10

$\left(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)$

$= x^2 - \frac{\sqrt{2}}{\sqrt{3}}$

$\left(x^2 - \frac{\sqrt{2}}{\sqrt{3}}\right)\left(x^2 + \frac{\sqrt{2}}{\sqrt{3}}\right) = x^4 - \frac{2}{3}$

$3x^4 - 2 = 0$

[To remove roots]

27. $2x^4 - 28x^2 + 90 = 0$

$y = x^2$

$2y^2 - 28x + 90 = 0$

$y^2 - 14x + 45 = 0$

$y = 9, 5$

$x^2 = 9$	$x^2 = 5$
$x = \pm 3$	$x = \pm\sqrt{5}$

28. Exam : 3. 6 - 1

$p(x) = 9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

$+ \overset{1}{\rightarrow} - \overset{2}{\rightarrow} + \overset{3}{\rightarrow} - \overset{4}{\rightarrow} + \; + \; + + \; +$

4 sign changes

No. of +ve roots cannot be more than 4

similarly for $p(-x)$

$- \; - \; - \; - \; - \overset{(1)}{\rightarrow} + \overset{(2)}{\rightarrow} - \; +$

2 sign changes

No. of -ve roots $\Rightarrow$ $\boxed{\text{atmost two root}}$

29. $x^3 - ax^2 + bx \cancel{+} c$

$\Sigma_1 = p\lambda + q\lambda + r\lambda = +a$

$\Sigma_2 = (p\lambda)(q\lambda) + (q\lambda)(r\lambda) + (r\lambda)(p\lambda) = b$

$\Sigma_3 = p\lambda \times q\lambda \times r\lambda$

$= c$

$\lambda = +\dfrac{a}{p+q+r}$ $\quad \lambda^3 = \dfrac{c}{pqr}$

$\left(\dfrac{a}{p+q+r}\right)^3 = \dfrac{c}{pqr}$

$a^3 (pqr) = c(p+q+r)^3$

30. $z = \dfrac{\sqrt{3}}{2} - i\dfrac{1}{2}$

polar form of $\dfrac{\sqrt{3}}{2} - i\dfrac{1}{2} = \cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}$

Rotation of z is $= e^{-i\frac{\pi}{6}}$

$ze^{i\theta} = e^{-i\frac{\pi}{6}} e^{i2\pi/3}$

$= e^{i(\frac{2\pi}{3} - \pi/6)}$

$= e^{i(\frac{3\pi}{6})}$

$= e^{i(\pi/2)}$

Section - III

31. $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$(2\times2) \quad (2\times3) \qquad (2\times3)$

$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$a + 4c = -7$ | $b + 4d = 2$

$2a + 5c = -8$ | $2b + 5d = 4$

$3a + 6c = -9$ | $3b + 6d = 6$

$a = 1 \quad b = 2 \quad c = -2$

$d = 0$

$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

32. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

$|A| = -(\cos^2\alpha + \sin^2\alpha)$

$= -1$

$\text{adj} A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

33. $x + 3y - 2z = 0$ [Example 1.36)]
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

$[A|B] = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$

$\sim \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 7 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2 = \rho[A|0] = 2 < 3$ [no. of unknown)

system consistent
infinitely many solution

$z = k \quad k \in R$

$\left(x = \frac{-10k}{7},\ y = \frac{8k}{7} \quad z = k\right)$

34. Example 1.1

$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$|A| = 0$

$\text{adj} A = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

$A(\text{adj} A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3$

$(\text{adj} A)A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3$

$A(\text{adj} A) = (\text{adj} A)A = |A|I_3$.

35. E.x: 2.7 (4)

$1 + z = (1 - z)[\cos 2\theta + i\sin 2\theta]$

$z + z\cos 2\theta + i\sin 2\theta = -1 + \cos 2\theta + i\sin 2\theta$

$z = \dfrac{-1 + \cos 2\theta + i\sin 2\theta}{1 + \cos 2\theta + i\sin 2\theta}$

$\boxed{\cos 2\theta - 1 = -2\sin^2\theta \quad | \quad \sin 2\theta = 2\sin\theta\cos\theta}$

$= \dfrac{-2\sin^2\theta + 2i\sin\theta\cos\theta}{2\cos^2\theta + 2i\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta(-\sin\theta + i\cos\theta)}{2\cos\theta[\cos\theta + i\sin\theta]}$

$= i\tan\theta \dfrac{[\cancel{i\cos\theta - \sin\theta}]}{[\cancel{i\cos\theta - \sin\theta}]}$

36. $A = z_1 = 7+9i$

$B = z_2 = -3+7i$

$C = z_3 = 3+3i$

$AB = |z_1 - z_2| = |7+9i+3-7i|$
$= |10+2i|$
$= \sqrt{100+4}$
$= \sqrt{104}$

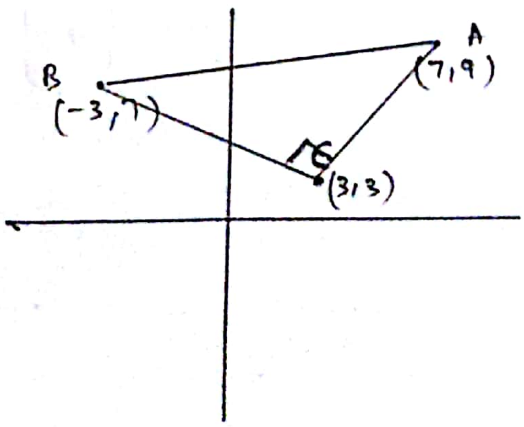
$BC = |z_2 - z_3| = |-3+7i-3-3i|$
$= |-6+4i|$
$= \sqrt{36+16}$
$= \sqrt{52}$

$CA = |z_3 - z_1| = |3+3i-7-9i|$
$= |-4-6i|$
$= \sqrt{52}$

$\because AB^2 = CA^2 + BC^2$
$104 = 52 + 52$

$\therefore \angle C = 90°$

$\therefore$ It forms right angled triangle.



37. E.x 2.5 - 8 [8 square units]

$z_1 = z \quad z_2 = iz \quad z_3 = z + iz$

$|z_1 - z_2| = |z||1-i|$
$= |z|\sqrt{2}$

$|z_2 - z_3| = |z|$

$|z_3 - z_1| = |iz| = |i||z| = |z|$

$AB = \sqrt{2}|z| \quad BC = |z| \quad CA = |z|$

$BC^2 + CA^2 = 2|z|^2 = AB^2$

$A = \frac{1}{2} \times b \times h$

$8 = \frac{1}{2} \times |z| \times |z|$

$16 = |z|^2$

$\boxed{4 = |z|}$

38. E.x : 2.7 1-(iv)

polar form of $i-1 = \sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$

$\dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \dfrac{\sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

$= \sqrt{2}\left[\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right]$

$= \sqrt{2}\left[\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right]$

$= \sqrt{2}\left[\cos\left(2k\pi + \frac{5\pi}{12}\right)\right]$

$k \in \mathbb{Z}$

39. E.x 3.5 4.

$$\sqrt{\frac{x}{a}} = y$$

$$2y + \frac{3}{y} = \frac{b}{a} + \frac{6a}{b}$$

$$2y^2 + 3 = y\frac{(b^2+6a^2)}{ab}$$

$$(2ay - b)(by - 3a) = 0$$

$$y = b/2a \qquad y = \frac{3a}{b}$$

when

$$\boxed{y = b/2a}$$

$$\sqrt{\frac{x}{a}} = b/2a = \frac{b^2}{4a^2}$$

$$\boxed{x = \frac{b^2}{4a}}$$

when $y = \frac{3a}{b}$

$$\sqrt{\frac{x}{a}} = \frac{3a}{b} = \frac{9a^2}{b^2}$$

$$\boxed{x = \frac{9a^3}{b^2}}$$

40. $\boxed{\text{Theorem } 3.2}$

section - IV

41. (a) Ex. 1.3 - 2

$$AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$BA = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$AB = BA = 4I_3$$

$$\boxed{B^{-1} = \frac{1}{4}A}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$BX = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$X = B^{-1}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\boxed{x = 2} \quad \boxed{y = 1} \quad \boxed{z = -1}$$

(b) E.X - 2.8 : 5

$z^3 + 27 = 0$

$z^3 = -27$

$z^3 = (-1)(27)$

$z = (-1)^{1/3}\, 3^{3/3}$

$= 3(-1)^{1/3}$

$z = 3[\cos\pi + i\sin\pi]^{1/3}$

$= 3[\cos(2k+1)\pi + i\sin(2k+1)\pi]^{1/3}$

$= 3\left[\cos(2k+1)\frac{\pi}{3} + i\sin(2k+1)\frac{\pi}{3}\right]$

$k = 0, 1, 2$

$k=0 \quad z = 3\cos\frac{\pi}{3}$

$k=1 \quad z = 3\cos\pi = -3$

$k=2 \quad z = 3\cos\frac{5\pi}{3}$

42 (a) $A|B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-4 \end{array}\right]$

(i) $\lambda - 3 = 0 \Rightarrow \lambda = 3 \quad \mu \neq 10$

$\rho(A) = 2 \quad \rho[A|B] = 3$

$\rho(A) \neq \rho[A|B]$

$\therefore$ inconsistent & no soln

(ii) $\lambda \neq 3 \quad \mu \in R$

$\rho(A) = \rho[A|B] = 3 =$ no. of unknown

$\therefore$ consistent & unique soln

(iii) $\lambda - 3 = 0 \quad \mu - 10 = 0$

$\lambda = 3 \quad \mu = 10$

$\rho(A) = \rho[A|B] = 2 < 3$ (no of unknown)

System consistent infinitely many solution

(b) $\text{Im}\left[\frac{2z+1}{iz+1}\right] = -2.$

$z = x+iy$

$\frac{2z+1}{iz+1} = \frac{2x+2iy}{ix-y+1} = \frac{2x+2iy}{(1-y)+ix}$

$= \frac{\overset{a}{2x} + \overset{b}{2iy}}{\underset{c}{(1-y)} + \underset{d}{ix}} \times \frac{(1-y)-ix}{(1-y)-ix}$

$\frac{a+ib}{c+id} = \left[\frac{ac+bd}{c^2+d^2}\right] + i\left[\frac{bc-ad}{c^2+d^2}\right]$

$\text{Im}\left[\frac{2z+1}{iz+1}\right] = -2$

$\frac{y(1-y) - (2x)(1-y)}{(1-y)^2 + x^2} = -2$

$y - y^2 - 2x + 2xy = -2[1 + y^2 - 2y + x^2]$

$y - y^2 - 2x + 2xy = -2 - 2y^2 + 4y - 2x^2$

$2x^2 + 2y^2 + 2xy - 2x - 4y + y + 2 = 0$

$2x^2 + 2y^2 + 2xy - 2x - 3y + 2 = 0$

43. (a) E.X : 1.6 - 2

$$\begin{bmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$[A|B] = \left[\begin{array}{ccc|c} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & -(k+2)(k-1) & -(k+2) \end{array}\right]$$

(i) $k = 1$

$\rho(A) = 1 \quad \rho[A|B] = 2$

$\rho(A) \neq \rho[A|B]$

no solution

(ii) $k \neq 1 \quad k \neq -2$

$\rho(A) = \rho[A|B] = 3$ [no. of unknown]

~~Infinitely many solution~~

unique solution

(iii) $k = -2$

$\rho(A) = \rho[A|B] = 2 <$ [no. of unknown]

Infinitely many soln.

(b) E X : 2.8 - 10

$z = (-1)^{1/4}$

$= (\cos\pi + i\sin\pi)^{1/4}$

$z = \cos\left(\frac{\pi}{4}(2k+1)\right) + i\sin\left((2k+1)\frac{\pi}{4}\right)$

$k = 0, 1, 2, 3$

$k = 0 \quad z = \frac{1}{\sqrt{2}}(1+i)$

$k = 1 \quad z = \frac{1}{\sqrt{2}}(-1+i)$

$k = 2 \quad z = -\frac{1}{\sqrt{2}}(1+i)$

$k = 3 \quad z = \frac{1}{\sqrt{2}}(1-i)$

Four roots are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$

 Ex. 1.40

44(a)
$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0$$

$$\begin{vmatrix} p & b & c \\ -(p-a) & q-b & 0 \\ -(p-a) & 0 & r-c \end{vmatrix} = 0$$

$$(p-a)(q-b)(r-c)\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} p/p-a & b/q-b & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\frac{p}{p-a} + \frac{b}{q-b} + 1 + \frac{c}{r-c} + 1 = 2$$

$$\frac{p}{p-a} + \frac{b+q-b}{q-b} + \frac{c+r-c}{r-c} = 2$$

$$\boxed{\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2}$$

(b) E.x 3.3 - 5

$1+2i \rightarrow$ one root

$1-2i \rightarrow$ other root

$\sqrt{3} \rightarrow$ one root

$-\sqrt{3} \rightarrow$ other root

sum of roots = 2

product of roots = 5

$$x^2 - 2x + 5 = 0$$

sum of roots = 0

product of roots = -3

$$x^2 - 3 = 0$$

$$(x^2-2x+5)(x^2-3) = 0$$

$$x^4 - 2x^3 + 2x^2 + 6x - 15$$

$$\begin{array}{r|l} & x^2 - x - 9 \\ \hline x^4 - 2x^3 + 2x^2 + 6x - 15 & x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 \\ & \\ \hline & 0 \\ \hline \end{array}$$

other factor $x^2 - x - 9$

$$x = \frac{1 \pm \sqrt{37}}{2}$$

roots are

$$1 \pm 2i, \pm\sqrt{3}, \frac{1 \pm \sqrt{37}}{2}$$

45(a) Example 1.28

$v(3)=64$

$v(6)=133$

$v(9)=208$

$9a+3b+c=64$

$36a+6b+c=133$

$81a+9b+c=208$

$$[A|B]=\left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 36 & 6 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$9a+3b+c=64$

$2b+c=41$

$\boxed{c=1}$

$\boxed{a=1/3}$

$\boxed{b=20}$

$\boxed{t=20}$

$v(20)=\frac{1}{3}\times(20)^2+20\times20+1$

$=\frac{400}{3}+400+1$

$=\frac{400+1200+3}{3}$

$=\frac{1603}{3}$

(b) Ex. 3.3 - 4

$2x^3-6x^2+3x+k=0$

$a=2 \quad b=-6 \quad c=3 \quad d=k$

$\alpha=2(\beta+\gamma)$

sum of root $=\alpha+\beta+\gamma=-b/a$

$=\frac{\alpha}{2}+\alpha=+3$

$=\frac{3\alpha}{2}=-3$

$\boxed{\alpha=2}$

$\alpha\beta\gamma=-\frac{d}{a}=\frac{-k}{2}$

$2\beta\gamma=-\frac{k}{2}$

$\beta\gamma=-\frac{k}{4}$

$\alpha\beta+\beta\gamma+\gamma\alpha=3/2$

$2\beta+\beta\gamma+2\alpha=3/2$

$2(\beta+\gamma)+\beta\gamma=3/2$

$\alpha+\beta\gamma=3/2$

$\alpha-\frac{k}{4}=3/2$

$2-\frac{k}{4}=3/2$

$\boxed{k=2}$

$\gamma=-\frac{1}{2\beta} \quad \beta-\frac{1}{2\beta}=1$

$\beta=\frac{1\pm\sqrt{3}}{2}$

$2,\ \frac{1+\sqrt{3}}{2},\ \frac{1-\sqrt{3}}{2}$

46.(a) 3.1 - 6

$\frac{\alpha}{\beta} = 3/2$ $\quad 2\alpha = 3\beta$

$\alpha = \frac{3}{2}\beta$

α, β, γ roots

by vieta's formula

$\frac{3}{2}\beta + \beta + \gamma = 9$

$\frac{5}{2}\beta + \gamma = 9$

$\gamma = \frac{18 - 5\beta}{2}$

$\frac{3}{2}\beta\beta + \beta\gamma + \frac{3}{2}\beta\gamma = \frac{c}{a} = 14$

$\frac{3}{2}\beta^2 + \frac{90\beta}{4} - \frac{25\beta^2}{4} = 14$

$6\beta^2 + 90\beta - 25\beta^2 = 56$

$19\beta^2 - 90\beta + 56 = 0$

$\beta = \frac{14}{19}$ or $\beta = 4$

$\beta = \frac{14}{19}$ other roots $\frac{21}{19}, \frac{14}{19}, \frac{136}{19}$

$\beta = 4$ other roots $6, 4 - 1$

(b) 2.7 - 6

$z = x + iy$

$\arg\left(\frac{x+iy-i}{x+iy+2}\right) = \frac{\pi}{4}$

$\arg\left(\frac{x+i(y-1)}{(x+2)+iy}\right) = \frac{\pi}{4}$

Using formula

$\frac{a+ib}{c+id} = \left[\frac{ac+bd}{c^2+d^2}\right] + i\left[\frac{bc-ad}{c^2+d^2}\right]$

$\frac{\pi}{4} = \arg\left[\frac{x(x+2)+(y-1)y}{(x+2)^2+y^2} + i\left[\frac{xy-(y-1)x}{(x+2)^2+y^2}\right]\right]$

$\frac{\pi}{4} = \tan^{-1}\left[\frac{xy-(y-1)x}{x(x+2)+(y-1)y}\right]$

$\tan\frac{\pi}{4} = \frac{xy - xy + x}{x^2+2x+y^2-y}$

$1 = \frac{xy - xy + x}{x^2+2x+y^2-y}$

$x^2 + y^2 + 3x - 3y + 2 = 0$

 (1.4 − 5)

47. (a) $x \to$ dosai
$y \to$ idlies
$z \to$ vadais

$\boxed{\text{but altered}}$

$$2x + 3y + 2z = 150$$
$$2x + 2y + 4z = 200$$
$$6x + 4y + 2z = 280$$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 6 & 4 & 2 \end{vmatrix}$$

$$= 2(4-16) - 3(4-24) + 2(8-12)$$
$$= 2(-12) - 3(-20) + 2(-4)$$
$$= -24 + 60 - 8$$
$$= 28$$

$$\Delta x = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 280 & 4 & 2 \end{vmatrix}$$

$$= 840$$

$$\Delta y = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 6 & 280 & 2 \end{vmatrix}$$

$$= 280$$

$$\Delta z = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 6 & 4 & 280 \end{vmatrix}$$

$$= 840$$

$$x = \frac{\Delta x}{\Delta} = \frac{840}{28} = 30$$
$$y = \frac{\Delta y}{\Delta} = \frac{280}{28} = 10$$
$$z = \frac{\Delta z}{\Delta} = \frac{840}{28} = 30$$

$$3x + 6y + 6z = 3(30) + 6(10) + 7(30)$$
$$= 90 + 60 + 210$$
$$= 360$$

Rs. 350 in hand but bill is 360 so they will not able to pay bill

(b) $x + \frac{1}{x} = 2\cos\alpha$

$$x^2 + 1 = 2x\cos\alpha$$
$$x^2 - 2x\cos\alpha + 1 = 0$$

by using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \cos\alpha \pm i\sin\alpha$$
$$y = \cos\beta \pm i\sin\beta$$
$$x^m = \cos m\alpha \pm i\sin m\alpha$$
$$y^n = \cos n\beta \pm i\sin n\beta$$
$$x^m y^n = \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$$
$$\frac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$$
$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$$

$$\frac{x^m}{y^n} = \cos(m\alpha - n\beta) + i\sin(m\alpha - n\beta)$$
$$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$$