

Business Mathematics And Statistics

Chapter-6

Applications of Differentiation

Demand Functions

Observations:-

* price and quantity of the demand function are in inverse variation.

* The graph of the demand function lies only in first quadrant.

* Slope of the demand curve is negative (-ve).

Supply Functions

Observations:-

* price and quantity of the supply function are in direct variation.

* The graph of supply function lies only in first quadrant.

* Slope of the supply curve is positive (+ve).

Some standard results

If $C(x) = f(x) + K$ be the total cost function, then

$$i) \text{ Average cost } [AC] = \frac{\text{Total cost}}{\text{output}} = \frac{C(x)}{x} = \frac{f(x) + K}{x}$$

$$ii) \text{ Average variable cost } [AVC] = \frac{\text{variable cost}}{\text{output}} = \frac{f(x)}{x}$$

Business Mathematics And Statistics

Chapter-6

Applications of Differentiation

Demand Functions

Observations:-

* price and quantity of the demand function are in inverse variation.

* The graph of the demand function lies only in first quadrant.

* Slope of the demand curve is negative (-ve).

Supply Functions

Observations:-

* price and quantity of the supply function are in direct variation.

* The graph of supply function lies only in first quadrant.

* Slope of the supply curve is positive (+ve).

Some standard results

If $C(x) = f(x) + K$ be the total cost function, then

$$i) \text{ average cost } [AC] = \frac{\text{total cost}}{\text{output}} = \frac{C(x)}{x} = \frac{f(x) + K}{x}$$

$$ii) \text{ average variable cost } [AVC] = \frac{\text{variable cost}}{\text{output}} = \frac{f(x)}{x}$$

$$\text{iii) Average fixed cost [AFC]} = \frac{\text{Fixed cost}}{\text{output}} = \frac{K}{x}$$

$$\text{iv) Marginal cost [MC]} = \frac{dc}{dx} = \frac{d}{dx} [C(x)] = C'(x).$$

$$\text{v) Marginal average cost [MAC]} = \frac{d}{dx} (AC)$$

$$\text{vi) Total cost [TC]} = \text{Average cost} \times \text{output}$$

$$\text{vii) Average cost [AC] is minimum, when } MC = AC.$$

Revenue Function :-

Some standard results:-

If $R(x) = Px$ be the revenue function, then

$$\text{i) Average revenue [AR]} = \frac{\text{Total Revenue}}{\text{output}} = \frac{R(x)}{x} = P$$

$$\text{ii) Marginal revenue [MR]} = \frac{dR}{dx} = \frac{d}{dx} (R(x)) = R'(x).$$

$$\text{iii) Marginal average revenue [MAR]} = \frac{d}{dx} (AR) = AR'(x).$$

Profit Function :-

Some standard results:-

If $P(x) = R(x) - C(x)$ be the profit function, then

$$\text{i) Average profit [AP]} = \frac{\text{Total profit}}{\text{output}} = \frac{P(x)}{x}$$

$$\text{ii) Marginal profit [MP]} = \frac{dP}{dx} = \frac{d}{dx} (P(x)) = P'(x).$$

iii) Marginal average profit $[MAP] = \frac{d}{dx} (AP) = AP'(x)$

iv) profit $[P(x)]$ is maximum when $MR = MC$.

Elasticity

$$\eta = \frac{x}{y} \cdot \frac{dy}{dx}$$

i) price elasticity of demand.

$$\eta_d = -\frac{P}{x} \cdot \frac{dx}{dp}$$

ii) price elasticity of supply.

$$\eta_s = \frac{P}{x} \cdot \frac{dx}{dp}$$

Some important results on price elasticity

i) If $|\eta| > 1$, then the quantity demand or supply is said to be elastic.

ii) If $|\eta| = 1$, then the quantity demand or supply is said to be unit elastic.

iii) If $|\eta| < 1$, then the quantity demand or supply is said to be inelastic.

Derivative test for increasing and decreasing function

Let $f(x)$ be a continuous function on $[a, b]$.

i) $f(x)$ is increasing in $[a, b]$ if $f'(x) \geq 0$.

ii) $f(x)$ is decreasing in $[a, b]$ if $f'(x) \leq 0$.

Remarks:

- i) $f(x)$ is strictly increasing in (a, b) if $f'(x) > 0$ for every $x \in (a, b)$.
- ii) $f(x)$ is strictly decreasing in (a, b) if $f'(x) < 0$ for every $x \in (a, b)$.
- iii) $f(x)$ is said to be a constant function if $f'(x) = 0$.

criteria for local maxima and local minima.

- i) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c .
- ii) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c .

optimum quantity q_0

$$* EOQ = q_0 = Rt_0 = \sqrt{\frac{2C_3R}{C_1}}$$

i) optimum number of orders per year.

$$n_0 = \frac{\text{demand}}{EOQ} = R \sqrt{\frac{C_1}{2C_3R}} = \sqrt{\frac{RC_1}{2C_3}} = \frac{1}{t_0}$$

ii) minimum average cost per unit time, $C_0 = \sqrt{2C_3C_3R}$.

iii) carrying cost = $\frac{q_0}{2} \times C_1$, ordering cost = $\frac{R}{q_0} \times C_3$

iv) At EOQ, ordering cost = carrying cost.

* If $u(x, y)$ is a continuous function of x and y then

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

Euler's theorem :-

If $u = f(x, y)$ is a homogeneous function of degree n ,

$$\text{then } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$$

partial elasticity of demand :-

$$i) \eta_{q, P_1} = \frac{E_q}{E_{P_1}} = \frac{-P_1}{Q} \frac{\partial Q}{\partial P_1}$$

$$ii) \eta_{q, P_2} = \frac{E_q}{E_{P_2}} = \frac{-P_2}{Q} \frac{\partial Q}{\partial P_2}$$

Relationship among marginal revenue (MR), Average Revenue (AR) and Elasticity of demand (η_d).

$$* MR = AR \left[1 - \frac{1}{\eta_d} \right] \text{ (or)}$$

$$* \eta_d = \frac{AR}{AR - MR}$$

chapter-7Financial Mathematics.

i) Amount of immediate annuity (or) ordinary annuity (or) certain annuity.

$$* A = \frac{a}{i} [(1+i)^n - 1]$$

ii) Present value of immediate annuity (or) ordinary annuity).

$$* P = \frac{a}{i} \left[1 - \frac{1}{(1+i)^n} \right]$$

iii) Amount of annuity due at the end of n period.

$$* A = \frac{a(1+i)}{i} [(1+i)^n - 1]$$

iv) present value of annuity due.

$$* P = \frac{a(1+i)}{i} \left[1 - \frac{1}{(1+i)^n} \right]$$

v) perpetual Annuity.

$$* P = \frac{a}{i}$$

Some useful results:

i) Investment:

money invested = number of shares x market value
of a share.

Income :

$$\text{Annual Income} = \text{no. of shares} \times \text{Face value of a share} \times \text{rate of dividend}$$

ii) Return percentage (or yield percentage) :

$$\text{Percentage of return} = \frac{\text{Income}}{\text{Investment}} \times 100$$

iv) number of shares :

$$\text{number of shares purchased} = \frac{\text{Investment}}{\text{market value of a share}}$$

note :

i) when the stock is purchased, brokerage is added to cost price.

ii) when the stock is sold, brokerage is subtracted from the selling price ...

chapter-8 descriptive statistics and probability.

ungrouped data

1) Direct method :

$$\bar{X} = \frac{\sum X}{n}$$

2) Short-cut method

$$\bar{X} = A + \frac{\sum d}{n} \text{ where } d = X - A$$

Grouped data

Direct method :

$$\bar{X} = \frac{\sum fX}{N} \text{ where } N = \sum f$$

Short-cut method

$$\bar{X} = A + \frac{\sum fd}{N}$$

* Direct method

$$\bar{X} = \frac{\sum fm}{N}$$

where $m = \text{midpoint}$.

$f = \text{the frequency}$.

$N = \sum f = \text{total frequency}$.

short-cut method

$$\bar{X} = A + \frac{\sum fd}{N}$$

where A is assumed mean or arbitrary value, $d = m - A$.

iii) Step deviation method

$$\bar{X} = A + \left(\frac{\sum fd}{N} \times c \right), \text{ where } d = \frac{(m-A)}{c} \text{ A is any arbitrary}$$

value or assumed mean and c is the magnitude of class interval.

MODE:-

mode is the value which repeats maximum number of times among the given observations.

i) ungrouped data:

$$* Q_1 = \text{Size of } \left(\frac{n+1}{4} \right)^{\text{th}} \text{ value.}$$

$$* Q_3 = \text{Size of } \left(\frac{3(n+1)}{4} \right)^{\text{th}} \text{ value.}$$

$$* D_1 = \text{Size of } \left(\frac{n+1}{10} \right)^{\text{th}} \text{ value.}$$

$$* D_2 = \text{Size of } 2 \left(\frac{n+1}{10} \right)^{\text{th}} \text{ value.}$$

$$* P_{60} = \text{Size of } 60 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ value.}$$

$$* P_{99} = \text{Size of } 99 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ value.}$$

iii) Grouped data (continuous case):

$$* Q_1 = L + \left(\frac{\frac{N}{4} - PCF}{f} \right) \times C$$

where L is the lower limit of the Q_1 class,

f is the frequency of the Q_1 class,

C is the magnitude of the Q_1 class,

PCF is the cumulative frequency of the pre Q_1 class

$$* Q_3 = L + \left(\frac{\frac{3N}{4} - PCF}{f} \right) \times C$$

where L is the lower limit of the third quartile class,

f is the frequency of the third quartile class,

C is the magnitude of the third quartile class,

PCF is the cumulative frequency of the pre Q_3 class

Geometric mean

Geometric mean is defined as the n^{th} root of the product of n observations or values.

$$* G.M = \text{Antilog} \left(\frac{\sum f \log x}{N} \right), n \text{ is number of observation.}$$

9) Indirect observation

$$G.M = \text{Anti log} \left(\frac{\sum f \log x}{N} \right); \text{ where } N = \sum f$$

continuous observation.

$$G.M = \text{Antilog} \left[\frac{\sum f \log m}{N} \right]; \text{ where } m \text{ is midpoint and } N = \sum f.$$

Harmonic mean.

$$* HM = \frac{N}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

i) In individual observation.

$$* HM = \frac{N}{\sum \left(\frac{1}{x} \right)}$$

ii) In discrete frequency distribution.

$$* HM = \frac{N}{\sum \left(\frac{f}{x} \right)} \text{ where } N = \text{total frequency} = \sum f.$$

iii) In continuous frequency distribution

$$* HM = \frac{N}{\sum \left(\frac{f}{m} \right)} \text{ where } m \text{ is midpoint and } N \text{ is total frequency.}$$

Relationship among the averages.

$$AM \geq GM \geq HM.$$

Quartile Deviation.

$$* Q.D = \frac{1}{2} (Q_3 - Q_1).$$

i) Relative measure for QD.

$$\text{coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

mean deviation.

ii) computation of mean deviation - Individual observations.

$$* \text{MD about mean} = \frac{\sum |x - \bar{x}|}{n} = \frac{\sum |D|}{n}$$

where $|D| = |x - \bar{x}|$ and n is the number of observations.

$$* \text{MD about median} = \frac{\sum |x - \text{median}|}{n} = \frac{\sum |D|}{n}$$

where $|D| = |x - \text{median}|$ and n is the number of observations.

iii) computation of mean deviation - Discrete series.

$$* \text{MD about mean} = \frac{\sum f |x - \bar{x}|}{N} = \frac{\sum f |D|}{N}$$

where $|D| = |x - \bar{x}|$ by ignoring negative signs and N is total frequencies.

$$* \text{MD about Median} = \frac{\sum f |x - \text{median}|}{N} = \frac{\sum f |D|}{N}$$

where $|D| = |x - \text{median}|$ by ignoring negative sign and N is total frequencies.

iv) calculation of mean deviation - continuous series.

$$* \text{MD about mean} = \frac{\sum f |M - \bar{x}|}{N} \text{ or}$$

$$\text{MD about mean} = \frac{\sum f |D|}{N}$$

where M is a mid value, $|D| = |M - \bar{x}|$ and N is the total frequencies.

$$* \text{MD about median} = \frac{\sum f |M - \text{median}|}{N} \text{ or}$$

$$\text{MD about mean} = \frac{\sum f |D|}{N}$$

where M is a mid value, $|D| = |M - \text{median}|$ and N is the total frequency.

iv) Relative measure for mean deviation.

$$* \text{co-efficient of MD about mean} = \frac{\text{mean deviation about mean}}{\text{mean}}$$

$$* \text{co-efficient of MD about median} = \frac{\text{mean deviation about median}}{\text{median}}$$

probability.

$$P = P(E) = \frac{\text{number of favourable cases}}{\text{Total number of exhaustive cases}} = \frac{m}{n}$$

properties.

$$i) 0 \leq P(E) \leq 1.$$

ii) Sum of all the probability equal to 1.

iii) If $P(E) = 0$ then E is an impossible event.

observation :-

i) $A \cup B \Rightarrow$ at least one of the events A or B occurs.

ii) $A \cap B \Rightarrow$ both events A and B occur.

iii) $\bar{A} \cap \bar{B} \Rightarrow$ Neither A nor B occur.

iv) $A \cap \bar{B} \Rightarrow$ Event A occurs and B does not occur.

conditional probability.

$$* P(B/A) = \frac{P(A \cap B)}{P(A)} ; P(A) \neq 0.$$

$$* P(A/B) = \frac{P(A \cap B)}{P(B)} ; P(B) \neq 0.$$