XII - STD BUSINESS MAHEMATICS AND STATISTICS

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I. Applications of Matrices and Determinants:

1. Rank of a matrix ρ(A):

(i) The rank of a matrix A is the order of the largest non zero minor of A.

(ii) If A is a matrix of order $m \times n$ then $\rho(A) < \text{minimum}\{m, n\}$

2.Determinant method (Cramer's rule):

When $\Delta \neq 0$, the unique solution is given by

$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta}$

3. Transition probability matrix:

(i) At equilibrium, $(A\ B)\ T = (A\ B)$ where $A + B = 1$

II. Integral Calculus – I

1. Properties of indefinite integrals:

(i) $\int a f(x)\,dx = a\int f(x)\,dx$

(ii) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

2.Standard results of indefinite integrals:

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
2. $\int e^x\,dx = e^x + c$
3. $\int \sin x\,dx = -\cos x + c$
4. $\int \sec^2 x\,dx = \tan x + c$
5. $\int \frac{1}{x}\,dx = \log|x| + c$
6. $\int a^x\,dx = \frac{1}{\log a}a^x + c, a > 0$ and $a \neq 1$
7. $\int \cos x\,dx = \sin x + c$
8. $\int \text{cosec}^2 x\,dx = -\cot x + c$
9. $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, n \neq -1$
10. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$
11. $\int u\,dv = uv - \int v\,du$
12. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
13. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
14. $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c$
15. $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$
16. $\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \ldots$
17. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
18. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
19. $\int e^{ax}[a f(x) + f'(x)]\,dx = e^{ax} f(x) + c$
20. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$
21. $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

3. Definite integral:

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

4. Properties of definite integrals:

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$
2. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
5. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
6. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
7. a) If $f(x)$ is an even function, then $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$

   b) If $f(x)$ is an odd function, then $\int_{-a}^a f(x)\,dx = 0$

5. Particular case of Gamma Integral:

If n is a positive integer, then $\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$

6. Properties of gamma function:

1. $\Gamma(n) = (n-1)\Gamma(n-1),\ n>1$
2. $\Gamma(n+1) = n!$, n is a positive integer
3. $\Gamma(n+1) = n\Gamma(n),\ n>0$
4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

7. Definite integral as the limit of a sum:

$$\int_a^b f(x)dx = \lim_{\substack{h\to 0\\ n\to\infty}} \sum_{r=1}^{n} h\ f(a+rh) \quad where\ h = \frac{b-a}{n}$$

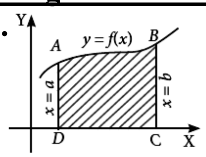
Note:

(i) $1+2+3+......n = \frac{n(n+1)}{2} = \sum_{r=1}^{n} r$

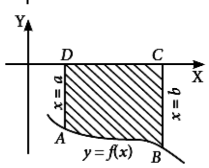
(ii) $1^2+2^2+3^2+......n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{r=1}^{r=n} r^2$

(iii) $1^3+2^3+3^3+......n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \sum_{r=1}^{r=n} r^3$

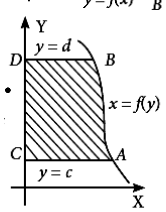
III. Integral Calculus – II

1. 

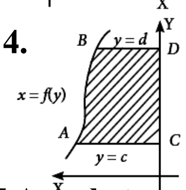
Area $A = \int_a^b y\,dx$

2. 

$A = \int_a^b -y\,dx$

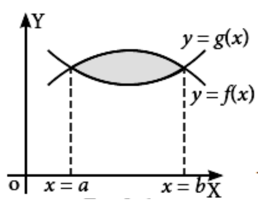
3. 

$A = \int_c^d x\,dy$

4. 

$A = \int_c^d -x\,dy$

5. Area between two curves



$A = \int_a^b [f(x) - g(x)]dx.$

6. Application of Integration in Economics and Commerce.

i) Total sale $= \int_0^r f(t)dt,\ 0 \le t \le r$

2. Elasticity of demand is $\eta_d = \frac{-p}{x}\frac{dx}{dp}$

3. Total inventory carrying cost $= c_1 \int_0^T I(x)dx$

4. Amount of annuity after N Payment is $A = \int_o^N pe^{rt}\,dt$

5. Cost function is $C = \int (MC)\,dx + k.$

6. Average cost function is $AC = \frac{C}{x}, x \ne 0$

7. Revenue function is $R = \int (MR)dx + k.$

8. Demand function is $P = \frac{R}{x}$

9. Profit function is =MR–MC $= R'(x) - C'(x)$

10. Consumer's surplus $= \int_0^{x_0} f(x)\,dx - x_0 p_0$

11. Producer's surplus $= x_0 p_0 - \int_0^{x_0} p(x)\,dx$

IV. Differential Equations

1. variables are separable

$f(x)dx = g(y)dy$ (or) $f(x)dx + g(y)dy = 0$

By direct integration we get the solution.

2. Homogeneous Differential Equations

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$$

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

The given differential equation becomes $v + x\frac{dv}{dx} = F(v)$

Separating the variables, we get

$$x\frac{dv}{dx} = F(v) - v \Rightarrow \frac{dv}{F(v)-v} = \frac{dx}{x}$$

By integrating we get the solution in terms of v and x.

Replacing v by $\frac{y}{x}$ we get the solution.

3. Linear diff. equations of first order:

i) If $\frac{dy}{dx} + Py = Q$ **then**

$$ye^{\int pdx} = \int Qe^{\int pdx}dx + c$$

ii) If $\frac{dx}{dy} + Px = Q$ **then**

$$xe^{\int pdy} = \int Qe^{\int pdy}dy + c$$

4. Second Order first degree differential equations with constant coefficients:

General solution is $y = C.F + P.I$

Nature of roots	Complementary function
Real and different $(m_1 \neq m_2)$	$Ae^{m_1 x} + Be^{m_2 x}$
Real and equal $m_1 = m_2 = m(say)$	$(Ax+B)e^{mx}$
Complex roots $(\alpha \pm i\beta)$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

V. Numerical Methods

1. $\Delta f(x) = f(x+h) - f(x)$
2. $\nabla f(x) = f(x) - f(x-h)$
3. $\nabla f(x+h) = \Delta f(x)$
4. $Ef(x) = f(x+h)$
5. $E^n f(x) = f(x+nh)$
6. Newton's forward interpolation formula:

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + ...$$

7. Newton's backward interpolation formula:

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + ...$$

8. Lagrange's interpolation formula:

$$y = f(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}y_1 + ... + \frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)...(x_n-x_{n-1})}y_n$$

VI. Random Variable and Mathematical Expectation

1. Probability Mass function:

$$P_X(x) = p(x) = \begin{cases} P(X=x_i) = p_i = p(x_i) & if\ x = x_i, i = 1,2,...,n,... \\ 0 & if\ x \neq x_i \end{cases}$$

It Must satisfy the following condition

(i) $p(x_i) \geq 0 \ \forall i$, (ii) $\sum_{i=1} p(x_i) = 1$

2. Discrete distribution function:

$$F_X(x) = P(X \leq x), \ for\ all\ x \in R$$

$$i.e., F_X(x) = \sum_{x_i \leq x} p(x_i)$$

3. Probability density function:

$$P(t_1 \leq X \leq t_2) = \int_{t_1}^{t_2} f_X(x)dx.$$

It Must satisfy the following condition

(i) $f(x) \geq 0 \ \forall x$ and (ii) $\int_{-\infty}^{\infty} f(x)dx = 1$.

4.Continuous distribution function (The distribution function (d.f) or The cumulative distribution function (c.d.f))

The function $F_X(x)$ or simply $F(x)$ has the following properties

(i) $0 \leq F(x) \leq 1, -\infty < x < \infty$

(ii) $F(-\infty) = \lim_{x\to-\infty} F(x) = 0$ and $F(+\infty) = \lim_{x\to\infty} F(x) = 1$.

(iii) $F(\cdot)$ is a monotone, non-decreasing function; that is, $F(a) \leq F(b)$ for $a < b$.

(iv) $F(\cdot)$ is continuous from the right; that is, $\lim_{h\to 0} F(x+h) = F(x)$.

(v) $F'(x) = \frac{d}{dx}F(x) = f(x) \geq 0$

(vi) $F'(x) = \frac{d}{dx}F(x) = f(x) \Rightarrow dF(x) = f(x)dx$

$dF(x)$ is known as probability differential of X.

(vii) $P(a \leq x \leq b) = \int_a^b f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$

$= P(X \leq b) - P(X \leq a)$

$= F(b) - F(a)$

5. Mathematical Expectation:

i) X be a discrete random variable then

$E(X) = \sum x\, p(x)$

ii) X is a continuous random variable then

$E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$

iii) **The mean of X, denoted by μ_x or $E(X)$.**

6. Variance:

i) The variance of X is defined by

$Var(X) = \sum [x - E(X)]^2 p(x)$

if X is discrete random variable with probability mass function $p(x)$.

ii) $Var(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f_X(x)dx$

if X is continuous random variable with probability density function $f_X(x)$.

iii) Expected value of $[X - E(X)]^2$ is called the variance of the random variable.

i.e., $Var(X) = E[X - E(X)]^2 = E(X^2) - [E(X)]^2$

where $E(X^2) = \begin{cases} \sum_x x^2 p(x), \text{ if } X \text{ is Discrete Random Variable} \\ \int_{-\infty}^{\infty} x^2 f(x)dx, \text{ if } X \text{ is Continuous Random Variable} \end{cases}$

iv) Standard deviation of X (S.D(X)) or σ_X $= +\sqrt{Var[X]}$

7. Properties of Mathematical expectation :

i) E(a) =a ,"a" constant ii) E(aX) =a(Ex)

iii) E(aX+b) =a(Ex)+b iv) If x≥0 then E(x)≥0

v) $V(a) = 0$ vi) $V(ax+b) = a^2V(x)$

VII. Probability Distributions

1.Binomial distribution:

i). The probability for exactly x success in n independent trials is given by $p(x)$

$$= \binom{n}{x} p^x q^{n-x} \text{ where } x = 0,1,2,.n \ \& \ q = 1-p$$

ii).The mean of the binomial distribution is np and variance are npq

2. Poisson distribution:

i). The Poisson probability distribution is $p(x)$

$$= e^{-\lambda}\frac{\lambda^x}{x!} \text{ Where } X = 0,1,2.. \ \& \ \lambda = np$$

ii). The mean and variance of the poisson distribution is λ.

iii).Poisson distribution can never be symmetrical.

3. Normal distribution:

i) The normal probability distribution is given by

$$f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)\left(e^{-1/2(x-\mu/\sigma)^2}\right)$$

ii) In normal distribution the mean, median and mode are equal

iii) Standard normal random variate is denoted as $Z = (X-\mu)/\sigma$

iv) The standard normal probability distribution is $1/\sqrt{2\pi}\left(e^{\frac{-z^2}{2}}\right)$

VIII. Sampling Techniques and Statistical Inference

1. Test of significance for single mean:

i)The test statistic (for large samples) is:

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

2. Confidence limits :

$$\bar{x} - Z_{\alpha/2}SE \le \mu \le \bar{x} + Z_{\alpha/2}SE \text{ (or) } \bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Normal Probability Table

Critical Values Z_α	Level of significance (α)			
	1%	2%	5%	10%
Two-tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 2.055$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -2.055$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

IX. Applied Statistics

1. **Method of Least Squares:**

The st.line Equation , Y= a+bx

Two Normal Equations, $\Sigma Y = na + b\Sigma X$;

$\Sigma XY = a\Sigma X + b\Sigma X^2$

2. Seasonal Index (S.I) = $\frac{\text{Seasonal Average}}{\text{Grand average}} X100$

3. Weighted Index Number

Price Index (P01) = $\frac{\sum p_1 w}{\sum p_0 w} x100$

Laspeyre's price index number $P_{01}^L = \frac{\sum p_1 q_0}{\sum p_0 q_0} X100$

Paasche's price index number $P_{01}^P = \frac{\sum p_1 q_1}{\sum p_0 q_1} X100$

Fisher's price index number $P_{01}^F = \sqrt{P_{01}^L \ x \ P_{01}^P} = \sqrt{\frac{\sum p_1 q_0 \ X \sum p_1 q_1}{\sum p_0 q_0 \ X \sum p_0 q_1}} X100$

4. Time Reversal Test : $P_{01} \times P_{10} = 1$. Factor Reversal Test: $P_{01} \ X \ Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$

5. Cost of Living Index Number

Aggregate Expenditure Method = $\frac{\sum p_1 q_0}{\sum p_0 q_0} X100$.

Family Budget Method = $\frac{\sum PV}{\sum V}$

6. The control limits for $\bar{X}$ chart in two different cases are

case (i) when $\bar{X}$ and SD are given	case (i) when $\bar{X}$ and SD are not given
$UCL = \bar{\bar{X}} + 3\frac{\sigma}{\sqrt{n}}$	$UCL = \bar{\bar{X}} + A_2\bar{R}$
$CL = \bar{\bar{X}}$	$CL = \bar{\bar{X}}$
$LCL = \bar{\bar{X}} - 3\frac{\sigma}{\sqrt{n}}$	$LCL = \bar{\bar{X}} - A_2\bar{R}$

The control limits for R chart in two different cases are

case (i) when SD are given	case (i) when SD are not given
$UCL = \bar{R} + 3\sigma_R$	$UCL = D_4\bar{R}$
$CL = \bar{R}$	$CL = \bar{R}$
$LCL = \bar{R} - 3\sigma_R$	$LCL = D_3\bar{R}$