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Chapter 1

Applications of Matrices and Determinants

EXERCISE 1.1

1. Find the adjoint of the following:

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

Solution

(i) Let $A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Cofactor of $A = \begin{bmatrix} 2 & -6 \\ -4 & -3 \end{bmatrix}$. $\therefore$ $adjA = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$$\therefore \text{ Cofactor of } A = \begin{bmatrix} 8-7 & -(6-3) & 21-12 \\ -(6-7) & (4-3) & -(14-9) \\ (3-4) & -(2-3) & (8-9) \end{bmatrix} = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$$

$$adj(A) = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}.$$

(iii) Let $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

We know that $adj(kA) = k^{n-1} adjA$

$$\therefore adjA = \left(\frac{1}{3}\right)^{3-1} adj\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\text{Cofactor of } \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}$$

$$adj(A) = \frac{1}{9}\begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}^T = \frac{1}{9}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}.$$

2. Find the inverse (if it exists) of the following :

(i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Solution

$$\text{Let } A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

$$|A| = 6-4 = 2 \neq 0$$

$$adj A = \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

(ii)

$$\text{Let } A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$|A| = 5(25-1)-(5-1)+(1-5) = 120-4-4 = 112$$

$$\text{Cofactor of } A = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix} \Rightarrow adj A = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj A) = \frac{1}{112}\times 4\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}.$$

(iii)

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{vmatrix} = 2$$

$$\text{Cofactor of } A = \begin{bmatrix} 1 & -3 & 9 \\ -(6-7) & 4-3 & -(-14-9) \\ (3-4) & -(2-3) & (8-9) \end{bmatrix} = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$$

$$adj(A) = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix};\ A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

3. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

Solution

$$\text{Given that } F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$|F(\alpha)| = \cos^2\alpha - 0 + \sin^2\alpha = 1 \neq 0$$

$\therefore [F(\alpha)]^{-1}$ exists.

$$\text{Cofactor of } F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & \cos^2\alpha+\sin^2\alpha & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$adjF(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1} = \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$= [F(\alpha)]^{-1}.$$

4. If $A=\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2-3A-7I_2=O_2$. Hence find A^{-1}.

Solution

$$A^2 = A\cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$A^2-3A-7I_2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -15 & -9 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2-3A-7I_2 = 0$$

$$A^2-3A = 7I_2$$

Pre multiply both sides by A^{-1}, we get

$$A-3I = 7A^{-1}$$

$$A^{-1} = \frac{1}{7}(A-3I)$$

$$A-3I = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}(A-3I) = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}.$$

5. If $A=\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1}=A^T$.

Solution

$$\text{To prove } A^{-1} = A^T$$
$$AA^{-1} = AA^T$$
$$\text{It is enough to prove } AA^T = I$$
$$AA^T = \frac{1}{81}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}\begin{bmatrix}-8 & 4 & 1\\ 1 & 4 & -8\\ 4 & 7 & 4\end{bmatrix}$$
$$= \frac{1}{81}\begin{bmatrix}81 & 0 & 0\\ 0 & 81 & 0\\ 0 & 0 & 81\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = I$$

Hence proved.

6. If $A = \begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I$.

Solution

$$adjA = \begin{bmatrix}3 & 4\\ 5 & 8\end{bmatrix}$$
$$A(adjA) = \begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}\begin{bmatrix}3 & 4\\ 5 & 8\end{bmatrix} = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} = 4\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = |A|I$$
$$(adjA)A = \begin{bmatrix}3 & 4\\ 5 & 8\end{bmatrix}\begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix} = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} = |A|I$$

From the above two results,

$$A(adjA) = (adjA)A = |A|I.$$

7. If $A = \begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B = \begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Solution

$$|A| = 15-14 = 1 \neq 0 \quad |B| = -2+15 = 13 \neq 0$$
$$A^{-1} = \begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}; \quad B^{-1} = \frac{1}{13}\begin{bmatrix}2 & 3\\ -5 & -1\end{bmatrix}$$
$$AB = \begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}\begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix} = \begin{bmatrix}7 & -5\\ 18 & -11\end{bmatrix}$$
$$|AB| = -77+90 = 13$$
$$(AB)^{-1} = \frac{1}{13}\begin{bmatrix}-11 & 5\\ -18 & 7\end{bmatrix};$$
$$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix}2 & 3\\ -5 & -1\end{bmatrix}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix} = \frac{1}{13}\begin{bmatrix}10-21 & -4+9\\ -25+7 & 10-3\end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix} \quad ..(2)$$

From (1) and (2) $\quad (AB)^{-1} = B^{-1}A^{-1}$.

8. If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

Solution

$$|adjA| = 2[24-0]-(4)[-6-14]+2[0+24] = 16$$

$$adj(adjA) = \begin{bmatrix} 24 & 20 & 24 \\ -(-8-0) & (4+4) & -(0-8) \\ (28-24) & -(-14+6) & (24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix} = 4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$A = \pm\frac{1}{\sqrt{|adjA|}}(adjA)$$

$$= \pm\frac{1}{\sqrt{16}}(4)\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \pm\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}.$$

9. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

Solution

$$|adjA| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix} = 2(36-18) = 36$$

$$A^{-1} = \pm\frac{1}{\sqrt{(adjA)}}(adjA)$$

$$= \pm\frac{1}{\sqrt{36}}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix} = \pm\frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}.$$

10. Find $adj(adj(A))$ if $adjA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

Solution

$$adjA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

$$\text{Cofactor of } adjA = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}$$

$$adj(adjA) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}.$$

11. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

Solution

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}.$$

$$A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} = \cos^2 x \begin{bmatrix} 1 & -\dfrac{\sin x}{\cos x} \\ \dfrac{\sin x}{\cos x} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} 1 & -\dfrac{\sin x}{\cos x} \\ \dfrac{\sin x}{\cos x} & 1 \end{bmatrix}\begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x - \sin^2 x & -\sin x \cos x - \sin x \cos x \\ \sin x \cos x + \sin x \cos x & \cos^2 x - \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}.$$

12. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

Solution

Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

Then $AB = C$

$$A = CB^{-1}$$

$$= \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}^{-1} = 7\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\frac{1}{(-10+3)}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}.$$

13. Given $A=\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}, B=\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$, find a matrix X such that $AXB=C$.

Solution

$$AXB = C$$
$$XB = A^{-1}C \Rightarrow X = A^{-1}CB^{-1}$$
$$X = \frac{1}{2}\begin{bmatrix}0 & 1\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\frac{1}{5}\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix} = \frac{1}{10}\begin{bmatrix}2 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$$
$$= \frac{1}{10}\begin{bmatrix}0 & 10\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}.$$

14. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}\left(A^2-3I\right)$.

Solution

$$2A^{-1} = A^2 - 3I$$

It is enough to prove $2I = A^3 - 3A$

$$A^2 = A\cdot A$$
$$= \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} = \begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$$
$$A^3 = \begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} = \begin{bmatrix}2 & 3 & 3\\ 3 & 2 & 3\\ 3 & 3 & 2\end{bmatrix}$$
$$A^3 - 3A = \begin{bmatrix}2 & 3 & 3\\ 3 & 2 & 3\\ 3 & 3 & 2\end{bmatrix} - \begin{bmatrix}0 & 3 & 3\\ 3 & 0 & 3\\ 3 & 3 & 0\end{bmatrix} = \begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix} = 2I$$

EXERCISE 1.2

1. Find the rank of the following matrices by minor method :

(i) $\begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}-1 & 3\\ 4 & -7\\ 3 & -4\end{bmatrix}$ (iii) $\begin{bmatrix}1 & -2 & -1 & 0\\ 3 & -6 & -3 & 1\end{bmatrix}$ (iv) $\begin{bmatrix}1 & -2 & 3\\ 2 & 4 & -6\\ 5 & 1 & -1\end{bmatrix}$

(v) $\begin{bmatrix}0 & 1 & 2 & 1\\ 0 & 2 & 4 & 3\\ 8 & 1 & 0 & 2\end{bmatrix}$

Solution

(i) Let $A = \begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$; $|A| = 4-4=0$ and hence $\rho(A)\neq 2$

$\therefore \rho(A) = 1$.

(ii) Let $A = \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

Consider a second order minor $\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0 \quad \therefore \rho(A) = 2$.

(iii) Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Consider a second order minor $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 \neq 0 \quad \therefore \rho(A) = 2$.

(iv) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

$$|A| = (-4+6) + 2(-2+30) + 3(2-20) = 2 + 56 - 54 = 4 \neq 0$$

$\therefore \rho(A) = 3$.

(v) Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

Consider a third order minor

$$\begin{vmatrix} 0 & 2 & 1 \\ 0 & 4 & 3 \\ 8 & 0 & 2 \end{vmatrix} = 0 - 0 + 8(6-4) = 16 \neq 0 \quad \therefore \rho(A) = 3.$$

2. Find the rank of the following matrices by row reduction method

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

$\therefore \rho(A) = 2$.

(ii) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -1 & 1 \\ 0 & -3 & 2 \end{bmatrix} R_3 \to \frac{1}{4} R_3$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & -7 & 5 \\ 0 & -3 & 2 \end{bmatrix} R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 7R_2 \\ R_4 \to R_4 - 3R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} R_4 \to 2R_4 - R_3$$

$\therefore \rho(A) = 3$.

(iii) Let $A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} R_3 \to R_3 - 2R_2$$

$$\therefore \rho(A) = 3.$$

3. Find the inverse of each of the following by Gauss-Jordan method:

(i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

Solution

(i) Let $A = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

$$[A \quad I] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & 0 & 1 \end{array}\right] R_1 \to \frac{1}{2} R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_2 \to R_2 - 5R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_1 \to R_1 + R_2$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right] R_2 \to 2R_2$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 1 \\ -5 & 2 \end{bmatrix}.$$

(ii) Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

$$[A \quad I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 6R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_2 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 4R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_3 \\ R_2 \to R_2 + R_3 \\ \\ \end{array}$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}.$$

(iii) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$$[A \quad I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_3 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 3R_3 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to (-1)R_3 \end{array}$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}.$$

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(i) $2x+5y=-2,\ x+2y=-3$ (ii) $2x-y=8,\ 3x+2y=-2$

(iii) $2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$

(iv) $x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13.$

Solution

(i) $2x+5y=-2,\quad x+2y=-3$

$$\begin{bmatrix}2&5\\1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\-3\end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = 4-5=-1\neq 0$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix}2&-5\\-1&2\end{bmatrix}=\begin{bmatrix}-2&5\\1&-2\end{bmatrix}$$

$$X = A^{-1}B=\begin{bmatrix}-2&5\\1&-2\end{bmatrix}\begin{bmatrix}-2\\-3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-11\\4\end{bmatrix}$$

$$\therefore x=-11,\ y=4.$$

(ii) $2x-y=8,\quad 3x+2y=-2$

$$\begin{bmatrix}2&-1\\3&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\2\end{bmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix}2&-1\\3&2\end{vmatrix}=4+3=7$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix}2&1\\-3&2\end{bmatrix}$$

$$AX = A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{7}\begin{bmatrix}2&1\\-3&2\end{bmatrix}\begin{bmatrix}8\\-2\end{bmatrix}=\frac{1}{7}\begin{bmatrix}14\\-28\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\-4\end{bmatrix}$$

$$x=2,\ y=-4.$$

(iii)
$$2x+3y-z = 9$$
$$x+y+z = 9$$
$$3x-y-z = -1$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 01 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 2(-1+1)-3(-1-3)-1(-1-3) = 16$$

$$\text{Cofactor of } A = \begin{bmatrix} 0 & 4 & -4 \\ -(-3-1) & (-2+3) & (-2-9) \\ (3+1) & -(2+1) & (2-3) \end{bmatrix}$$

$$adjA = \begin{bmatrix} 0 & 4 & -4 \\ 4 & 1 & 11 \\ 4 & -3 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore\ x = 2,\quad y = 3,\quad z = 4.$$

(iv)

$$x+y+z-2 = 0$$

$$6x-4y+5z-31 = 0$$

$$5x+2y+2z = 13$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{vmatrix} = 1(-8-10)-1(12-25)+1(12+20) = 27$$

$$\text{Cofactor of } A = \begin{bmatrix} -18 & 13 & 32 \\ 0 & -3 & 3 \\ 9 & 1 & -10 \end{bmatrix}$$

$$adjA = \begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{27}\begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$X = \frac{1}{27}\begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}\begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 81 \\ -54 \\ 27 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\therefore x = 3,\ y = -2,\ z = 1.$$

2. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1$, $3x+2y+z=7$, $2x+y+3z=2$.

Solution

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$AB = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly $$BA = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider $$AB = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = 4I$$

$$\frac{1}{4}AB = I$$

$$B^{-1} = \frac{1}{4}A = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

Given system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$BX = C \text{ where } B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$X = B^{-1}C$$

$$= \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$x = 2, \quad y = 1, \quad z = -1$$

3. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem).

Solution

Let x be the salary and y be the fixed increment.

$$\text{Then} \quad x + 3y = 19800$$

$$x + 9y = 23400$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19800 \\ 23400 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = 9 - 3 = 6 \neq 0$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 19800 \\ 23400 \end{bmatrix} = \frac{600}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 33 \\ 39 \end{bmatrix}$$

$$= 100\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 33 \\ 39 \end{bmatrix} = 300\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 11 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = 300\begin{bmatrix} 60 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18000 \\ 600 \end{bmatrix}$$

$$x = ₹18000, \quad y = ₹600.$$

Starting salary = ₹18,000. Annual increment is ₹ 600.

4. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

Solution

Let one man can finish the work in x days and one woman can finish it in y days.

A man's one day work $= \frac{1}{x}$ and a woman's one day work $= \frac{1}{y}$.

In 3 days a man completes $\frac{3}{x}$ work and a woman completes $\frac{3}{y}$ work.

$\therefore$ 4 men and 4 women complete the work

$$\Rightarrow \quad \frac{12}{x} + \frac{12}{y} = 1 \qquad \text{... (1)}$$

In 4 days a man completes $\frac{4}{x}$ work and a woman completes $\frac{4}{y}$ work.

2 men and 5 women complete the work means

$$\frac{8}{x} + \frac{20}{y} = 1 \qquad \text{... (2)}$$

Take $X = \frac{1}{x}, \quad Y = \frac{1}{y}$

$$\therefore 12X + 12Y = 1$$

$$8X + 20Y = 1$$

$$\begin{bmatrix} 12 & 12 \\ 8 & 20 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{144} \begin{bmatrix} 20 & -12 \\ -8 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{144} \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$X = \frac{1}{18}, \; Y = \frac{1}{36}$$

$$x = 18, \; y = 36.$$

A man and a woman alone can do the work in 18 days and 36 days respectively.

5. The prices of three commodities A, B and C are ₹x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem).

Solution

From the given data

$$2x-4y+5z = 15000$$
$$3x+y-2z = 1000$$
$$-x+3y+z = 4000$$

$$\Rightarrow \begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$$AX = B$$

$$|A| = \begin{vmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{vmatrix} = 68$$

$$adjA = \begin{bmatrix} 7 & -1 & 10 \\ 19 & 7 & -2 \\ 3 & 19 & 14 \end{bmatrix}^T = \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix} = \frac{1000}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{1000}{68}\begin{bmatrix} 136 \\ 68 \\ 204 \end{bmatrix} = 1000\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

$$\therefore x = 2000, \; y=1000, \; z=3000$$

The prices per unit of A, B and C are respectively ₹2000, ₹1000, ₹3000.

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule:

   (i) $5x-2y+16=0, \; x+3y-7=0$

   (ii) $\frac{3}{x}+2y=12, \; \frac{2}{x}+3y=13$

   (iii) $3x+3y-z=11, \; 2x-y+2z=9, \; 4x+3y+2z=25$

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0,\ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0.$

Solution

(i) $5x-2y+16=0,\ x+3y-7=0$

$$5x-2y=-16$$
$$x+3y=7$$
$$\Delta=\begin{vmatrix}5 & -2\\ 1 & 3\end{vmatrix}=15+2=17$$
$$\Delta_x=\begin{vmatrix}-16 & -2\\ 7 & 3\end{vmatrix}=-48+14=-34$$
$$\Delta_y=\begin{vmatrix}5 & -16\\ 1 & 7\end{vmatrix}=35+16=51$$
$$x=\frac{\Delta_x}{\Delta}=-\frac{34}{17}=-2$$
$$y=\frac{\Delta_y}{\Delta}=\frac{51}{17}=3$$
$$x=-2,\ y=3.$$

(ii) $\frac{3}{x}+2y=12,\ \frac{2}{x}+3y=13$

Let $\frac{1}{x}=X$

Then $3X+2y=12$

$2X+3y=13$

$$\Delta=\begin{vmatrix}3 & 2\\ 2 & 3\end{vmatrix}=9-4=5$$
$$\Delta_X=\begin{vmatrix}12 & 2\\ 13 & 3\end{vmatrix}=36-26=10$$
$$\Delta_y=\begin{vmatrix}3 & 12\\ 2 & 13\end{vmatrix}=39-24=15$$
$$X=\frac{\Delta_X}{\Delta}=\frac{10}{5}=2\ \therefore\ x=\frac{1}{2}$$
$$y=\frac{\Delta_y}{\Delta}=\frac{15}{5}=3$$
$$x=\frac{1}{2},\ y=3.$$

(iii) $3x+3y-z=11,\ 2x-y+2z=9,\ 4x+3y+2z=25$

$$\Delta=\begin{vmatrix}3 & 3 & -1\\ 2 & -1 & 2\\ 4 & 3 & 2\end{vmatrix}=-22$$

$$\Delta_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = -44$$

$$\Delta_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & -2 \\ 4 & 25 & 2 \end{vmatrix} = -66$$

$$\Delta_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = -88$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-44}{-22} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-66}{-22} = 3$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-88}{-22} = 4$$

$$x = 2,\ y = 3,\ z = 4.$$

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0,\ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

Let $X = \frac{1}{x},\ Y = \frac{1}{y}$ and $Z = \frac{1}{z}$.

$$3X-4Y-2Z = 1$$
$$X+2Y+Z = 2$$
$$2X-5Y-4Z = -1$$

Then $\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = -15.\ \Delta_X = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = -15$

$$\Delta_Y = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix} = -5$$

$$\Delta_Z = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = -5$$

$$X = \frac{\Delta_X}{\Delta} = \frac{-15}{-15} = 1 \Rightarrow x = 1$$

$$Y = \frac{\Delta_Y}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow y = 3$$

$$Z = \frac{\Delta_Z}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow z = 3.$$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem).

Solution

Let x and y be the number of questions answered correctly and incorrectly.

$$\text{Then} \quad x+y = 100 \qquad \dots (1)$$

$$x-\frac{1}{4}y = 80 \qquad \dots (2)$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{1}{4} \end{vmatrix} = -\frac{5}{4}$$

$$\Delta_x = \begin{vmatrix} 100 & 1 \\ 80 & -\frac{1}{4} \end{vmatrix} = -25-80 = -105$$

$$x = \frac{-105}{-\frac{5}{4}} = 21\times 4 = 84.$$

The student has answered 84 questions correctly.

3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem).

Solution

Let x litres of 50% acid and y litres of 25% acid solution to be mixed.

$$\text{Then} \quad x+y = 10 \qquad \dots (1)$$

$$50\% \text{ of } x + 25\% \text{ of } y = 40\% \text{ of } (x+y)$$

$$\frac{50}{100}x+\frac{25}{100}y = \frac{40}{100}\times 10$$

$$50x+25y = 40\times 10$$

$$2x+y = 16 \qquad \dots (2)$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$$

$$\Delta_x = \begin{vmatrix} 10 & 1 \\ 16 & 1 \end{vmatrix} = 10-16 = -6$$

$$\Delta_y = \begin{vmatrix} 1 & 10 \\ 2 & 16 \end{vmatrix} = 16-20 = -4$$

$$x = \frac{\Delta x}{\Delta} = \frac{-6}{-1} = 6$$

$$y = \frac{\Delta y}{\Delta} = \frac{-4}{-1} = 4$$

i.e., 6 litres of 50% acid and 4 litres of 25% acid solution to be mixed to get 10 litres of 40% of acid solution.

4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem).

Solution

$$\text{Given} \quad \frac{1}{A}+\frac{1}{B}=\frac{1}{10}$$

$$\frac{1}{A}-\frac{1}{B}=\frac{1}{30}$$

$$\text{Let} \quad x=\frac{1}{A},\ y=\frac{1}{B}$$

$$\text{Then} \quad x+y=\frac{1}{10}$$

$$x-y=\frac{1}{30}$$

$$\Delta=\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}=-1-1=-2$$

$$\Delta_x=\begin{vmatrix}\frac{1}{10} & 1\\ \frac{1}{30} & -1\end{vmatrix}=-\frac{1}{10}-\frac{1}{30}=\frac{-4}{30}=\frac{-2}{15}$$

$$\Delta_y=\begin{vmatrix}1 & \frac{1}{10}\\ 1 & \frac{1}{30}\end{vmatrix}=\frac{1}{30}-\frac{1}{10}=\frac{-2}{30}=\frac{-1}{15}$$

$$x=\frac{\Delta x}{\Delta}=\frac{1}{15}\Rightarrow A=15 \text{ min}$$

$$y=\frac{\Delta y}{\Delta}=\frac{1}{30}\Rightarrow B=30 \text{ min.}$$

Pump A, B fill the tank by 15 minutes and 30 minutes respectively.

5. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?

Solution

Let the cost of one dosai, one idly and one vadai be ₹ x, y and z respectively.

Then $\quad 2x+3y+2z = 150 \quad ; \quad 2x+2y+4z=200 \quad ; \quad 5x+4y+2z=250$.

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 3 & 1 \\ 2 & 2 & 2 \\ 5 & 4 & 1 \end{vmatrix} = 20$$

$$\Delta_x = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix} = 50\times2\begin{vmatrix} 3 & 3 & 1 \\ 4 & 2 & 2 \\ 5 & 4 & 1 \end{vmatrix} = 600$$

$$\Delta_y = 50\times2\begin{vmatrix} 2 & 3 & 1 \\ 2 & 4 & 2 \\ 5 & 5 & 1 \end{vmatrix} = 100\times(2) = 200$$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 50\times2\begin{vmatrix} 2 & 3 & 3 \\ 1 & 1 & 2 \\ 5 & 4 & 5 \end{vmatrix} = 600$$

$$x = \frac{\Delta_x}{\Delta} = \frac{600}{20} = 30$$

$$y = \frac{\Delta_y}{\Delta} = \frac{200}{20} = 10$$

$$z = \frac{\Delta_z}{\Delta} = \frac{600}{20} = 30$$

$\therefore$ The cost of one dosai is ₹30 and idly is ₹10 and vadai is ₹30.

The cost of 3 dosai, 6 idlies and 6 vadais

$$= 3\times30+6\times10+6\times30$$
$$= 90+60+180$$
$$= ₹330 < ₹350$$

Yes, they will be able to manage to pay the bill.

EXERCISE 1.5

1. Solve the following systems of linear equations by Gaussian elimination method:
   (i) $2x-2y+3z=2,\ \ x+2y-z=3,\ \ 3x-y+2z=1$
   (ii) $2x+4y+6z=22,\ 3x+8y+5z=27,\ -x+y+2z=2$

Solution

(i) $2x-2y+3z=2,\ \ x+2y-z=3,\ \ 3x-y+2z=1$

$$[A \quad B] = \begin{bmatrix} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & 0 & -5 & -20 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to 6R_3 - 7R_2 \end{matrix}$$

$$\therefore \; -5z = -20 \Rightarrow z = 4$$

$$-6y + 5z = -4$$

$$\Rightarrow -6y = -24 \Rightarrow y = 4$$

$$x + 2y - z = 3 \Rightarrow x = -1$$

$$x = -1, \; y = 4, \; z = 4.$$

(ii) $2x+4y+6z=22, \; 3x+8y+5z=27, \; -x+y+2z=2$

$$[A \quad B] = \begin{bmatrix} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 11 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{bmatrix} R_1 \to \frac{R_1}{2}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 11 \\ 0 & 2 & -4 & -6 \\ 0 & 3 & 5 & 13 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 3 & 5 & 13 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 / 2 \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 11 & 22 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 3R_2 \end{matrix}$$

$$\therefore \; 11z = 22 \Rightarrow z = 2$$

$$y - 2z = -3 \Rightarrow y = 1$$

$$x + 2y + 3z = 11 \Rightarrow x = 3$$

$$\text{Thus } x = 3, \; y = 1, \; z = 2.$$

2. If ax^2+bx+c is divided by $x+3, x-5,$ and $x-1$, the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method).

Solution

$$\text{Let } f(x) = ax^2 + bx + c$$

$$f(-3) = 21, \; f(5) = 61 \text{ and } f(1) = 9$$

$$9a-3b+c = 21$$
$$25a+5b+c = 61$$
$$a+b+c = 9$$

$$[A \quad B] = \begin{bmatrix} 9 & -3 & 1 & 21 \\ 25 & 5 & 1 & 61 \\ 1 & 1 & 1 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -20 & -24 & -164 \\ 0 & -12 & -8 & -60 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 25R_1 \\ R_3 \to R_3 - 9R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 5 & 6 & 41 \\ 0 & 3 & 2 & 15 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 / -4 \\ R_3 \to R_3 / -4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 5 & 6 & 41 \\ 0 & 0 & -8 & -48 \end{bmatrix} R_3 \to 5R_3 - 3R_2$$

$$\Rightarrow \quad -8c = -48 \Rightarrow c = 6$$
$$5b+6c = 41 \Rightarrow b = 1$$
$$a+b+c = 9 \Rightarrow a = 2$$
$$\text{Thus } a = 2, \; b = 1, \; c = 6$$

4. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method).

Solution

$$y = ax^2 + bx + c$$

The curve through the points $(-6,8), (-2,-12)$ and $(3,8)$.

$$\text{So} \quad 36a - 6b + c = 8$$
$$4a - 2b + c = -12$$
$$9a + 3b + c = 8$$

$$\text{Now } [A \quad B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right]$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 36 & -6 & 1 & 8 \\ 9 & 3 & 1 & 8 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 12 & -8 & 116 \\ 0 & 30 & -5 & 140 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 9R_1 \\ R_3 \to 4R_3 - 9R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & -1 & 28 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 / 4 \\ R_3 \to R_3 / 5 \end{matrix}$$

$$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 0 & 3 & -30 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

$$\Rightarrow 3c = -30 \Rightarrow c = -10$$

$$3b - 2c = 29 \Rightarrow b = 3$$

$$4a - 2b + c = -12 \Rightarrow a = 1$$

The equation is $y = x^2 + 3x - 10$.

The point $P(7,60)$ satisfies this equation. Hence the boy will meet his friend.

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

   (i) $x - y + 2z = 2,\ 2x + y + 4z = 7,\ 4x - y + z = 4$

   (ii) $3x + y + z = 2,\ x - 3y + 2z = 1,\ 7x - y + 4z = 5$

   (iii) $2x + 2y + z = 5, x - y + z = 1,\ 3x + y + 2z = 4$

   (iv) $2x - y + z = 2,\ 6x - 3y + 3z = 6,\ 4x - 2y + 2z = 4$

Solution

(i) $x - y + 2z = 2,\ 2x + y + 4z = 7,\ 4x - y + z = 4$

Augmented matrix $[A \quad B] = \begin{bmatrix} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - R_2 \end{matrix}$$

Now $\rho(A) = \rho[A/B] = 3$(= No of unknowns)

$\therefore$ The system has a unique solution.

$$-7z = -7 \Rightarrow z = 1$$

$$y = 1$$

$$x - y + 2z = 2$$

$$\Rightarrow x = 1$$

Therefore, is $x=1,\ y=1,\ z=1$.

(ii) $3x+y+z=2,\ x-3y+2z=1,\ 7x-y+4z=5$

$$[A\quad B] = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\rho(A) = \rho[A/B] = 2 (< \text{No. of unknowns})$$

The system is consistent and has infinitely many solutions.

The system reduces to

$$x-3y+2z = 1$$
$$10y-5z = -1$$
$$\text{Take } z = k \in \mathbb{R}$$
$$y = \frac{1}{10}(5k-1)$$
$$x = \frac{1}{10}(7-5k)$$

$\therefore$ The solution is $x = \frac{1}{10}(7-5k),\ y=\frac{1}{10}(5k-1),\ z=k,\ k\in\mathbb{R}$.

(iii) $2x+2y+z=5, x-y+z=1,\ 3x+y+2z=4$

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$$

$$[A\quad B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\Rightarrow \quad \rho(A) = 2, \ \rho([A\ B]) = 3$

$\therefore$ The system is inconsistent.

(iv)
$$2x - y + z = 2$$
$$6x - 3y + 3z = 6$$
$$4x - 2y + 2z = 4$$

$$[A \quad B] = \begin{bmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$\rho(A) = \rho([A, B]) = 1$ (< no. of unknowns)

The system is consistent and has infinitely many solutions.

The system reduces to a single equation i.e.,

$$2x - y + z = 2$$

Take $y = s, \ z = t, \ s, t \in R$.

$$2x = 2 + s - t$$
$$x = \frac{1}{2}[s - t + 2]$$

$\therefore$ Solution is $x = \frac{1}{2}[s - t + 2], \ y = s, \ z = t, \ s, t \in \mathbb{R}$

2. Find the values of k for which the equations $kx - 2y + z = 1, \ x - 2ky + z = -2, \ x - 2y + kz = 1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

Solution

$$[A \quad B] = \begin{bmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{bmatrix} \begin{matrix} R_1 \leftrightarrow R_3 \\ \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & -2+2k & 1-k^2 & 1-k \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - kR_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & 1-k^2+(1-k) & -2-k \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 + R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & (1-k)(2+k) & -2-k \end{bmatrix}$$

(i) **No solution :** When $k=1$,

$$\rho(A) \neq \rho([AB])$$

$\therefore$ The system has no solution, when $k=1$.

(ii) **Unique solution :** When $k \neq 1$ and $k \neq -2$

$$\rho(A) = \rho([A\ B]) = 3 \text{ (= No. of unknowns).}$$

The system has unique solution when $k \neq 1$ and $k \neq -2$.

(iii) **Infinitely many solutions :** When $k=-2$,

$$\rho(A) = \rho([A,B]) = 2$$

$\therefore$ The system has infinitely many solutions, when $k=-2$

3. Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions

Solution

$$[A\ \ B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{bmatrix} \begin{matrix} \\ R_2 \to 2R_2 - 7R_1 \\ R_3 \to R_3 - R_2 \end{matrix}$$

(i) **For no solution** : When $\lambda=5$, $\mu \neq 9$

$$\rho(A) = 2,\ \rho([A\ B]) = 3.$$

When $\lambda=5, \mu \neq 9$, the system has no solution.

(ii) **For unique solution :** When $\lambda \neq 5$, $\mu \in \mathbb{R}$.

$$\rho(A) = \rho([AB]) = 3$$

When $\lambda \neq 5$, $\mu \in \mathbb{R}$ the system has a unique solution.

(iii) **For infinite solutions :** When $\lambda=5$ and $\mu=9$,

$$\rho(A) = \rho([A\,B]) = 2$$

When $\lambda = 5$ and $\mu = 9$, the system has infinite number of solutions.

EXERCISE 1.7

1. Solve the following system of homogeneous equations.

(i) $3x+2y+7z=0,\ 4x-3y-2z=0,\ 5x+9y+23z=0$

(ii) $2x+3y-z=0,\ x-y-2z=0,\ 3x+y+3z=0$

Solution

(i) $3x+2y+7z=0,\ 4x-3y-2z=0,\ 5x+9y+23z=0$

$$[A\quad B] = \begin{bmatrix} 3 & 2 & 7 & 0 \\ 4 & -3 & -2 & 0 \\ 5 & 9 & 23 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 17 & 34 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to 3R_2 - 4R_1 \\ R_3 \to 3R_3 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 + R_2 \end{matrix}$$

$$\rho(A) < 3$$

The system has infinitely many solutions and reduces to

$$3x+2y+7z = 0$$
$$-17y-34z = 0$$

Take $z = k \in \mathbb{R}$

$$y = -2k$$
$$x = -k$$

Thus $x = -k,\ y = -2k,\ z = k,\ k \in \mathbb{R}$

(ii) $2x+3y-z=0,\ x-y-2z=0,\ 3x+y+3z=0$

$$[A\quad B] = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 0 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 9 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to 5R_3 - 4R_2 \end{matrix}$$

$\Rightarrow \rho(A) = 3$ and hence it has only trivial solution

i.e., $x=0,\ y=0,\ z=0$.

2. Determine the values of λ for which the following system of equations $x+y+3z=0,\ 4x+3y+\lambda z=0,\ 2x+y+2z=0$ has

(i) a unique solution (ii) a non-trivial solution

Solution

(i)
$$[A\quad B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & \lambda & 0 \end{bmatrix} R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda-12 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda-8 & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

For having trivial solution $\lambda \neq 8$ (unique solution).

For having a non-trivial solution $\lambda = 8$.

EXERCISE 1.8

Choose the correct or the most suitable answer from the given four alternatives :

1. If $|adj(adjA)| = |A|^9$, then the order of the square matrix A is

(1) 3 (2) 4 (3) 2 (4) 5

Solution

$$|adj(adjA)| = |A|^{(n-1)^2} = |A|^9 \Rightarrow (n-1)^2 = 9 \Rightarrow n = 4$$

$\therefore$ Order A is 4.

[Option:2]

2. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(1) A (2) B (3) I_3 (4) B^T

Solution

$$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T = (A^{-1}A^T)\left((A^T)^T(A^{-1})^T\right)$$

$$= A^{-1}A^T\left(A(A^{-1})^T\right) = (A^{-1}A)\left(A^T(A^{-1})^T\right) = I$$

[Option:3]

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\dfrac{|adjB|}{C} =$

(1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) 1

Solution

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \therefore |A| = 6-5 = 1$$

$$|C| = |3A| = 3^2|A| = 3^2 \times 1 = 9$$

$$\frac{|adjB|}{|C|} = \frac{|adj(adjA)|}{|C|} = \frac{|A|^{(2-1)^2}}{9|A|} = \frac{1}{9}.$$

[Option:2]

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then $A =$

(1) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

Solution

Find the answer in substitution method.

[Option:3]

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$

(1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

Solution

As per the options are interms of A^{-1}

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= 2A^{-1}.$$

[Option:4]

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

(1) -40 (2) -80 (3) -60 (4) -20

Solution

$$|adj\ A| = 10, \quad |adj\ B| = -8$$

$$|adj\,(AB)| = |adj\ B|\,|adjA| = -80$$

[Option:2]

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1) 15 (2) 12 (3) 14 (4) 11

Solution

$$P = adjA = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & 4 \end{bmatrix}$$

$$|adjA| = \begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & 4 \end{vmatrix}$$

$$|A|^2 = \begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & 4 \end{vmatrix}$$

$$|adj\ A| = |A|^{3-1} \Rightarrow 4^2 = \begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & 4 \end{vmatrix}$$

Simplifying we get $x = 11$. **[Option:4]**

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(1) 0 (2) -2 (3) -3 (4) -1

Solution

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 3(2) - 1(-2) - 1(4+2) = 2$$

$$a_{23} = \frac{(\text{cofactor of } 2)}{|A|} = \frac{-\begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}}{2} = -\frac{2}{2} = -1.$$ **[Option:4]**

9. If A, B and C are invertible matrices of some order, then which one of the following is not true?

(1) $adjA = |A|A^{-1}$ (2) $adj(AB) = (adjA)(adjB)$

(3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Solution

$$adj(AB) = (adj\ B)\ (adj\ A)$$

$\therefore$ Option (2) is incorrect. **[Option:2]**

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

(1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

Solution

$$(AB)^{-1} = B^{-1}A^{-1}$$

Apply substitution method. **[Option:1]**

11. If A^TA^{-1} is symmetric, then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

Solution

$$\left(A^T A^{-1}\right) = \left(A^T A^{-1}\right)^T \Rightarrow \left(A^{-1}\right)^T A = A^T A^{-1} \Rightarrow \left(A^{-1}\right)^T AA = A^T A^{-1} A$$

$$\Rightarrow \left(A^T\right)^{-1} A^2 = A^T$$

$$\Rightarrow A^2 = A^T \cdot A^T = \left(A^T\right)^2$$

[Option:2]

12. If A is non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $\left(A^T\right)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

Solution

$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$$

[Option:4]

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

(1) $\frac{-4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

Solution

If $A^T = A^{-1}$ then $AA^T = I$

$$\therefore \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + \frac{4}{5} \times \frac{3}{5} = 0$$

$$5(3x) + 12 = 0$$

$$x = -\frac{4}{5}$$

[Option:1]

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $\left(\cos^2\theta\right)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

Solution

$$|A| = 1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$$

$$B = A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{\sec^2\frac{\theta}{2}} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} = \left(\cos^2\frac{\theta}{2}\right)A^T$$

[Option:2]

15. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $A(adjA)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then $k=$

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

Solution

$$|A|I = kI$$
$$k = |A|=1$$
$$\therefore k = 1 .$$

[Option:4]

16. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

Solution

$$\lambda A^{-1} = A \Rightarrow \lambda I = A^2 \Rightarrow |\lambda I|=|A|^2 \Rightarrow \lambda^2=|A|^2 \Rightarrow \lambda^2=19^2 \Rightarrow \lambda=19$$

[Option:3]

17. If $adjA=\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}$ and $adjB=\begin{bmatrix}1 & -2\\ -3 & 1\end{bmatrix}$ then $adj(AB)$ is

(1) $\begin{bmatrix}-7 & -1\\ 7 & -9\end{bmatrix}$ (2) $\begin{bmatrix}-6 & 5\\ -2 & -10\end{bmatrix}$ (3) $\begin{bmatrix}-7 & 7\\ -1 & -9\end{bmatrix}$ (4) $\begin{bmatrix}-6 & -2\\ 5 & -10\end{bmatrix}$

Solution

$$adj(AB) = (adjB)(adjA)=\begin{bmatrix}-1 & -2\\ -3 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}=\begin{bmatrix}2-8 & 3+2\\ -6+4 & -9-1\end{bmatrix}$$
$$=\begin{bmatrix}-6 & 5\\ -2 & -10\end{bmatrix}.$$

[Option:2]

18. The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

Solution

R_2 and R_3 are constant multiple of R_1.

$\therefore$ rank is 1

[Option:1]

19. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1=\begin{vmatrix}m & b\\ n & d\end{vmatrix}, \Delta_2=\begin{vmatrix}a & m\\ c & n\end{vmatrix}, \Delta_3=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$, then the values of x and y are respectively,

(1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

Solution

$$a\log x+b\log y = m,\ c\log x+d\log y=n$$

$$\log x = \frac{\begin{vmatrix} m & b \\ n & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{\Delta_1}{\Delta_3}$$

$$x = e^{\frac{\Delta_1}{\Delta_3}}$$

[Option:4]

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$.

(iv) $A(adjA) = (adjA)A = |A| I$.

(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4)(i), (ii) and (iv)

Solution

Since statement (iii) is incorrect. **[Option:4]**

21. If $\rho(A) = \rho([A \mid B])$, then the system $AX = B$ of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

Solution

[Option:2]

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

Solution $\begin{vmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{vmatrix} = 0 \Rightarrow \sin^2\theta - \cos^2\theta = 0 \Rightarrow \sin^2\theta = \cos^2\theta$

$$\theta = \frac{\pi}{4}$$

[Option:4]

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7, \mu \ne -5$ (2) $\lambda = -7, \mu = 5$ (3) $\lambda \ne 7, \mu \ne -5$ (4) $\lambda = 7, \mu = -5$

Solution

To get infinitely many solutions

$$\rho(A) = \rho(A, B) \ne 3$$

$$\Rightarrow \lambda = 7,\ \mu = -5.$$

[Option:4]

24. Let $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and $4B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & x\\ -1 & 1 & 3\end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

Solution

The a_{13} element of $\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}\begin{bmatrix}\cdots & \cdots & -\frac{1}{4}\\ \cdots & \cdots & \frac{x}{4}\\ \cdots & \cdots & \frac{3}{4}\end{bmatrix}$ is 0

$$\frac{-2}{4}-\frac{x}{4}+\frac{3}{4} = 0$$

$$\Rightarrow x = 1.$$

[Option:4]

25. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then $adj(adjA)$ is

(1) $\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ (2) $\begin{bmatrix}6 & -6 & 8\\ 4 & -6 & 8\\ 0 & -2 & 2\end{bmatrix}$ (3) $\begin{bmatrix}-3 & 3 & -4\\ -2 & 3 & -4\\ 0 & 1 & -1\end{bmatrix}$ (4) $\begin{bmatrix}3 & -3 & 4\\ 0 & -1 & 1\\ 2 & -3 & 4\end{bmatrix}$

Solution

$$adj(adjA) = |A|^{n-2} A = |A|^{3-2} A = |A|A$$

$$= 1.A \quad (\because |A|=1)$$

$$= A$$

[Option:1]