RAVI MATHS TUITION CENTER ,GKM COLONY, CH- 82. PH: 8056206308

IMPORTANT 2 MARKS WITH ANSWERS FOR MID TERM

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12th Standard 2019 EM

Maths

Reg.No. : | | | | | | |

Time : 01:00:00 Hrs

Total Marks : 60

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1) If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

Given $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix}$

$= \begin{bmatrix} 22 & 9 \\ -2 & 1 \end{bmatrix}$

$\therefore A^2-3A-7I_2$

$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22-15-7 & 9-9+0 \\ -3+3+0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0_2$

Hence proved.

$\therefore A^2-3A-7I_2 = 0$

Post;- multiplying by A^{-1} we get,

$A^2\text{-}A^{-1}\text{-}3AA^{-1}\text{-}7I_2A^{-1} = 0.A^{-1}$

$\Rightarrow A(AA^{-1})-3(AA^{-1})-7(A^{-1})=0$

$[\because I_2A^{-1}=A^{-1}$ and | $(0)A^{-1}=0]$

$\Rightarrow AI-3I-7A-1 = 0$ $[\because AA^{-1}=I]$

$\Rightarrow AI-3I=7A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{7}[A-3I]$ $[\therefore AI=A]$

$\Rightarrow A^{-1} = \frac{1}{7} = \left[\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$

$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 5-3 & 3-0 \\ -1-0 & -2-3 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$.

2) If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = |A|I_2$.

Given A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

adj A= $\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

[Interchange the elements in the leading diagonal and change the sign of the elements in the off diagonal]

|A|=24-20=4

∴ A(adj A) = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

= $\begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ....(1)

(adj A)(A)= $\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

= $\begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ....(2)

$|A|I_2$ =4 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ....(3)

From (1), (2) and (3), it is proved that

A (adj A) = (adj A) A = $|A|I_2$

3)

If adj(A) = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

Given adj A= $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$

We know that A =± $\frac{1}{\sqrt{|adjA|}}$ .adj (adj A) ....(1)

|adj A| =2 $\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} + 4\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix}$

[Expanded along R_1]

=2(24-0)+4(-6-14)+2(0+24)

=2(24)+4(-20)+2(24)=48-80+48

=96-80=16

Now, adj (adj A)

$$= \begin{bmatrix} +\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} & +\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix} \\ -\begin{vmatrix} -4 & 2 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 2 \\ -2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} \\ +\begin{vmatrix} -4 & 2 \\ 12 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ -3 & -7 \end{vmatrix} & +\begin{vmatrix} 2 & -4 \\ -3 & 12 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(24-0)-(6-14)+(0+24) \\ -(-8-0)+(4+4)-(0-8) \\ +(28-24)-(-14+6)+(24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$=4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

Substituting (2) and (3) in (1) we get,

$$A=\frac{1}{\sqrt{16}}.4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$A=\pm\frac{4}{4}\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \pm\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

4) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

Given adj (A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

We know that $A^{-1} = \pm\frac{1}{\sqrt{|adjA|}}$ (adj A) ....(1)

$|adj\ A| = 0+2\begin{vmatrix} 6 & -6 \\ -3 & 6 \end{vmatrix} +0$

[Expanded along R_1]

=2(36-18)=2(18)=36

$\therefore A^{-1} = \pm\frac{1}{\sqrt{36}}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

$= \pm\frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}.$

5) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

Given A= $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

|A| =1+tan²x

∴ $A^{-1}=\frac{1}{|A|}$ adjA

$=\frac{1}{1+tan^2}\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

[Interchange the elements in the leading diagonal and change the sign of elementsin the off diagonal]

$A^T=\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

∴ A^TA^{-1}

$=\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}\frac{1}{1+tan^2x}\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

$=\frac{1}{1+tan^2x}\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}\begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

$=\frac{1}{1+tan^2x}\begin{bmatrix} 1-tan^2x & -tanx-tanx \\ tanx+tanx & -tan^2x+1 \end{bmatrix}$

$=\begin{bmatrix} \frac{1-tan^2x}{1+tan^2x} & \frac{-2tanx}{1+tan^2x} \\ \frac{2tanx}{1+tan^2x} & \frac{1-tan^2x}{1+tan^2x} \end{bmatrix}$

$A^TA^{-1}=\begin{bmatrix} cos2x & -sin2x \\ sin2x & cos2x \end{bmatrix}$

[∵ $\sin 2x=\frac{2tanx}{1+tan^2x}$ and $\cos 2x=\frac{1-tan^2x}{1+tan^2x}$].

6) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

Let the encryption matrix be A= $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

|A|=-1+2=1-0

∴ A^{-1}= $\frac{1}{|A|} adjA = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

Hence the decryption matrix is $\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

Coded row matrix	Decoding matrix	Decoded row matrix
[2 -3]	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=[2+6 2+3]=[8 5]
[20 4]	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=[20-8 20-4] =[12 16]

So, the sequence of decoded row matrices is

[8 5], [12 16]

Now the 8^{th} English alphabet is H.

5^{th} English alphabet is E.

12^{th} English alphabet is L.

and the 16^{th}. English alphabet is P.

Thus the receiver reads the message as "HELP".

7) Solve the following system of linear equations by matrix inversion method:

2x + 5y = −2, x + 2y = −3

2x+5y=-2, x+2y=-3

The matrix form of the system is

$\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

⇒ AX =B where

A= $\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

X= $\begin{pmatrix} x \\ y \end{pmatrix}$

⇒ =A^{-1}B

|A|= $\begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix}$ =4-5=-1-0

∴ A^{-1}= $\frac{1}{|A|} adjA = \frac{1}{-1}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$

= $\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

∴ X=A^{-1}B= $\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

= $\begin{bmatrix} 4-15 \\ -2+6 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$

∴ Solution set is {-11,4}

8) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

Given A=$\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, B=$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

AB=$\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

=$\begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-3 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$

=$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$=4. I_3

BA=$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

=$\begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$

=$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$=4. I_3.

So, we get AB=BA=4. I_3

⇒ $\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = 1$

⇒ $B^{-1} = \frac{1}{4}$ =1

Writing the given set of equations in matrix form we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \left[\frac{1}{4}A\right]\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}1\\ 7\\ 2\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}-5+7+6\\ 7+7-10\\ 1-7+2\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\ 4\\ -4\end{bmatrix}=\begin{bmatrix}2\\ 1\\ -1\end{bmatrix}$$

∴ x=2, y=1, z=-1

Hence, the solution set is {2, 1, - l}.

9) If $ax^2 + bx + c$ is divided by x + 3, x − 5, and x − 1, the remainders are 21,61 and 9 respectively. Find a,b and c. (Use Gaussian elimination method.)

Let $P(x) = ax^2+bx+c$

Given P(-3) = 21

[∵ P(x) ÷x+3, the remainder is 21]

⇒ a(-3)2+b(-3)+c=21

⇒ 9a-3b+c=21

Also, P(5)=61

⇒ a(5)2+b(5)+c=61

[using remainder theorem]

⇒ 25x+5b+c=61 ....(2)

and P(1)=9

⇒ a(1)2+b(1)+c=9

⇒ a+b+c=9 ....(3)

Reducing the augment matrix to an equivalent row-echelon form using elementary row operations, we get

$$\begin{bmatrix}9 & - & 1 & 21\\ 25 & 5 & 1 & |\,61\\ -1 & 1 & 1 & 9\end{bmatrix}\xrightarrow{R_1\leftrightarrow R_3}\begin{bmatrix}1 & 1 & 1 & 9\\ 25 & 5 & 1 & |\,61\\ 9 & -3 & 1 & 21\end{bmatrix}$$

$$\xrightarrow{R_3\rightarrow R_3-9R_1 R_2\rightarrow R_2-25R_1}\begin{bmatrix}1 & 1 & 1 & 9\\ 0 & -20 & -24 & |\,-164\\ 0 & -12 & -8 & -60\end{bmatrix}$$

$$\xrightarrow{R_2\rightarrow R_2\div R_3\rightarrow R_3\div 4}\begin{bmatrix}1 & 1 & 1 & 9\\ 0 & -5 & -6 & |\,-41\\ 0 & -3 & -2 & -15\end{bmatrix}$$

$$\xrightarrow{R_3\rightarrow R_3-\frac{3}{5}R_2}\begin{bmatrix}1 & 1 & 1 & 9\\ 0 & -5 & -6 & |\,-41\\ 0 & 0 & \frac{8}{5} & \frac{48}{5}\end{bmatrix}$$

$$\xrightarrow{R_3\rightarrow 5R_3}\begin{bmatrix}1 & 1 & 1 & 9\\ 0 & -5 & -6 & |\,-41\\ 0 & 0 & 8 & 48\end{bmatrix}$$

Writing the equivalent equations from the row-echelon matrix we get,

a+b+c = 9 ....(1)

- 5b-6c = -41 .....(2)

8c=48

$\Rightarrow c=\frac{48}{8}=6$

Substituting c=6 in (2) we get,

$\Rightarrow -5b-6(6)=-41$

$\Rightarrow -5b=-41+36=-5$

$\Rightarrow -5b=-41+36=-5$

$\Rightarrow b=\frac{-5}{-5}=1$

Substituting b = 1, c = 6 in (1) we get,

a+1+6=9

$\Rightarrow a+7=9$

$\Rightarrow a=9-7$

$\Rightarrow a=2$

$\therefore$ a=2, b=1, and c=6

10) By using Gaussian elimination method, balance the chemical reaction equation:

$C_2\,H_5 + O_2 \rightarrow H_2O + CO_2$

Given $C_2H_6+O_2 \longrightarrow H_20+CO_2$

We have to find positive integers x_1 x_2, x_3 and x_4 such that

$x_1C_2H_6+x2O_6 \longrightarrow x_3H_2O+x_4CO_2$ ....(1)

The number of carbon atoms on the LHS of (1) should be equal to the number of carbon atoms on the RHS of (1)

$\therefore 2x_1=1x_4$

$\Rightarrow 2x_1-x_4=0$ ......(2)

Considering hydrogen atoms we get,

$6x_1=2x_3$

$\Rightarrow 6x_1-2x_3=0$

$\Rightarrow 3x_1-x_3=0$ ....(3)

Also, considering oxygen atoms we get,

$2x_2=1x_3+2x_4$

$\Rightarrow 2x_2-x_3-2a_4=0$ ....(4)

Equations (2), (3) and (4) forni a homogeneous system of linear equations in 4 unknowns

$\therefore$ The augmented matrix [A|0] is

$$\begin{bmatrix} 2 & 0 & 0 & -1 & & 0 \\ 3 & 0 & -1 & 0 & | & 0 \\ 0 & 2 & -1 & 2 & & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \rightarrow R_1 \div 2} \begin{bmatrix} 1 & 0 & 0 & \frac{-1}{2} & & 0 \\ 3 & 0 & -1 & 0 & | & 0 \\ 0 & 2 & -1 & -2 & & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow R_2 - 3R_1} \begin{bmatrix} 1 & 0 & 0 & \frac{-1}{2} & & 0 \\ 0 & 0 & 1 & \frac{3}{2} & | & 0 \\ 0 & 2 & -1 & -2 & & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \\ 0 & 2 & 0 & \frac{-7}{2} & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{-1}{2} & 0 \\ 0 & 2 & 0 & \frac{-7}{2} & 0 \\ 0 & 0 & -1 & \frac{3}{2} & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \to 2R]{R_1 \to 2R_2 R_2 \to 2R_2} \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 4 & 0 & -7 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{array}\right]$$

Here ρ (A)=, the number of unknowns

$\therefore$ The system is consistent with one parameter family of solutions, so let $x_4 = t$

Writing the equations, from the row-echelon form we get

$2x_1 - x_4 = 0$

$\Rightarrow 2x_1 = x_4$

$\Rightarrow 2x = t$

$\Rightarrow x_1 = \frac{t}{2}$

$4x_2 - 7x_4 = 0$

$\Rightarrow 4x_2 = 7t$

$\Rightarrow x_2 = \frac{7t}{4}$

$-2x_3 + 3x_4 = 0$

$\Rightarrow 2x_3 = 3x_4$

$\Rightarrow x_3 = \frac{3t}{4}$

Since x_1, x_2, x_3 and x_4 are positive integers, let us choose t=4

$\therefore x_1 = \frac{4}{2} = 2$

$$\therefore x_2 = \frac{7(4)}{4} = 7$$

$$\therefore x_3 = \frac{3(4)}{2} = 6 \text{ and } x_4 = t = 4$$

So, the balanced equation is

$2C_2H_6 + 7O_2 \longrightarrow 6H_2O + 4CO_2$

11) Find the values of the real numbers x and y, if the complex numbers (3−i)x−(2−i)y+2i +5 and 2x+(−1+2i)y+3+ 2i are equal.

Given (3 -i) x - (2 - i) y + 2i + 5

= 2x + (-1 + 2i) y + 3 + 2i

⇒ 3x - ix - 2y + iy + 2i + 5 = 2x - y + 2iy + 3 + 2i

choosing the real and imaginary parts

(3x-2y + 5) + i (-x + y + 2) = 2x - y + 3 + i (2y+ 2)

Equating the real and imaginary parts both sides, we get

3x- 2y+ 5 = 2x-y+3

⇒ 3x - 2y + 5 - 2x +y - 3 = 0

⇒ x-y = -2 ... (1)

-x+y+2 = 2y+2

⇒ -x+y+2-2y-2 = 0

⇒ -x-y=0 ⇒ x+y = 0 ... (2)

(1)-(2) we get,

x-y =-2

(-) x+y =0

____

2y =-2

y=1

Substituting y = 1 in (2) we get.

x+1 =0 ⇒ x=-1

∴ x=-1 and y=1

12) The complex numbers u,v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v=3−4i and w=4+3i, find u in rectangular form.

Given v=3-4i, w=4+3i and $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$

$\therefore \frac{1}{u} = \frac{1}{3-4i} + \frac{1}{4+3i}$

$= \frac{3+4i}{(3-4i)(3+4i)} + \frac{4-3i}{(4+3i)(4-3i)}$

$= \frac{3+4i}{9-(4i)^2} + \frac{4-3i}{16-(3i)^2} = \frac{3+4i}{9+16} + \frac{4-3i}{16+9}$

$= \frac{3+4i}{25} + \frac{4-3i}{25} = \frac{3+4i+4-3i}{25}$

$\frac{1}{u} = \frac{7+i}{25}$

$\therefore u = \frac{25}{7+i} \times \frac{7-i}{7-i} = \frac{25(7-i}{7^2-(i^2)}$

$= \frac{25(7-i)}{49+1} = \frac{25(7-i)}{50} = \frac{1}{2}$ (7-i)

$\therefore u = \frac{1}{2}$ (7-i)

13) For any two complex number z_1 and z_2 such that $|z_1|=|z_2|=1$ and z1z2 $\neq$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is real number.

Given $|z_1|=|z_2|=1$

$\Rightarrow z_1 \bar{z} = 1$

$\Rightarrow z_1 = \frac{1}{\bar{z}_1}$

and $z_1 \bar{z}_2$ - -1

Also $z_2 \bar{z}_2 = 1$

$\Rightarrow z_2 = \frac{1}{\bar{z}_2}$

Consider $\frac{z_1+z_2}{1+z_1 z_2}$

$$\therefore \frac{z_1+z_2}{1+z_1z_2} = \frac{\frac{1}{\bar{z}_1}+\frac{1}{\bar{z}_2}}{1+\frac{1}{\bar{z}1}.\frac{1}{\bar{z}_2}} = \frac{\frac{\bar{z}_2+\bar{z}_1}{\bar{z}_1\bar{z}_2}}{\frac{\bar{z}_1\bar{z}_2+1}{\bar{z}_1\bar{z}_2}}$$

$$= \frac{\bar{z}_2+\bar{z}_1}{\bar{z}_1\bar{z}_2+1} = \frac{\bar{z}_1\bar{1}_2}{1+z_1z_2}$$

$$= \left(\frac{z_1\bar{1}_2}{1+z_1z_2}\right)$$

$$\therefore \frac{z_1+z_2}{1+z_1z_2} = \frac{\frac{1}{\bar{z}_1}+\frac{1}{\bar{z}_2}}{1+\frac{1}{\bar{z}1}.\frac{1}{\bar{z}_2}} = \frac{\frac{\bar{z}_2+\bar{z}_1}{\bar{z}_1\bar{z}_2}}{\frac{\bar{z}_1\bar{z}_2+1}{\bar{z}_1\bar{z}_2}}$$ is real [$\because z=\bar{z} \Rightarrow$ is real]

14) If |z|= 3, show that $7 \le |z+6-8i| \le 13$.

Given |z|=3

Show that $7 \le |z+6-8i| \le 13$

$|z+6-8i| \le |z|+|6-8i|$

[Triangle law of inequality]

$\le 3\sqrt{6^2+(-8)^2} \le 3+\sqrt{36+64} \le \; + \sqrt{1w}$

$|z+6-8i| \le 3+10 \le 13$

Also $||z+6-8i| \ge |x|-|6-8i| \ge |3-\sqrt{6^2+(-8)^2}|$

$\ge |3-\sqrt{36+84}| \; \ge |3-10| \ge |-7| \ge 7$ ....(2)

From (1) and (2) we get,

$7 \le |z+6-8i| \le 13$.

15) Find the square roots of 4+3i

let Z=|4+3I|

$=\sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25}$

$\sqrt{a+ib} = \pm\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}$

[Here |z|=5, a=4, b=3]

$\sqrt{4+3i} = \pm\sqrt{\frac{5+4}{2}} + i\frac{3}{|3|}\sqrt{\frac{5-4}{2}}$

$=\pm\sqrt{\frac{9}{2}} + i\frac{3}{3}\sqrt{\frac{1}{2}}$

$=\pm\frac{3}{\sqrt{2}} + i.\frac{1}{\sqrt{2}}$

16) If z=x+iy is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of z is real axis.

Given z=x+iy

Consider $\left|\frac{z-4i}{z+4i}\right| = 1 \Rightarrow \left|\frac{x+iy-4i}{x+iy+4i}\right|$=1

$\Rightarrow \left|\frac{x+i(y-4)}{x+i(y+4)}\right|$

$\Rightarrow \frac{\sqrt{x^2+(y-4)^2}}{\sqrt{x^2+((y+4)^2}}$=1

$\Rightarrow \sqrt{x^2+(y-4)^2} = \sqrt{x^2+(y+4)^2}$

Squaring both sides we get,

$x^2+(y-4)^2=x^2+(y+4)^2$

$\Rightarrow x^2+y^2-8y+16 = x^2+y^2+8y+16$

⇒ 8y+8y=0

⇒ 16y=0

⇒ y=0 [∵ 16-0]

y = 0 is the equation of real axis Locus of z is the real axis.

17) If $cos\alpha + cos\beta + cos\gamma = sin\alpha + sin\beta + sin\gamma = 0$ then show that

(i) $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha+\beta+\gamma)$

(ii) $sin3\alpha + sin3\beta + sin3\gamma + sin3\gamma = 3sin(\alpha+\beta+\gamma)$

Given cosα+cosβ+cosγ =sinα+sinβ+sinγ

∴ (cosα+cosβ+cosγ)+i(sinα+sinβ+sinγ)=0

⇒ (cosα+i sinα) + (cosβ+i sinβ)+(cosγ +i sinγ)=0

⇒ a+b+c=0 where a=cosα+i sinα, b=cosβ+i sinβ, c=cosγ +i sinγ

If a+b+c=0, then $a^3+b^3+c^3=3abc$

∴ $(cosα+i sinα)^3 + (cosβ+i sinβ)^3 + (cosγ +i sinγ)^3$ = 3[(cosα+i sinα) + (cosβ+i sinβ) + (cosγ +i sinγ)

=3[(cos(α+β+γ) + i sin(α+β+γ)]

⇒ (cos3α+cosβ+cosγ) + i[sin3α+sin3β+sin3γ)]

=3(cos(α+β+γ) + i sin(α+β+γ)

Equating the real and imaginary parts, we get cosα+cosβ+cosγ = 3sin(α+β+γ) And sinα+sinβ+sinγ =sin(α+β+γ).

18)

Find the value of $\left(\dfrac{1+sin\frac{\pi}{10}+icos\frac{\pi}{10}}{1+sin\frac{\pi}{10}-icos\frac{\pi}{10}}\right)^{10}$

LHS = $\left(\dfrac{1+sin\frac{\pi}{10}+icos\frac{\pi}{10}}{1+sin\frac{\pi}{10}-icos\frac{\pi}{10}}\right)^{10}$

Let z=$sin\frac{\pi}{10}+icos\frac{\pi}{10}$

$\therefore \frac{1}{z}=sin\frac{\pi}{10}-icos\frac{\pi}{10}$

∴ LHS = $\left[\dfrac{1+z}{1+\frac{1}{z}}\right]^{10}=\left[\dfrac{1+z}{\frac{z+1}{z}}\right]^{10}$ =z^{10}

=$\left[sin\frac{\pi}{10}+icos\frac{\pi}{10}\right]^{10}$

=$i^{10}\left[cos\frac{\pi}{10}-isin\frac{\pi}{10}\right]^{10}$

=$i^{10}\left[cos10\frac{\pi}{10}-isin10\frac{\pi}{10}\right]$ [By De Moivers theorum]

=i^{10}[cosπ-i sinπ]=-1(-1-i(0))=1

19) If $\omega \neq 1$ is a cube root of unity, show that $\left(1-\omega+\omega^2\right)^6+\left(1+\omega-\omega^2\right)^6=128$

$(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$

LHS =$(1-\omega+\omega^2)6 + (1+\omega-\omega^2)^6$

=$(1+\omega^2-\omega)^6 + (-\omega^2+\omega^2)^6$

[∵ $1+\omega+\omega^2=0$

⇒ $1+\omega=-\omega^2$

⇒ $1+\omega^2=-\omega$

=$(-\omega-\omega)^6 + (-2\omega^2)^6$

=$(-2\omega)^6+(-2\omega^2)^6$

=$2^6.\omega^6+2^6.\omega^{12}$

=$2^6[(\omega^3)^2+(\omega^3)^4]$

=$2^6[1+1]$ [∵ $\omega^3=1$]

$2^6 \times 2^1 = 2^7 = 128$ =RHS

20) Simplify the following

$\sum_{n=1}^{10} i^{n+50}$.

$i^{1+50}+i^{2+50}+ ....+i^{10+50}$

$=i^{51}+i^{52}+ ....+i^{60}$

Taking i^{50} common we get,

$i^{50} [i+i^2+i^3+i^4) + (i^5+i^6+i^7+i^8) +i^9+i^{10}]$

$=i^{50}[0+(i^{4+1}+i^{4+2}+i^{4+3}+i^{4+4}) + (i^{8+1}+i^{8+2})]$

$=i^{50} [0+0+i+i^2]$ $[\because i+i^2+i^3+i^4=0]$

$=i^{50} [i-1] =i^{48+2}(i-1)$

$=i^2(i-1)$ $[\therefore i^{48}=1]$

$=-1(i-1) =-i+1=1-i$

21) Construct a cubic equation with roots 1,2, and 3

Given roots are 1,2 and 3

Here a = 1, β = 2 and ɣ = 3

A cubic polynomial equation whose roots are

α,β,ɣ is

$x^3-(\alpha+\beta+\gamma)+x^2(\alpha\beta+\beta\gamma+\gamma\alpha)x-\propto\beta\gamma=0$

$\Rightarrow x^3-(1+1+2)x^2(2+6+3)x-6=0$

$\Rightarrow x^3-6x^2+11x-6=0$

22) Solve the equation $3x^2-16x^2+23x-6=0$ if the product of two roots is 1.

Given cubic equation $3x^2$-$16x^2$+23x-6=0

Let ∝,$\frac{1}{\alpha}$ and ɤ be the roots of the equation

[∵ product of two roots is 1]

$(1) \rightarrow x^2 - \frac{16}{3}x^2 + \frac{23}{3} - 2 = 0 \qquad \ldots(1)$

comparing (1) with

$x^3 - \left(\frac{\alpha+\beta+\gamma}{\alpha}\right) + \left(\alpha\frac{1}{\alpha} + \frac{1}{\alpha}.\gamma + \gamma\alpha\right)$

$-\alpha\frac{1}{\alpha}.\gamma = 0 \qquad \ldots(2)$

We get,

$\alpha + \frac{1}{\alpha} + \gamma = \frac{16}{3} \qquad \ldots(3)$

$1 + \frac{\gamma}{\alpha} + \gamma\alpha = \frac{23}{3}$

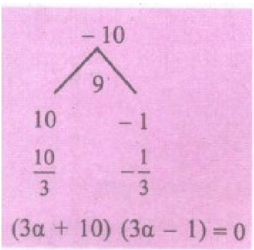
$\alpha.\frac{1}{\alpha}.\gamma = 2 \Rightarrow \gamma = 2 \quad \ldots(4)$

Substituting ɤ=2 in (3)

$\alpha + \frac{1}{\alpha} + 2 = \frac{16}{3}$

$\Rightarrow \alpha + \frac{1}{\alpha} = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3}$

$\Rightarrow \frac{\alpha^2+1}{\alpha} = \frac{10}{3}$



$3x^2 + 3 = 10\alpha$

$3\alpha^2 - 10\alpha + 3 = 0$

$\alpha = \frac{-10}{3} or \quad \alpha = \frac{1}{3}$

$(3\alpha + 10)(3\alpha - 1) = 0$

$\alpha = \frac{-10}{3}$ is not possible $\Rightarrow \alpha = \frac{1}{3}$

[$\because \alpha = \frac{-10}{3}$ will not satisfy(5)]

∴ The roots are 3,$\frac{3}{2}$,2.

23) Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio 3:2.

Let ∝,β,४ be the roots of the equation

Given $\frac{\alpha}{\beta} = \frac{3}{2} \Rightarrow 2\alpha = 3\beta \Rightarrow \alpha = \frac{3}{2}\beta$

$\therefore \frac{3}{2}\beta, \beta, \gamma$ are the roots of the given equation

Then by vieta's formula,

$\frac{3}{2}\beta + \beta + \gamma = \frac{-b}{a} = \frac{-(-9)}{1} = 9$

$\frac{5}{2}\beta + \gamma = 9 \Rightarrow \gamma = 9 - \frac{5}{2}\beta$

$\Rightarrow \gamma = \frac{18-5\beta}{2} \qquad ...(2)$

Also $\frac{3}{2}\beta(\beta) + \beta\gamma + \left(\frac{3}{2}\beta\right)\gamma = \frac{c}{a} = \frac{14}{1} = 14$

$\Rightarrow \frac{3}{2}\beta^2 + \frac{5}{2}\beta\left(\frac{18-5\beta}{2}\right) = 14 \quad [using \quad (2)]$

$\Rightarrow \frac{3}{2}\beta^2 + \frac{90\beta}{4} - \frac{25\beta^2}{4} = 14$

Multiplying by 4, $6\beta^2 + 90\beta - 25\beta^2 = 56$

$19\beta^2 - 90\beta + 56 = 0$

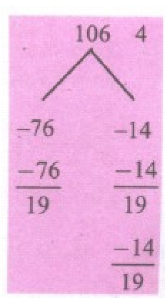
$\Rightarrow (\beta - 4)(19\beta - 14) = 0$

$\Rightarrow \beta = 4$

$\beta = \frac{14}{19}$

When β =4, the other roots are $\frac{3}{2}(4), 4, \frac{18-5}{2}(4)$

$\Rightarrow 6, 4, -1$



When $\beta = \frac{14}{19}$, the other roots are $\frac{3}{2}\beta, \beta\frac{18-5\beta}{2}[by(2)]$

$\Rightarrow \frac{3}{2}\left(\frac{14}{19}\right), \frac{14}{19}, \frac{18-5\left(\frac{14}{19}\right)}{2} \Rightarrow \frac{21}{19}, \frac{14}{19}, \frac{136}{19}$

24) If α, β, γ and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are α + β + γ + δ and αβ४δ .

Given polynomial equation is

$2x^4+5x^3-7x^2+8=0$

Here a=2, b=5,c=-7,d=0,e=8

By vieta's formula,

$\alpha+\beta+\gamma+\delta=\frac{-b}{a}=\frac{-5}{2}$

$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=\frac{c}{a}=\frac{-7}{2}$

$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=\frac{-d}{a}=0$

$\alpha\beta\gamma\delta=\frac{e}{a}=\frac{8}{2}=4$

Given roots of quadratic equation are

∝+β+ɤ+δ and ∝βɤδ

∴ sum of the roots=(∝+β+ɤ+δ)(∝βɤδ)

$=\left(\frac{-5}{2}+4\right)=\frac{-5+8}{2}=\frac{3}{2}$

$=(\alpha+\beta+\gamma+\delta)(\alpha\beta\gamma\delta)$

$=\left(\frac{-5}{2}\right)(4)=\frac{-20}{2}=-10$

∴ The sum required quadrate equation is x^2-x

(sum of the roots)+product of the roots=0

$\Rightarrow x^2-x\left(\frac{3}{2}\right)-10=0$

$\Rightarrow 2x^2-3x-20=0$

25) Find a polynomial equation of minimum degree with rational coefficients, having 2+$\sqrt{3}$ i as a root.

Since $2+i\sqrt{3}$ is a root of the polynomial equation, its conjugate 2-i$\sqrt{3}$ is also a root of the equation:

∴ Sum of the roots $=2+i\sqrt{3}+2-i\sqrt{3}=4$

Product of the roots $=(2+i\sqrt{3})(2-i\sqrt{3})$

$=2^2+(\sqrt{3})^2$

$[\because (a+ib)(a-ib)=a^2+b^2]$

=4+3=7

Hence, the polynomial equation of minimum degree with rational co-efficients is

x^2 - x (sum of the roots) + product of the roots = 0

$\Rightarrow x^2-x(4)+7=0$

$\Rightarrow x^2-4x+7=0$

26) Solve: (2x-1)(x+3)(x-2)(2x+3)+20=0

Rearrange the terms as

$(2x-1)(2x+3)(x+3)(x-2)+20=0$

$\Rightarrow (4x^2+6x-2x-3)(x^2-2x+3x-6)+20=0$

$\Rightarrow (4x^2+4x-3)(x^2+X-6)+20=0$

put $x^2+x=y$

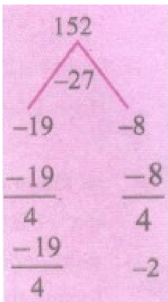
$\Rightarrow (4y-3)(y-6)+20=0$

$\Rightarrow 4y^2-24y-3y+18+20=0$

$\Rightarrow 4y^2-27y+38=0$

$\Rightarrow (y-2)(4y-19)=0$

$y = 2, \frac{19}{4}$



Case (i)

When y=2

$x^2+x=2$

$x^2+x-2.=0$

$\Rightarrow (x+2)(x-1)=0$

$\Rightarrow x=-2,1$

Case (ii)

When $y = \frac{19}{4}, x^2 + x = \frac{19}{4}$

$\Rightarrow 4x^2 + 4x = 19$

$\Rightarrow 4x^2 - 4x - 19 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{16-4(4)(-19)}}{8}$

$\Rightarrow x = \frac{-4 \pm \sqrt{16+304}}{8}$

$\Rightarrow x = \frac{-4 \pm \sqrt{320}}{8}$

$\Rightarrow x = \frac{4 \pm 8\sqrt{5}}{8}$

$\Rightarrow x = \frac{-4(-1 \pm 2\sqrt{5})}{8}$

$\Rightarrow x = -1 \pm 2\sqrt{5}$

Hence the roots are $-2, 1, -1 \pm 2\sqrt{5}$

27) Solve the equation $9x-36x^2+44x-16=0$ if the roots form an arithmetic progression.

Here, a = 9, b = - 36, c = 44, d= -16

Since the rootsform an arithmetic progression,

Let the roots be a - d, a and a + d

Sum of the roots $= \frac{-b}{a}$

$\Rightarrow (a-d)+(a)+(a+d) = \frac{-(-36)}{9} = 4$

$\Rightarrow \quad 3a = 4 \Rightarrow a = \frac{4}{3}$

and product of the roots $= \frac{-d}{a}$

$= \frac{-(-16)}{9} = \frac{16}{9}$

$\Rightarrow (a-d)(a)(a+d) = \frac{16}{9}$

$\left(a^2-d^2\right)(a) = \frac{16}{9}$

$\left(\frac{16}{9}-d^2\right)\left(\frac{4}{3}\right) = \frac{16}{9} \qquad [\because a = \frac{4}{3}]$

$\frac{16}{9}-d^2 = \frac{16}{9}\times\frac{3}{4} = \frac{4}{3}$

$\frac{16}{9}-\frac{4}{3} = d^2 = \frac{16}{9}\times\frac{3}{4} = \frac{4}{3}$

$\frac{16}{9}-\frac{4}{3} = d^2$

$\Rightarrow d^2 = \frac{16-12}{9} = \frac{4}{9}$

$\Rightarrow d = \pm\sqrt{\frac{4}{9}} = \frac{2}{3}$

∴ The roots are a-d,a,a+d

$\Rightarrow \frac{4}{3}-\frac{2}{3}, \frac{4}{3}, \frac{4}{3}+\frac{2}{3} \Rightarrow \frac{2}{3}, \frac{4}{3}, 2$

28) Solve the following equations,

$\sin^2 x-5\sin x+4=0$

$\sin^2 x-5\sin x+4=0$

put y=sin x

⇒ y2-5y+4=0

⇒ (y-4)(y-1)=0

⇒ y=4,1

Case(i)

When y=4, sin x=4 and no solution for sin x = 4 since the range thesine function is [-1, 1]

Case (ii)

When

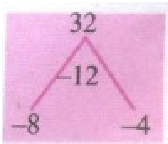
y = 1, sin x = 1

⇒ sinx=sin$\frac{\pi}{2}$ $[\because sin\frac{\pi}{2} = 1]$

$\Rightarrow x = 2n\pi+\frac{\pi}{2}, n \in$ Z

$[\because sin \quad x = sin \quad \alpha \Rightarrow x = 2n\pi + n \in Z]$.

29) Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$



$4^x - 3(2^{x+2}) + 2^5 = 0$

$\Rightarrow \left(2^2\right)^x - 3(2^x)(2^2) + 2^5 = 0$

$\left(2^x\right)^2 - 3\left(2^x\right)(2^2) = 0$

$\left(2^x\right)^2 - 12\left(2^x\right) + 32 = 0$

Put $2^x = y$

$y^2 - 12y + 32 = 0$

$(y-8)(y-4)=0$

$y=8,4$

Case (i) when $y = 8, 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$

Case (ii) when $y = 4, 2^x = 4 \Rightarrow 2^x = 2^2 x = \pm 2.$

∴ The roots are 2,3, -2.

30) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

The roots of $x^3+2x2+3x+4=0$ are ∝,β,ɣ

∴ ∝+β+ɣ=-co-efficient of x^2=-2 ...(1)

∝β+βɣ+ɣ∝=co-effficient of x=3 ...(2)

-∝βɣ=+4⟹ ∝βɣ=-4 ...(3)

From the cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-4} = \frac{-3}{4}$

$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{-2}{-4} = \frac{1}{2}$

$\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)\left(\frac{1}{\gamma}\right) = \frac{1}{\alpha\beta\gamma} = \frac{1}{-4} = -\frac{1}{4}$

∴ The required cubic equation is

$x^3 - \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)x^2 + \left(\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}\right)x - \left(\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}\right)$

$\Rightarrow x^3 + \frac{3}{4}x^2 + \frac{1}{2}x + \frac{1}{4} = 0$

Multiplying by 4 we get,

$4x^3+3x^2+2x+1=0$

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1) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify thatA(adj A)=(adj A)A = |A| I_3.

We find that |A| = $\begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{vmatrix}$ = 8(21 - 16) + 6(-18 + 8) + 2(24 - 14) = 40 - 60 + 20 = 0

By the definition of adjoint, we get

adj A = $\begin{bmatrix} (21-16) & -(-18+8) & (24-14) \\ -(-18+8) & (24-4) & -(32+12) \\ (24-14) & -(-32+12) & (56-36) \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

So, we get

A(adj A) = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

= $\begin{bmatrix} 40-60+20 & 80-120+40 & 80-120+40 \\ -30+70-40 & -60+140-80 & -60+140-80 \\ 10-40+30 & 20-80+60 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = $0I_3$ = $|A|I_3$,

Similarly, we get

(adj A)A = $\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

= $\begin{bmatrix} 40-60+20 & -30+70-40 & 10-40+30 \\ 80-120+40 & -60+140-80 & 20-80+60 \\ 80-120+40 & -60+140-80 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = $0I_3$ = $|A|I_3$.

Hence, A(adj A) = (adj A)A = $|A|I_3$.

2) If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

Since $A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$.

$A^2 + xA + yI_2 = O_2 \Rightarrow \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} + x\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 22+4x+y & 27+3x \\ 18+2x & 31+5x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

So, we get $22 + 4x + y = 0$, $31 + 5x + y = 0$, $27 + 3x = 0$ and $18 + 2x = 0$.

Hence $x = -9$ and $y = 14$. Then, we get $A^2 + xA + yI_2 = O_2$.

Post-multiplying this equation by A^{-1}, we get $A - 9I + 14A^{-1} = O_2$. Hence, we get

$A^{-1} = \frac{1}{14}(9I_2 - A) = \frac{1}{14}\left(9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\right) = \frac{1}{14}\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$.

3) Solve the following system of equations, using matrix inversion method:

$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

The matrix form of the system is AX = B, where

$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$.

We find $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6) = 10 + 15 + 15 = 40 - 0$.

So, A^{-1} exists and

$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{40}\begin{bmatrix} +(4+1) & -(-2-3) & +(-1+6) \\ -(-6+3) & +(-4-9) & -(-2-9) \\ +(3+6) & -(2-3) & +(-4-3) \end{bmatrix}^T = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Then, applying $X = A^{-1}B$, we get

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

So, the solution is $(x_1 = 1, x_2 = 2, x_3 = -1)$.

4) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

Here the number of unknowns is 3. So, if the system is consistent and has a non-trivial solution, then the rank of the coefficient matrix is equal to the rank of the augmented matrix and is less than 3.

So the determinant of the coefficient matrix should be 0.

Hence we get

$$\begin{vmatrix} 3\lambda - 8 & 3 & 3 \\ 3 & 3\lambda - 8 & 3 \\ 3 & 3 & 3\lambda - 8 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} 3\lambda - 2 & 3\lambda - 2 & 3\lambda - 2 \\ 3 & 3\lambda - 8 & 3 \\ 3 & 3 & 3\lambda - 8 \end{vmatrix} = 0 \text{ (by applying } R_1 \rightarrow R_1 + R_2 + R_3\text{)}$$

$$\text{or } (3\lambda - 2)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3\lambda - 8 & 3 \\ 3 & 3 & 3\lambda - 8 \end{vmatrix} = 0 \text{ (by taking out } (3\lambda - 2) \text{ from } R_1\text{)}$$

$$\text{or } (3\lambda - 2)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3\lambda - 11 & 3 \\ 3 & 3 & 3\lambda - 11 \end{vmatrix} = 0 \text{ (by applying } R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - 3R_1\text{)}$$

or $(3\lambda - 2)(3\lambda - 11)^2$ 0. So $\lambda = \frac{2}{3}$ and $\lambda = \frac{11}{3}$.

We now give an application of system of linear homogeneous equations to chemistry. You are already aware of balancing chemical reaction equations by inspecting the number of atoms present on both sides. A direct method is explained in the following example.

5) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

We are searching for positive integers x_1, x_2, x_3 and x_4 such that

$x_1C_5H_8 + x_2O_2 \longrightarrow x_3CO_2 + x_4H_2O$. ......(1)

The number of carbon atoms on the left-hand side of (1) should be equal to the number of carbon atoms on the right-hand side of (1). So we get a linear homogenous equation

$5x_1 = x_3 \Rightarrow 5x_1 - x_3 = 0$. ....(2)

Similarly, considering hydrogen and oxygen atoms, we get respectively,

$8x_1 = 2x_4 \Rightarrow 4x_1 - x_4 = 0$. ....(3)

$2x_2 = 2x_3 + x_4 \Rightarrow 2x_2 - 2x_4 = 0$. ....(4)

Equations (2), (3), and (4) constitute a homogeneous system of linear equations in four unknowns.

The augmented matrix is [A | B] = $\left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$.

By Gaussian elimination method, we get

$$[A \mid B] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \rightarrow 4R_3 - 5R_1} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right].$$

Therefore, $\rho(A) = \rho([A \mid B]) = 3 < 4$ = Number of unknowns.

The system is consistent and has infinite number of solutions.

Writing the equations using the echelon form, we get $4x_1 - x_4 = 0$, $2x_2 - 2x_3 - x_4 = 0$, $-4x_3 + 5x_4 = 0$.

So, one of the unknowns should be chosen arbitrarily as a non-zero real number.

Let us choose $x_4 = t$, $t - 0$.Then, by back substitution, we get $x_3 = \frac{5t}{4}$, $x_2 = \frac{7t}{4}$, $x_1 = \frac{t}{4}$.

Since x_1, x_2, x_3 and x_4 are positive integers, let us choose $t = 4$.

Then, we get $x_1 = 1$, $x_2 = 7$, $x_3 = 5$ and $x_4 = 4$.

So, the balanced equation is $C_5H_8 + 7O_2 \longrightarrow 5CO_2 + 4H_2O$.

6) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p - a, q - b, r - c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

Assume that the system px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution.

So, we have $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}$ = 0, Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ in the above equation,

we get $\begin{vmatrix} p & b & c \\ a-p & q-b & c \\ a-p & b & r-c \end{vmatrix}$ = 0. That is, $\begin{vmatrix} p & b & c \\ -(p-a) & q-b & c \\ -(p-a) & b & r-c \end{vmatrix}$ = 0.

Since p - a, q - b, r - c, we get (p - a)(q - b)(r - c) $\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$ = 0.

So, we have $\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$ = 0.

Expanding the determinant, we get $\frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c}$ = 0.

That is, $\frac{p}{p-a} + \frac{q-(q-b)}{q-b} + \frac{r-(r-c)}{r-c} = 0 \Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 2$.

7) Find,the rank of the matrix math $\begin{bmatrix} 4 & 4 & 0 & 3 \\ -2 & 3 & -1 & 5 \\ 1 & 4 & 8 & 7 \end{bmatrix}$.

Let A= $\begin{bmatrix} 4 & 4 & 0 & 3 \\ -2 & 3 & -1 & 5 \\ 1 & 4 & 8 & 7 \end{bmatrix}$

A $\xrightarrow{R_1 \leftrightarrow R_3}$ $\begin{bmatrix} 1 & 4 & 8 & 3 \\ -2 & 3 & -1 & 5 \\ 4 & 4 & 0 & 7 \end{bmatrix}$

$\xrightarrow{R_2 \to R_2 + 2R_1 R_3 \to R_3 - 4R_1}$ $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 11 & 15 & 19 \\ 0 & -12 & -32 & -25 \end{bmatrix}$

A is in row - echelon form and it has 3 non-zero rows.

$\therefore \rho$ (A) =3

8) Verify that $(A^{-1})^T = (A^T)^{-1}$ for A= $\begin{bmatrix} -2 & -3 \\ 5 & -6 \end{bmatrix}$.

$|A|=\begin{bmatrix}-2 & -3\\ 5 & -6\end{bmatrix}$ =12+15 =27

$\therefore A^{-1}=\frac{1}{|A|}adjA=\frac{1}{27}\begin{bmatrix}-6 & 3\\ -5 & 2\end{bmatrix}$

$(A^{-1})^T=\frac{1}{27}\begin{bmatrix}-6 & -5\\ 3 & 2\end{bmatrix}$ ....(1)

$A^T=\begin{bmatrix}-2 & 5\\ -3 & -6\end{bmatrix}$

$|A^T|=\begin{bmatrix}-2 & 5\\ -3 & -6\end{bmatrix}$ =12+15=27

$\therefore (A^T)^{-1}=\frac{1}{|A^T|}adj(A^T)=\frac{1}{27}\begin{bmatrix}-6 & -5\\ 3 & -2\end{bmatrix}$ ....(2)

From (1) and (2), $(A^{-1})^T=(A^T)^{-1}$

9) Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$

We find that $\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=\frac{1+2i}{1+1}=\frac{2i}{2}=i$

and $\frac{1-i}{1+i}=\left(\frac{1+i^2}{1-i}\right)^{-1}=\frac{1}{1}=-i$

Therefore, $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1-i}\right)^2$ $=i^3-(-i)^3=-i-i=-2i$

10) Find z^{-1}, if z=(2+3i)(1− i).

We find that z = (2+3i)(1−i)=(2+3)+(3−2)i=5+i

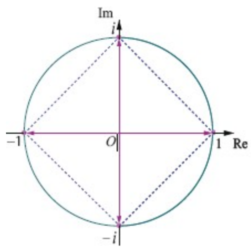
$\Rightarrow z^{-1}=\frac{1}{z}=\frac{1}{5+i}$

Multiplying the numerator and denominator by the conjugate of the denominator, we get

$z^{-1}=\frac{(5-1)}{(5+i)(5-i)}=\frac{5-i}{5^2+I^2}=\frac{5}{26}-i\frac{1}{26}$

$\Rightarrow z^{-1}=\frac{5}{26}-i\frac{1}{26}$

11) Find the fourth roots of unity.



Fourth roots of unity

Let $z^4=1$

In polar form, the equation z =1can be written as

$z = cos(0 + 2k\pi) + isin(0 + 2k\pi) = e^{12k\pi}$, k=0,1,2,...

Therefore,$(z)^{\frac{1}{4}} = cos\left(\frac{2k\pi}{4}\right) + isin\left(\frac{2k\pi}{4}\right) = e^{i\frac{2k\pi}{4}}$, k=0,1,2,3.

Taking k = 0,1,2,3, we get

k=0, z=cos0+isin0=1

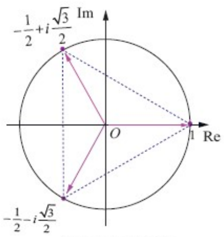
k=1, $z = cos\left(\frac{\pi}{2}\right) + isin\left(\frac{\pi}{2}\right) = i$

k=2, $z = cos\pi + isin\pi = -1$

k=3, $z = cos\frac{3\pi}{2} + isin\frac{3\pi}{2} = -cos\frac{\pi}{2} - isin\frac{\pi}{2} = i$

Fourth roots of unity are 1,i, −1, −i ⇒ 1,ω , ω^2 and ω^3 ,where $\omega = e^{i\frac{2\pi}{4}} = i$

12) Find the cube roots of unity.



Cube roots of unity

Let $z^3=1$.

In polar form, the equation z =1 can be written as

$z = cos(0 + 2k\pi) + isin(0 + 2k\pi) = e^{12k\pi}$, k=0,1,2.

Taking k = 0,1,2 , we get,

k=0, z=cos0+isino=1.

k=1,$z = cos\frac{2\pi}{3} + isin\frac{2\pi}{3} = cos\left(\pi - \frac{\pi}{3}\right) + isin\left(\pi - \frac{\pi}{3}\right)$

$= -cos\frac{\pi}{3} + isin\frac{\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

k=2, $z = cos\frac{4\pi}{3} + isin\frac{4\pi}{3} = cos\left(\pi + \frac{\pi}{3}\right) + isin\left(\pi + \frac{\pi}{3}\right)$

$= -cos\frac{\pi}{3} - isin\frac{\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Therefore, the cube roots of unity are

$1, \frac{-i + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2} \Rightarrow 1$ and ω^2 where $e^{i\frac{2\pi}{3}} = \frac{-1 + i\sqrt{3}}{2}$

13) Simplify $\left(\frac{1 + cos2\theta + isin2\theta}{1 + cos2\theta - isin2\theta}\right)^{30}$

Let $z = cos2\theta + isin2\theta$

As $|z|=|z|^2=z\bar{z}$ =1, we get $\bar{z} = \frac{1}{z} = cos2\theta - isin2\theta$

Therefore, $\frac{1 + cos2\theta + isin2\theta}{1 + cos2\theta - isin2\theta} = \frac{1 + z}{1 + \frac{1}{z}} = \frac{(1 + z)z}{z + 1} = z$

Therefore, $\left(\frac{1 + cos2\theta + isin2\theta}{1 + cos2\theta - isin2\theta}\right)^{30} = z^{30} = (cos2\theta + isin2\theta)^{30}$

$= cos60\theta + isin60\theta$

14) If $z = (cos\theta + isin\theta)$, show that $z^n + \frac{1}{z^n} = 2cosn\theta$ and $z^n - \frac{1}{z^n} = 2isinn\theta$

Let $z = (cos\theta + isin\theta)$

By de Moivre's theorem ,

$zn = (cos\theta + isin\theta)^n = cosn\theta + isinn\theta$

$\frac{1}{z^n} = z^{-n} = cosn\theta - isinn\theta$

Therefore, $z^n + \frac{1}{z^n} = (cosn\theta + isinn\theta) + (cosn\theta - isinn\theta)$

$z^n + \frac{1}{z^n} = 2cosn\theta$

Similarly,

$z^n - \frac{1}{z^n} = (cosn\theta + isinn\theta) - (cosn\theta - isinn\theta)$

$z^n - \frac{1}{z^n} = 2isinn\theta$

15) Find the locus of Z if |3z - 5| = 3 |z + 1| where z=x+iy.

Given |3z - 5| = 3 |z + 1

⇒ |3(x+iy)-5| = 3|x+iy+1|

⇒ |(3x-5)+3y| = 3|(x+1)+iy|

⇒ $\sqrt{(3x-5)^2 + 3^2} = 3\left[\sqrt{(x+1)^2 + y^2}\right]$

Squaring both sides we get,

$(3x - 5)^2 + 9 = 9 [(x + 1)^2 + y^2]$

⇒ $9x^2 - 30x + 25 + 9 = 9 [x^2 + 2x + 1 + y^2]$

⇒ 48x - 16 = 0

⇒ 3x-1 = 0

16) Find the locus of z if Re$\left(\frac{\bar{z}+1}{\bar{z}-i}\right)$ =0.

Let z = x+iy ⇒ $\bar{z}$ =x+iy

$\therefore \frac{\bar{z}+1}{z-1} = \frac{z-iy+1}{x-iy-i} = \frac{(x+1)iy}{x-i(y+1)}$

$= \frac{(x+1)-iy}{x-i(y+1)} \times \frac{x+i(y+1)}{x+i(y+1)}$

Choosing the real part alone we get,

$\frac{x(x+1)+y(y+1)}{x^2+(y+1)^2}$ =0

⇒ x(x+1) + y(y+1) =0

⇒ $x^2+x+y^2+y=0$ which is the locus of z.

17) If α and β are the roots of the quadratic equation $17x^2+43x-73 = 0$, construct a quadratic equation whose roots are α + 2 and β + 2.

Since α and β are the roots of $17x^2+43x-73=0$, we have $\alpha+\beta=\frac{-43}{17}$ and $\alpha\beta=\frac{-73}{17}$.

We wish to construct a quadratic equation with roots are α + 2 and β + 2 .Thus, to construct such a quadratic equation, calculate,

the sum of the roots = $\alpha+\beta+4=\frac{-4}{17}+4=\frac{25}{17}$ and

the product of the roots $=\alpha\beta+2(\alpha+\beta)+4=\frac{-73}{17}+2\left(\frac{-43}{17}\right)+4=\frac{-91}{17}$

Hence a quadratic equation with required roots is $x^2-\frac{25}{17}x-\frac{91}{17}=0$

Multiplying this equation by 17, gives $17x^2-25x-91=0$

which is also a quadratic equation having roots α + 2 and β + 2

18) Find the condition that the roots of $x^3+ax^2+bx+c=0$ are in the ratio p:q:r.

Since two roots are in the ratio p:q:r, we can assume the roots as pλ,qλ and rλ .

Then, we get

$\Sigma_1 = p\lambda+q\lambda+r\lambda = -a$ ....(1)

$\Sigma_2 = (p\lambda)(q\lambda)+(q\lambda)(r\lambda)+(r\lambda)(p\lambda)$ ....(2)

$\Sigma_3 = (p\lambda)(q\lambda)(r\lambda)=-c$ ....(3)

Now, we get

(1) $\Rightarrow \lambda = -\frac{a}{p+q+r}$ ......(4)

(3) $\Rightarrow \lambda^3 = \frac{c}{pqr}$ ....(5)

Substituting (4) in (5), we get

$\left(\frac{a}{p+q+r}\right)^3=-\frac{c}{pqr} \Rightarrow pqra^3 = c(p+q+r)^3$.

19) If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$ in terms of p.

The discriminant $\Delta=((4p)^2-4(4)(p+2)=16(p^2-p-2)=16(p+1)(p-2)$. So we get

$\Delta < 0$ if $-1 < p < 2$

$\Delta = 0$ if $p=-1$ or $p=2$

$\Delta > 0$ if $-° < p < -1$ or $2 < p < °$

Thus the given polynomial has

imaginary roots if $-1 < p < 2$;

equal real roots if $p=-1$ or $p=2$;

distinct real roots if $-° < p < -1$ o $2 < p < °$.

20) Solve the equation $2x^3+11x^2-9x-18=0$.

We observe that the sum of the coefficients of the odd powers and that of the even powers are equal. Hence −1 is a root of the equation. To find other roots, we divide $2x^3+11x^2-9x-18$ by x+1 and get $2x^2+9x-18$ as the quotient. Solving this we get $\frac{3}{2}$ and -6 as roots. Thus $-6,-1,\frac{3}{2}$ are the roots or solutions of the given equation.

21) Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

Let the roots be in A.P. Then, we can assume them in the form α-d, α, α+d

Applying the Vieta's formula $(\alpha-d)+\alpha+(\alpha+d)=\frac{p}{1}=p \Rightarrow 3\alpha=-p \Rightarrow \alpha=-\frac{p}{3}$.

But, we note that αis a root of the given equation. Therefore, we get

$\left(\frac{p}{3}\right)^3+p\left(\frac{p}{3}\right)^3+q\left(\frac{p}{3}\right)^3+r=0 \Rightarrow 9pq=2p^3+27r$.

22) If the roots of $x^3+px^2+qx+r=0$ are in H.P. prove that $9pqr = 27r^3+2p$.

Let the roots be in H.P. Then, their reciprocals are in A.P. and roots of the equation

$\left(\frac{1}{x}\right)^3+p\left(\frac{1}{x}\right)^2+q\left(\frac{1}{x}\right)$+r=0 ⇔ $rx^3+qx^2+px+1=0$ ....(1)

Since the roots of (1) are in A.P., we can assume them as α-d,α,α+d.

Applying the Vieta's formula, we get

$\Sigma_1=(\alpha-d)+\alpha+(\alpha+d)=-\frac{q}{r} \Rightarrow 3\alpha=-\frac{q}{r} \Rightarrow \alpha=-\frac{q}{3r}$.

But, we note that α is a root of (1). Therefore, we get

$\left(-\frac{q}{3r}\right)^2+q\left(-\frac{q}{3r}\right)^2+p\left(-\frac{q}{3r}\right)+1=0 \Rightarrow q^3+3q^3-9pqr+27r^2=0 \Rightarrow 2q^3+27r^2$.

23) Find the roots of $2x^3+3x^2+2x+3$

According to our notations, $a_n=2$ and $a_0=3$. If $\frac{p}{q}$ is a root of the polynomial, then as (p,q)=1, p must divide 3 and q must divide 2. Clearly, the possible values of p are 1,−1,3,−3 and the possible values of q are 1,−1,2,−2. Using these p and q we can form only the fractions $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{3}{1}$. Among these eight possibilities, after verifying by substitution, we get $\frac{-3}{2}$ is the only rational root. To find other roots, we divide the given polynomial $2x^3+3x^2+2x+3$ by 2x+3 and get x^2+1 as the quotient with zero remainder. Solving $x^2+1=0$, we get i and −i as roots. Thus $\frac{-3}{2}$,-i,i are the roots of the given polynomial equation.

24) Find solution, if any, of the equation $2\cos^2x-9\cos x+4=0$

The left hand side of this equation is not a polynomial in x . But it looks like a polynomial. In fact, we can say that this is a polynomial in cos x . However, we can solve the equation (1) by using our knowledge on polynomial equations. If we replace cos x by y , then we get the polynomial equation $2y^2-9y+4=0$ for which 4 and $\frac{1}{2}$ are solutions.

From this we conclude that x must satisfy cos x = 4 or cos x=$\frac{1}{2}$. But cosx=4 is never possible, if we take cosx=$\frac{1}{2}$, in fact, for all n ∈ Z, $x=2n\pi\pm\frac{\pi}{3}$ are solutions for the given equation (1).

If we repeat the steps by taking the equation $\cos^2x-9\cos x+20=0$, we observe that this equation has no solution.

25) Solve:$(x-1)^4+(x-5)^4=82$

Put y=$\frac{x-1+x-5}{2}=-3$

⇒ x=y+3

∴ $(x-1)^4+(x-5)^4=82$

⇒ $(y+3-1)^4+(y+3-5)^4=82$

⇒ $(y+2)^4+(y-2)^4=82$

⇒ $2(y^4+24y^2+16)=82$

⇒ $y^4+24y^2+16=41$

⇒ $y^4+24y^2-25=0$

⇒ $(y^2+25)(y^2-1)=0$

⇒ y=±5i, y=±1

∴ x=3±5i, 4,2.