10th – Maths (New Syllabus)

Chapter 1 – Relations and Funtions

All Definitions, Progress checks, Notes, Do you knows, Thinking corners, Points to Remember and 1 mark with Answers

Definition Of Cartesian product (Or) Cross product.

If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A,\ b \in B$ is called the Cartesian Product of A and B, and is denoted by $A \times B$.

Thus, $A \times B = \{(a,b) \mid a \in A, b \in B\}$.

Note

- $A \times B$ is the set of all possible ordered pairs between the elements of A and B such that the first coordinate is an element of A and the second coordinate is an element of B.
- $B \times A$ is the set of all possible ordered pairs between the elements of A and B such that the first coordinate is an element of B and the second coordinate is an element of A.
- If $a = b$, then $(a, b) = (b, a)$.
- The "cartesian product" is also referred as "cross product".

Note

- In general $A \times B \neq B \times A$, but $n(A \times B) = n(B \times A)$
- $A \times B = \phi$ if and only if $A = \phi$ or $B = \phi$
- If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$

Thinking Corner

When will $A \times B$ be equal to $B \times A$?

When A = B

Fig. 1.4

Progress Check

1. For any two non-empty sets A and B, $A \times B$ is called as **Cartesian product**.
2. If $n(A \times B) = 20$ and $n(A) = 5$ then $n(B)$ is **4**.
3. If $A = \{-1, 1\}$ and $B = \{-1, 1\}$ then geometrically describe the set of points of $A \times B$. **Square.** $\{(-1,-1), (-1,1), (1,-1), (1,1)\}$
4. If A, B are the line segments given by the intervals $(-4, 3)$ and $(-2, 3)$ respectively, represent the cartesian product of A and B. **Rectangle.** $\{(-4,-2), (-4,3), (3,-2), (3,3)\}$

Note

- The set of all points in the cartesian plane can be viewed as the set of all ordered pairs (x, y) where x, y are real numbers. In fact, $\mathbb{R} \times \mathbb{R}$ is the set of all points which we call as the cartesian plane.

Note

- The above two verified properties are called distributive property of cartesian product over union and intersection respectively. In fact, for any three sets A, B, C we have (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

1.3.1 Cartesian Product of three Sets

If A, B, C are three non-empty sets then the cartesian product of three sets is the set of all possible ordered triplets given by

$$A \times B \times C = \{(a,b,c) \text{ for all } a \in A, b \in B, c \in C\}$$

Note

- In general, cartesian product of two non-empty sets provides a shape in two dimensions and cartesian product of three non-empty sets provide an object in three dimensions.

Progress Check

Let A = {1, 2, 3, 4} and B = {a, b, c}.

1. Which of the following are relations from A to B?	2. Which of the following are relations from B to A?
✓ (i) { (1, b), (1, c), (3, a), (4, b) }	(i) { (c, a), (c, b), (c, 1) } X
X (ii) { (1, a), (b, 4), (c, 3) }	(ii) { (c, 1), (c, 2), (c, 3), (c, 4) } ✓
X (iii) { (1, a), (a, 1), (2, b), (b, 2) }	(iii) { (a, 4), (b, 3), (c, 2) } ✓

Definition **Of Relation.**

Let A and B be any two non-empty sets. A 'relation' R from A to B is a subset of $A \times B$ satisfying some specified conditions. If $x \in A$ is related to $y \in B$ through R, then we write it as $x\,\mathrm{R}\,y$. $x\,\mathrm{R}\,y$ if and only if $(x,y) \in \mathrm{R}$.

The domain of the relation $\mathrm{R} = \{x \in A \mid x\,\mathrm{R}\,y, \text{ for some } y \in B\}$

The co-domain of the relation R is B

The range of the relation $\mathrm{R} = \{y \in B \mid x\mathrm{R}y, \text{ for some } x \in A\}$

From these definitions, we note that domain of $\mathrm{R} \subseteq A$, co-domain of $\mathrm{R} = B$ and range of $\mathrm{R} \subseteq B$.

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'Null relation'

Let us consider the following examples. Suppose $A = \{-3,-2,-1\}$ and $B = \{1,2,3,4\}$. A relation from A to B is defined as $a - b = 8$ i.e., there is no pair (a,b) such that $a - b = 8$. Thus R contain no element and so $\mathrm{R} = \phi$.

A relation which contains no element is called a "Null relation".

DO YOU KNOW?

If $n(A) = p$, $n(B) = q$, then the total number of relations that exist between A and B is 2^{pq}.

usually use the symbol f to denote a functional relation.

Definition

A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x,y) \in f$.

That is, $f = \{(x,y) \mid \text{for all } x \in X, y \in Y\}$.

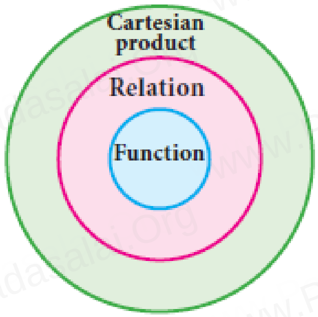

Fig. 1.12(a)

A function f can be thought as a mechanism (or device) (Fig.1.12(b)), which gives a unique output $f(x)$ to every input x.

DO YOU KNOW?

A function is also called as a mapping or transformation.

Note

If $f : X \to Y$ is a function then

- The set X is called the domain of the function f and the set Y is called its co-domain.
- If $f(a) = b$, then b is called 'image' of a under f and a is called a 'pre-image' of b.
- The set of all images of the elements of X under f is called the 'range' of f.
- $f : X \to Y$ is a function only if
  - (i) every element in the domain of f has an image.
  - (ii) the image is unique.
- If A and B are finite sets such that $n(A) = p$, $n(B) = q$ then the total number of functions that exist between A and B is q^p.

Progress Check

1. Relations are subsets of Cartesian products. Functions are subsets of Relations.
2. True or False: All the elements of a relation should have images. True
3. True or False: All the elements of a function should have images. True
4. True or False: If $R : A \to B$ is a relation then the domain of $R = A$. False
5. If $f : \mathbb{N} \to \mathbb{N}$ is defined as $f(x) = x^2$ the pre-image(s) of 1 and 2 are 1 and ---.
6. The difference between relation and function is Image.
   (Relation : One pre-image – one or more images) (Function : One pre-image – One image)
7. Let A and B be two non-empty finite sets. Then which one among the following two collection is large?
   (i) The number of relations between A and B. ✓
   (ii) The number of functions between A and B.

Thinking Corner

Is the relation representing the association between planets and their respective moons a function? **No (Because Different Planets have different numbers of Moons)**

Note

- Any equation represented in a graph is usually called a 'curve'.

Note

- The range of a function is a subset of its co-domain.

Types of Functions

In this section, we will discuss the following types of functions with suitable examples.

(i) one – one (ii) many – one (iii) onto (iv) into

A function $f : A \to B$ is called **one – one function** if distinct elements of A have distinct images in B.

A one-one function is also called an **injection**.

A function $f : A \to B$ is called **many-one function** if two or more elements of A have same image in B.

In other words, a function $f : A \to B$ is called many-one if f it is not one–one.

A function $f : A \to B$ is said to be **onto function** if the range of f is equal to the co-domain of f.

Fig. 1.25

In other words, every element in the co-domain B has a pre-image in the domain A.

An onto function is also called a **surjection**.

Note

- If $f : A \to B$ is an onto function then, the range of $f = B$. That is, $f(A) = B$.

A function $f : A \to B$ is called an **into function** if there exists atleast one element in B which is not the image of any element of A.

That is the range of f is a proper subset of the co-domain of f.

In other words, a function $f : A \to B$ is called 'into' if it is not 'onto'.

bijection. This process is widely used in the study of secret codes called '**cryptography**'.

If a function $f : A \to B$ is both one–one and onto, then f is called a bijection from A to B.

Note

- A one – one and onto function is also called a one – one correspondence.

Thinking Corner

Can there be a one to many function? - No

1.6.1 Vertical line test

"A curve drawn in a graph represents a function, if every vertical line intersects the curve in at most one point."

1.7.6 Horizontal Line Test

Previously we have seen the vertical line test. Now let us see the horizontal line test. A function represented in a graph is one–one, if every horizontal line intersects the curve in at most one point.

1.8 Special cases of function

There are some special cases of a function which will be very useful. We discuss some of them below

(i) Constant function

A function $f : A \to B$ is called a constant function if the range of f contains only one element. That is, $f(x) = c$, for all $x \in A$ and for some fixed $c \in B$.

(ii) Identity function

Let A be a non–empty set. Then the function $f : A \to A$ defined by $f(x) = x$ for all $x \in A$ is called an identity function on A and is denoted by I_A.

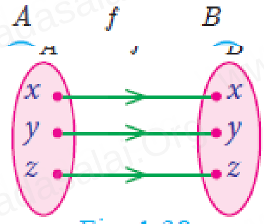

Fig. 1.38

(iii) Real valued function

A function $f : A \to B$ is called a real valued function if the range of f is a subset of the set of all real numbers $\mathbb{R}$. That is, $f(A) \subseteq \mathbb{R}$.

Progress Check

State True or False.

1. All one – one functions are onto functions. False
2. There will be no one – one function from A to B when $n(A) = 4$, $n(B) = 3$. True
3. All onto functions are one – one functions. False
4. There will be no onto function from A to B when $n(A) = 4$, $n(B) = 5$. True
5. If f is a bijection from A to B, then $n(A) = n(B)$. True
6. If $n(A) = n(B)$, then f is a bijection from A to B. False
7. All constant functions are bijections. False

Thinking Corner

Is an identity function one – one function? Yes

Definition

Let $f : A \to B$ and $g : B \to C$ be two functions (Fig.1.42). Then the composition of f and g denoted by $g \circ f$ is defined as the function $g \circ f(x) = g(f(x))$ for all $x \in A$.

Thinking Corner

If $f(x) = x^m$ and $g(x) = x^n$ does $f \circ g = g \circ f$? Yes

$x^{mn} = x^{mn}$

Note

- Generally, $f \circ g \neq g \circ f$ for any two functions f and g. So, composition of functions is not commutative.

Note

- Composition of three functions is always associative. That is, $f \circ (g \circ h) = (f \circ g) \circ h$

Progress Check

State your answer for the following questions by selecting the correct option.

1. Composition of functions is commutative
   (a) Always true (b) Never true (c) Sometimes true ✓
2. Composition of functions is associative
   (a) Always true ✓ (b) Never true (c) Sometimes true

Note

- Modulus function is not a linear function but it is composed of two linear functions x and $-x$.
- Linear functions are always one-one functions and has applications in Cryptography as well as in several branches of Science and Technology.

Progress Check

1. Is a constant function a linear function? Yes
2. Is quadratic function a one – one function? No
3. Is cubic function a one – one function? Yes
4. Is the reciprocal function a bijection? Yes
5. If $f : A \to B$ is a constant function, then the range of f will have One elements.

DO YOU KNOW?

The Composition $g \circ f(x)$ exists only when range of f is a subset of domain of g.

Points to Remember

- The Cartesian Product of A with B is defined as $A \times B = \{(a,b) \mid \text{for all } a \in A, b \in B\}$
- A relation R from A to B is always a subset of $A \times B$. That is $R \subseteq A \times B$
- A relation R from X to Y is a function if for every $x \in X$ there exists only one $y \in Y$.
- A function can be represented by
  - (i) an arrow diagram
  - (ii) a tabular form
  - (iii) a set of ordered pairs
  - (iv) a graphical form
- Some types of functions
  - (i) One-one function
  - (ii) Onto function
  - (iii) Many-one function
  - (iv) Into function
- Identity function $f(x) = x$
- Reciprocal function $f(x) = \frac{1}{x}$
- Constant function $f(x) = c$
- Linear function $f(x) = ax + b$, $a \neq 0$
- Quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$
- Cubic function $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$
- For three non-empty sets A, B and C, if $f : A \to B$ and $g : B \to C$ are two functions, then the composition of f and g is a function $g \circ f : A \to C$ will be defined as $g \circ f(x) = g(f(x))$ for all $x \in A$.
- If f and g are any two functions, then in general, $f \circ g \neq g \circ f$
- If f, g and h are any three functions, then $f \circ (g \circ h) = (f \circ g) \circ h$

Exercise 1.6

Multiple choice questions

1. If $n(A \times B) = 6$ and $A = \{1,3\}$ then $n(B)$ is

   (1) 1 (2) 2 (3) 3 (4) 6

2. $A = \{a,b,p\}$, $B = \{2,3\}$, $C = \{p,q,r,s\}$ then $n[(A \cup C) \times B]$ is

   (1) 8 (2) 20 (3) 12 (4) 16

3. If $A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.

   (1) $(A \times C) \subset (B \times D)$ (2) $(B \times D) \subset (A \times C)$

   (3) $(A \times B) \subset (A \times D)$ (4) $(D \times A) \subset (B \times A)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is

   (1) 3 (2) 2 (3) 4 (4) 8

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is

   (1) $\{2,3,5,7\}$ (2) $\{2,3,5,7,11\}$

   (3) $\{4,9,25,49,121\}$ (4) $\{1,4,9,25,49,121\}$

6. If the ordered pairs $(a+2, 4)$ and $(5, 2a+b)$ are equal then (a,b) is

   (1) (2, –2) (2) (5,1) (3) (2,3) (4) (3, –2)

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

   (1) m^n (2) n^m (3) $2^{mn} - 1$ (4) 2^{mn}

8. If $\{(a,8),(6,b)\}$ represents an identity function, then the value of a and b are respectively

   (1) (8,6) (2) (8,8) (3) (6,8) (4) (6,6)

9. Let $A = \{1,2,3,4\}$ and $B = \{4,8,9,10\}$. A function $f : A \to B$ given by $f = \{(1,4),(2,8),(3,9),(4,10)\}$ is a

   (1) Many-one function (2) Identity function

   (3) One-to-one function (4) Into function

10. If $f(x) = 2x^2$ and $g(x) = \dfrac{1}{3x}$, then $f \circ g$ is

    (1) $\dfrac{3}{2x^2}$ (2) $\dfrac{2}{3x^2}$ (3) $\dfrac{2}{9x^2}$ (4) $\dfrac{1}{6x^2}$

11. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to

    (1) 7 (2) 49 (3) 1 (4) 14

12. Let f and g be two functions given by

$f = \{(0,1),(2,0),(3,-4),(4,2),(5,7)\}$

$g = \{(0,2),(1,0),(2,4),(-4,2),(7,0)\}$ then the range of $f \circ g$ is

(1) {0,2,3,4,5} (2) {–4,1,0,2,7} (3) {1,2,3,4,5} (4) {0,1,2}

13. Let $f(x) = \sqrt{1+x^2}$ then

(1) $f(xy) = f(x).f(y)$ (2) $f(xy) \geq f(x).f(y)$

(3) $f(xy) \leq f(x).f(y)$ (4) None of these

14. If $g = \{(1,1),(2,3),(3,5),(4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are

(1) (–1,2) (2)(2, –1) (3) (–1, –2) (4) (1,2)

15. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is

(1) linear (2) cubic (3) reciprocal (4) quadratic

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Errors if any, Pl. notify to the mail.

Exercise 1.6

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3)	(3)	(1)	(2)	(3)	(4)	(3)	(1)	(3)	(3)	(1)	(4)	(3)	(2)	(4)